CHAPTER 293

A TRANSPORT RATE FORMULA FOR MIXED-SIZE SANDS

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Abstract

Experiments were conducted on the transport of mixed grain size sands due to nonlinear waves, over both rippled and flat beds. The sand mixture was composed of a fine sand with median diameter of 0.2 mm and a coarse sand with median diameter of 0.87 mm. Conditions for the initiation of sheet flow were investigated and transport rates were measured. It was found that the transport rate of fine sand is significantly reduced by the existence of the coarse sand, whereas the coarse sand behaved almost as if there were no other sand present. A transport rate formula for mixed sands was presented and verified by using the measured data.

1. Introduction

Sandy beaches are the focus of many attentions because of their role in preserving the coastal environment, their ability in reducing disasters, and their attractiveness for recreational activities. They are, however, subjected to erosion problems due to the action of natural forces or artificial impacts. As a countermeasure, beach nourishment is one of the most acceptable methods for protecting sandy beaches. The grain size of the sand used for filling the beach is, however, decided by the available sources of material and in many cases does not coincide with the grain size of the native sand. Also, using a sand coarser than the native sand is in general more desirable because it can better stabilize the beach.

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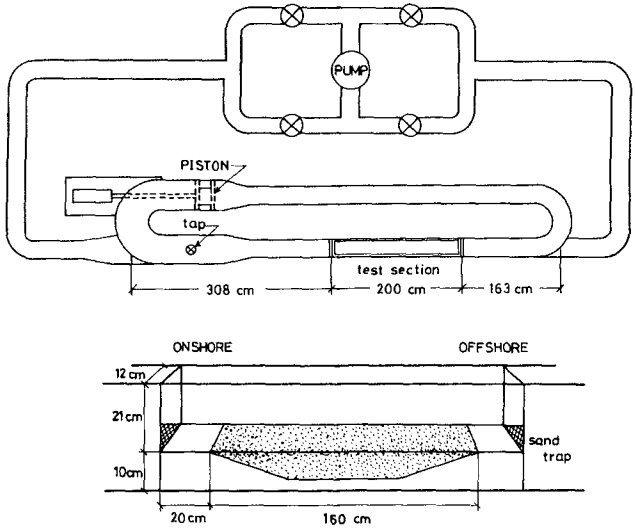

Figure 1: **Oscillatory/steady flow water tunnel and its test section.**

Needless to mention that even the grain size of the native sand itself may be nonuniform. Therefore, estimation of the transport rates of mixed-size sands is of crucial importance in predicting the behavior of both natural and artificial beaches.

In this study experiments were conducted on the transport of mixed grain size sands due to nonlinear waves over rippled and flat beds. The transport rate formula of Dibajnia and Watanabe (1992), which has originally been derived for 0.2 mm sand, was first generalized to cover the transport of granular sediments with other grain sizes and densities. A method was then presented to apply this formula to estimating the transport rate of mixed size sands. Conditions for the initiation of sheet flow were also investigated.

2. Experiments

The present experiments were carried out in a loop-shape oscillatory flow water tunnel at the University of Tokyo. The tunnel and its test section are shown in Fig. 1. Two kinds of sand, fine and coarse, were mixed together. The fine sand had a median grain size of 0.2 mm and the fall velocity of 2.3 cm/s. For the coarse sand these values were 0.87 mm and 8.7 cm/s, respectively. The percentage of the weight of each sand in the total weight of a mixed sample was defined as the percentage of that sand, P_i. Three types of mixed sands with percentages of fine sand, $P_{0.2}$, equal to 25%, 50%, and 75% were prepared. The oscillatory velocity profiles generated in the tunnel were calculated by the first

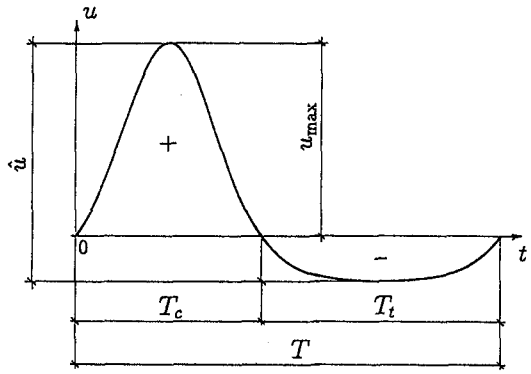

Figure 2: **Typical velocity profile in the experiments.**

Table 1: Experimental conditions for transport rate measurements.

T (s)	3	3	3	3	3	3	3	3	3	5	5	5
d_0 (cm)	23	28	35	38	43	48	50	55	71	62	78	98
$P_{0.2} = 100\%$				⊙	⊙						⊙	
$P_{0.2} = 75\%$	⊙	⊙	⊙	⊙	⊙		⊙	⊙	⊙			
$P_{0.2} = 50\%$	⊙	⊙	⊙	⊙	⊙	⊙	⊙	⊙		⊙	⊙	⊙
$P_{0.2} = 25\%$	⊙	⊙	⊙		⊙		⊙		⊙			
$P_{0.2} = 0\%$				⊙		⊙						

⊙ : Points where transport rate is measured.

order Cnoidal wave theory. Two wave periods, $T = 3$ and 5 s and one nonlinearity index $u_{max}/\hat{u} = 0.7$ (see Fig. 2) were selected. By changing the velocity amplitude, transport of sands over rippled beds as well as under sheet flow conditions were observed and net transport rates were measured. The experimental conditions for which transport rates were measured are summarized in Table 1. In this table, d_0 and T are the excursion length of water particle and the priod of oscillation, respectively. Conditions for initiation of sheet flow were also investigated. The present experimental data have originally been reported in Suzuki *et. al* (1994) and Inui *et. al* (1995).

3. Initiation of Sheet Flow

For a uniform sand, the transport mode usually changes from the bedload transport over an initial flat bed to the suspended load transport over the ripples and, as the velocity amplitude continue to increase, to the sheet flow transport over the reflattened bed. Several experiments were performed to observe the

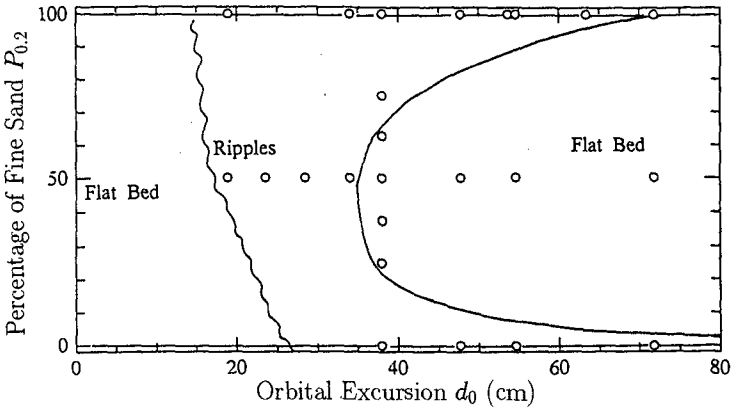

Figure 3: **Occurrence of different transport modes.**

behavior of mixed sand beds. The procedure was to start each experiment from a flat bed and see whether ripples appear or not. The results for $T = 3$ s were summarized in Fig. 3. It is interesting to see that for mixed sands, sheet flow occurs under lower velocities than those required for uniform sands. However, this was the case when the bed has initially been flat. Additional few experiments were performed with an initially rippled bed. As shown in Fig. 4, it was found that a higher velocity than that in the case of uniform sand is required to wash out the ripples from the mixed sand bed. It was thus concluded that the mixture of fine and coarse sand provides a firm structure which is not deformed as easily as uniform sands are.

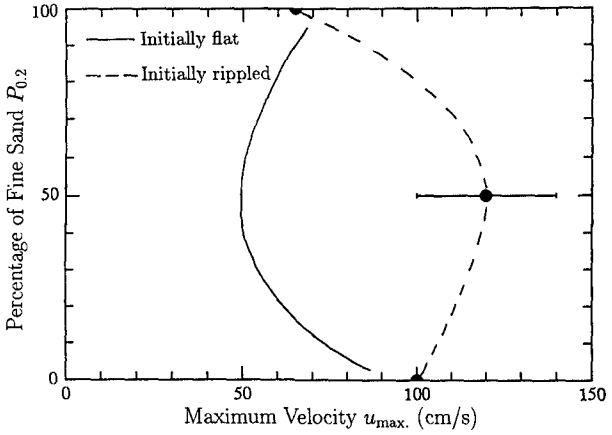

Figure 4: **Dependence of the initiation of sheet flow on the initial bed form.**

4. Generalized Transport Rate Formula

A transport rate formula involving unsteady aspects of the sand transport phenomenon has been presented by Dibajnia and Watanabe (1992) for estimating the sheet flow sand transport rates. The formula has later been extended by them (Dibajnia *et. al*, 1994) to cover suspended load over ripples as well as bed load transports. Their formula, however, is based on the experimental data obtained by using a sand with median grain size of 0.2 mm only, and needs further verification for sands of other sizes before it is applied to the transport of mixed sands.

In their experiments on sheet flow, Dibajnia and Watanabe (1992) noticed that once there was a positive velocity (Fig. 2) large enough to raise up sand particles to such a level that they could not reach the bottom before the negative velocity occured, then these particles tended to be carried to the negative direction. Dibajnia and Watanabe (1992), therefore, defined a dimensionless parameter, Γ, as

$$\Gamma = \frac{u_c T_c(\Omega_c^3 + \Omega_t'^3) - u_t T_t(\Omega_t^3 + \Omega_c'^3)}{(u_c + u_t)T} \tag{1}$$

and assumed the net transport rate to be a function of Γ. For the dimensionless net transport rate, Φ, Dibajnia (1995) proposed

$$\Phi = \frac{q_{\text{net}}(1-\epsilon)}{Wd} = 0.0023 \cdot \text{sign}(\Gamma) \cdot |\Gamma|^{0.5} \tag{2}$$

In the above relations q_{net} is the net volumetric transport rate, ϵ the sediment porosity, d the sediment grain size, W the sediment fall velocity, and u_c and u_t are representative velocity amplitudes for the positive and negative portions of the velocity profile, respectively, as

$$u_c^2 = \frac{2}{T_c}\int_0^{T_c} u^2 dt, \quad u_t^2 = \frac{2}{T_t}\int_{T_c}^{T} u^2 dt \tag{3}$$

In Eq. (1) Ω_c' represents the amount of suspended sand remaining from the positive half cycle, to be carried by the negative velocity. Similarly Ω_t' stands for the amount of sand still in suspension after the negative half cycle, which will be transported to the positive direction. On the other hand, Ω_c represents that amount of sand which is entrained and carried only by the positive velocity; and Ω_t indicates the amount of sand entrained, transported, and settled during the negative half cycle. Values of Ω were estimated in terms of another parameter, ω, equal to the ratio of the time required for a sand particle, suspended during a half cycle, to reach the bottom to the time duration of that half cycle. The reader is referred to the original paper for more details.

Now consider the simple case of sheet flow sand transport under half of a sinusoidal cycle. In this condition, Eq. (1) reduces to

Table 2: Transport rate data of Sawamoto and Yamashita (1986)

sediment type	specific gravity	d (cm)	W (cm/s)	T (s)	u_{max} (cm/s)	Q (cm^2/s)
sand A	1.65	0.02	2.5	3.8	125.3	2.013
sand A	1.65	0.02	2.5	3.8	114.6	1.575
sand A	1.65	0.02	2.5	3.8	101.5	1.180
sand A	1.65	0.02	2.5	3.8	88.7	0.876
sand A	1.65	0.02	2.5	3.8	74.4	0.642
sand B	1.65	0.07	10.5	3.8	125.3	1.308
sand B	1.65	0.07	10.5	3.8	114.6	1.063
sand B	1.65	0.07	10.5	3.8	101.5	0.641
sand B	1.65	0.07	10.5	3.8	88.7	0.500
sand B	1.65	0.07	10.5	3.8	74.4	0.353
sand C	1.65	0.16	22.5	3.8	125.3	1.240
sand C	1.65	0.16	22.5	3.8	114.6	1.001
sand C	1.65	0.16	22.5	3.8	101.5	0.587
sand C	1.65	0.16	22.5	3.8	88.7	0.357
sand C	1.65	0.16	22.5	3.8	74.4	0.271
coal dust	0.58	0.15	7.6	3.8	125.3	4.750
coal dust	0.58	0.15	7.6	3.8	114.6	3.215
coal dust	0.58	0.15	7.6	3.8	101.5	2.405
coal dust	0.58	0.15	7.6	3.8	88.7	1.835
coal dust	0.58	0.15	7.6	3.8	74.4	1.080
coal dust	0.58	0.15	7.6	3.8	56.2	0.441
coal dust	0.58	0.15	7.6	3.8	44.3	0.158

$$\Gamma = \Omega^3 \tag{4}$$

which means that a method to evaluate Ω can best be verified by using averaged transport rates measured under half cycle of sinusoidal oscillations. Such data have been reported by Sawamoto and Yamashita (1986) for sheet flow sand transport, and are shown here in Table 2. The data are for quartz sands of three different sizes (0.2, 0.7, and 1.8 mm) and for coal dust with density of 1.58. According to Dibajnia and Watanabe (1992), Ω may be evaluated as

$$\begin{aligned} \Omega_j &= \omega_j \cdot \frac{WT_j}{d} = \frac{1}{2}\frac{u_j^2}{sgWT_j} \cdot \frac{WT_j}{d} \\ &= \frac{1}{2}\frac{u_j^2}{sgd} = \Theta \quad : \quad \text{Mobility number} \end{aligned} \tag{5}$$

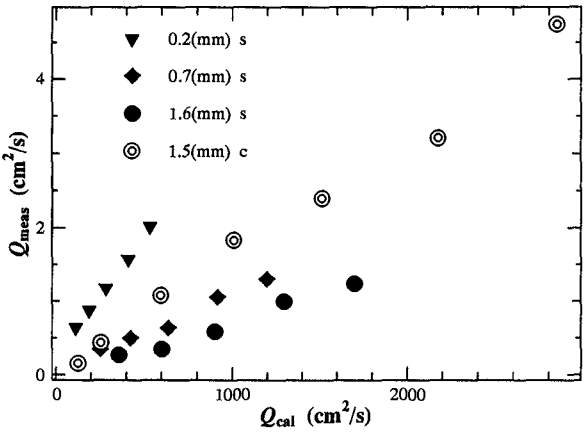

Figure 5: **Comparison of calculated half cycle averaged transport rates with measurements (previous method).**

where $s = (\rho_s - \rho)/\rho$ in which ρ and ρ_s are the densities of water and sediment, respectively, g is the acceleration of gravity, and the subscript j should be replaced by either c or t. Figure 5 shows the comparison between measured and calculated transport rates when Eq. (5) was applied. In this figure $Q = q(1-\epsilon)$ where q is the half cycle averaged transport rate. The proportionality coefficient in Eq. (2) is not considered. It is seen that each sediment type defines a different line of agreement. This indicates that the proportionality coefficient should depend on the grain size and density of sediment whenever the transport rate is assumed to be a function of either Mobility or Shields numbers.

Sawamoto and Yamashita (1986) found that the half cycle averaged sheet flow transport rate is proportional to the third power of the ratio of the friction velocity to the sediment fall velocity. Considering this, a new way to evaluate Ω is as follows.

$$\begin{aligned} \Omega_j &= \omega_j \cdot T_j \sqrt{\frac{sg}{d}} = \frac{1}{2} \frac{u_j^2}{sgWT_j} \cdot T_j \sqrt{\frac{sg}{d}} \\ &= \frac{1}{2} \frac{u_j^2}{W\sqrt{sgd}} \propto \left(\frac{u_j}{W}\right)^2 \end{aligned} \tag{6}$$

Figure 6 shows the results after applying Eq. (6). All the data follow the same line of agreement, indicating that the new method for calculating Ω is valid for granular sediments with different grain sizes as well as different densities. The proportionality constant in Fig. 6 is about 0.0035, and is valid for half cycle averaged transport rates. For the net transport rate, original data of Dibajnia and Watanabe (1992) were used ($d = 0.2$ mm) to obtain the value of the constant.

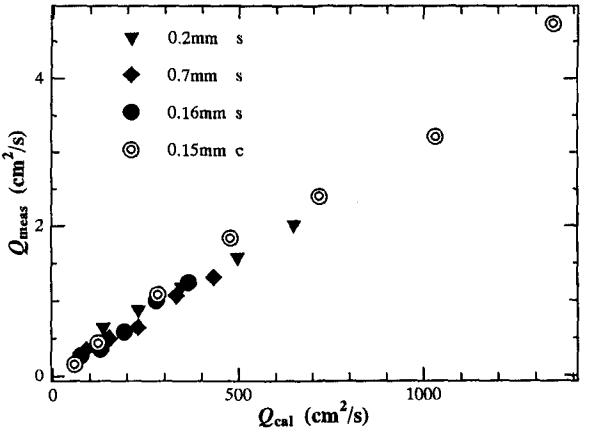

Figure 6: **Comparison of calculated half cycle averaged transport rates with measurements (new method).**

The generalized transport formula was thus obtained (Takasawa *et. al*, 1996) as

$$\Phi = \frac{q_{\rm net}(1-\epsilon)}{Wd} = 0.0015 \cdot \mathrm{sign}(\Gamma) \cdot |\Gamma|^{0.5} \tag{7}$$

$$\begin{cases} \text{if} \quad \omega_j \le \omega_{\rm cr} & \begin{cases} \Omega_j &= \omega_j \cdot T_j \sqrt{\dfrac{sg}{d}} \\ \Omega'_j &= 0 \end{cases} \\ \text{if} \quad \omega_j > \omega_{\rm cr} & \begin{cases} \Omega_j &= \omega_{\rm cr} \cdot T_j \sqrt{\dfrac{sg}{d}} \\ \Omega'_j &= (\omega_j - \omega_{\rm cr}) \cdot T_j \sqrt{\dfrac{sg}{d}} \end{cases} \end{cases} \tag{8}$$

$$\omega_j = \frac{1}{2} \frac{u_j^2}{sgWT_j} \tag{9}$$

where Γ should be obtained as before from Eq. 1. In order to verify the above formulation, net sand transport rate data for grain sizes other than 0.2 mm are required. The only such a data set available to the authors was that reported by Watanabe and Isobe (1990), giving the net sand transport rate over ripples under nonlinear waves for sands of 0.2 and 0.87 mm median diameter. Value of $\omega_{\rm cr}$ in Eq. (8) is equal to 1 for sheet flow. In presence of ripples, $\omega_{\rm cr}$ is equal to 0.03 for 0.2 mm sand and equal to 0.05 for 0.87 mm sand. Threshold velocity should also be considered. Comparison between measured and calculated nondimensional net transport rates for the experiments of Watanabe and Isobe (1990) is shown in Fig. 7. In this figure, the transport rates of the 0.87 mm sand are multiplied by a factor of ten to be more visible. Satisfactory agreement is observed.

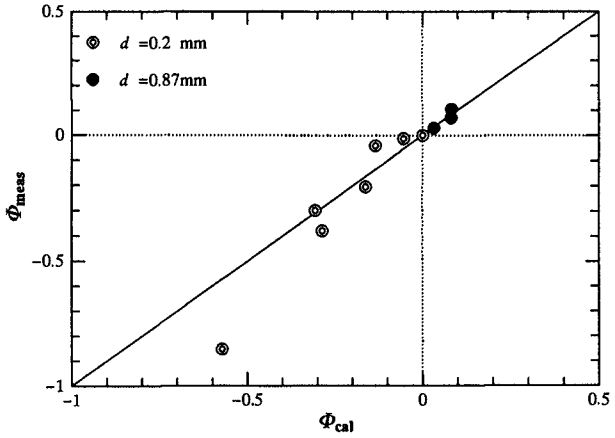

Figure 7: **Comparison of calculated net transport rates over ripples with measurements.**

5. Transport Rate of Mixed Sands

Let us first assume that the transport of each of the sands in a mixture, is not affected by the existence of the other sand. Then the transport rate of each sand could be obtained by using the following formulation.

$$\Phi_i = \frac{M_i}{(\rho_s - \rho) W_i d_i} = 0.0015 \ \mathrm{sign}(\Gamma) \cdot |\Gamma|^{0.5} \cdot P_i \tag{10}$$

$$\begin{cases} \text{if} \quad \omega_j \le \omega_{cr} & \begin{cases} \Omega_j &= \omega_j \cdot T_j \sqrt{\dfrac{sg}{d_i}} \\ \Omega'_j &= 0 \end{cases} \\ \text{if} \quad \omega_j > \omega_{cr} & \begin{cases} \Omega_j &= \omega_{cr} \cdot T_j \sqrt{\dfrac{sg}{d_i}} \\ \Omega'_j &= (\omega_j - \omega_{cr}) \cdot T_j \sqrt{\dfrac{sg}{d_i}} \end{cases} \end{cases} \tag{11}$$

$$\omega_j = \frac{1}{2} \frac{u_j^2}{sgW_iT_j} \tag{12}$$

In the above relations M is the immersed weight of sand corresponding to Φ, subscript i is to address each grain size, and it is assumed that sands have the same density ρ_s. The above formulation was applied to the present experimental data. The results were compared with the measurements as shown in Fig. 8. The dash line in this figure is to separate sheet flow data from those obtained over ripples, and the solid line is the line of perfect agreement. The coarse sand data are again multiplied by a factor of ten for clarity. It is noticed that the fine sand

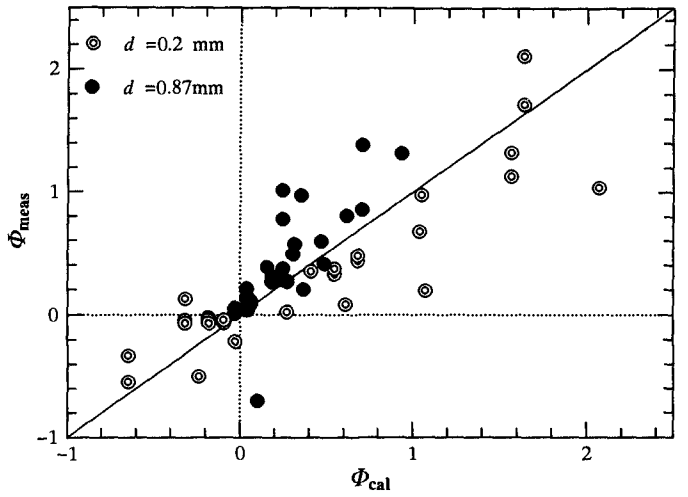

Figure 8: **Calculated net transport rates of mixed sand (Eq. 10) compared with measurements.**

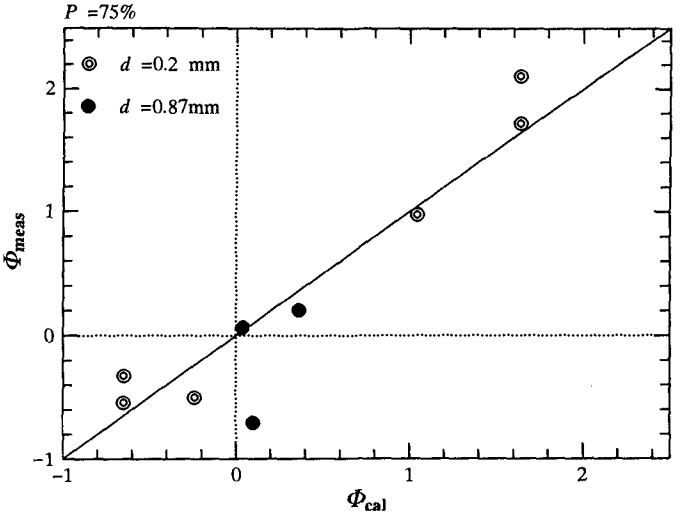

Figure 9: **Calculated net transport rates of mixed sand (Eq. 10) compared with measurements for $P = 0.75\%$.**

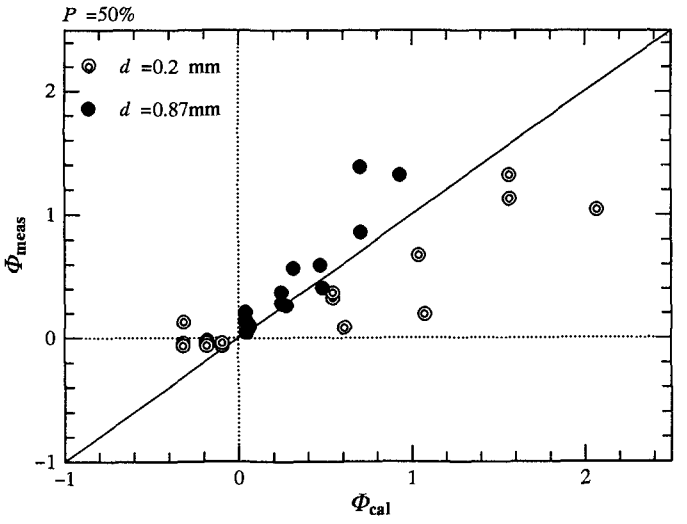

Figure 10: **Calculated net transport rates of mixed sand (Eq. 10) compared with measurements for** $P = 0.50\%$.

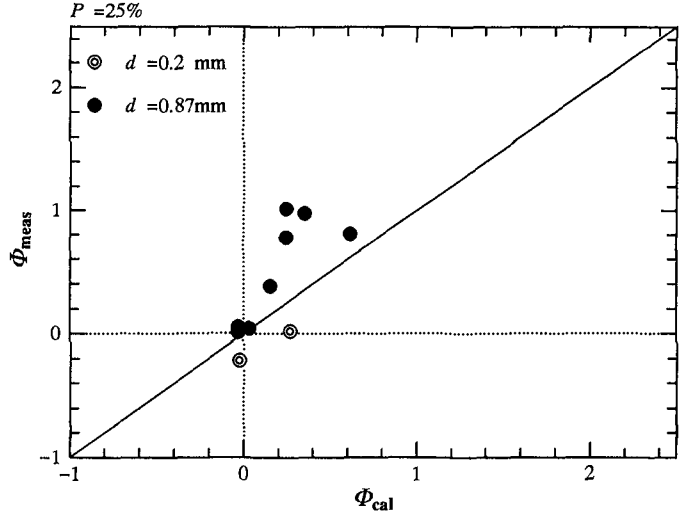

Figure 11: **Calculated net transport rates of mixed sand (Eq. 10) compared with measurements for** $P = 0.25\%$.

data lay below the solid line, whereas the data for coarse sand are located above this line. This simply means that while the transport rate of fine sand has in fact been reduced, the coarse sand has kept its activity. The data were separated according to the percentage of sand and plotted again in Figs. 9, 10, and 11. For $P = 75\%$, *i.e.* when each sand has occupied the major part of the mixture, the data more or less follow the solid line of agreement. Interesting results are observed for $P = 25\%$, where measured transport rates of coarse sand show much larger values than those estimated. Despite the fact that coarse sand amounted only to 25% of the mixture, it has behaved as if it were alone; a clear indication of the armouring phenomenon.

The reduction in the transport rate of fine sand was explained by assuming that a percentage of the total flow energy equal to the percentage of fine sand is consumed to carry the fine sand. In the present formulation, ω is the parameter involving the flow kinetic energy. In general, Eqs. (10) and (12) may be rewritten as

$$\Phi_i = \frac{M_i}{(\rho_s - \rho) W_i d_i} = 0.0015 \ \mathrm{sign}(\Gamma) \cdot |\Gamma|^{0.5} \tag{13}$$

$$\omega_j = \frac{1}{2} \frac{u_j^2}{sgW_i T_j} \cdot P_i^{\frac{1}{\alpha_i}} \tag{14}$$

where α is expected to depend on the ratio of the mean diameter of each sand to that of the other sand. For the present sand mixture it was found that $\alpha_{0.2} = 1$

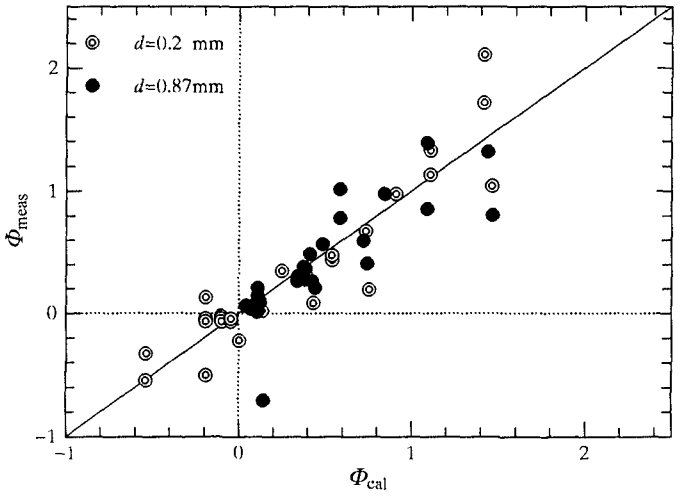

Figure 12: **Comparison of net transport rates of mixed sand, calculated by the presented method, with measurements.**

and $\alpha_{0.87} = 4$ are appropriate. Equations (1), (3), (11), (13), and (14) were thus applied to estimating the net transport rates of the present experiments. Calculated transport rates reasonably agreed with the measurements as shown in Fig. 12.

6. Concluding Remarks

The transport rate formula of Dibajnia and Watanabe (1992) was generalized to cover the transport of uniform granular sediments of any size and density.

Experiments were performed on transport of mixed-size sands. It was shown that fine and coarse sands when mixed together may make a firm structure and thus the bed may not be deformed as easily as in the case of uniform sands. Measurements of transport rate showed that armouring of fine sand by the coarse sand causes a siginificant reduction in the transport rate of the fine sand. But the transport rates of the coarse sand itself were almost unaffected by the presence of the fine sand. A method to apply the above generalized transport formula to estimating the transport rate of mixed sands was presented. Application of this method to beach profile simulation will be presented in a separate paper.

In the present experiments, the oscillatory velocity amplitude had to be limited because of the limited length of the test section (maximum velocity was about 95 cm/s) and higher velocities could not be tried. Therefore, further experiments under sheet flow conditions but with higher velocities are required to confirm whether the armouring function of the coarse sand will persist or not.

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