CHAPTER 246

Asymmetric boundary layer flow above sand ripples under progressive waves

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Abstract

Boundary layer flow above sand ripples under both oscillatory flow and progressive waves is examined by two dimensional numerical simulation. The simulation shows the difference of the flow between under an oscillatory flow and progressive waves. The boundary layer flow under oscillatory flow is symmetric, on the other hand, the flow under progressive waves is asymmetric.The mechanism of the asymmetric flow is explained by the convection effects of progressive waves.

Introduction

To understand the boundary layer flow above sand ripples is important for the mass transport rate and the sediment transport.The boundary layer flow above sand ripples is characterized by vortices which are generated at ripple crests.The generation, development, movement and dissipation of the vortices should be carefully examined for the understanding of the boundary layer flow.

Many experimental and numerical studies for the boundary layer flow above ripples have been carried out. There are several types of numerical simulation such as the vortex method, the turbulent simulation by k-ε and the direct numerical simulation (DNS) . The discrete vortex method is performed by Longuet-Higgins (1981) , Smith and Stansby (1985) and Ismail and Syuto (1985) . Sato, Uehara and Watanabe (1985) simulated the boundary layer flow using the k-ε model, and Penasantana, Watanabe and Isobe (1990) examined the accuracy of the three types of k-ε turbulent model. Both Blondeaux and Vittori (1990) and Hamanaka and Sato (1992) simulate the boundary layer flow by DNS using the spectral method or the pseudo spectral method, and examine the characteristics of the vorticity.

In most of the numerical studies, the main flow,i.e,the flow above boundary layer, is an oscillatory flow. Nadaoka and Yagi (1988) report the difference of the

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boundary layer flow between under an oscillatory flow and progressive waves over a horizontal bed. Takigawa, Kim and Gonotani (1989) , and Toue , Nadaoka and Katsui (1993) also show the differences above ripple beds. According to Nadaoka et al (1988) ,the difference are caused by the vertical convective effects of progressive waves. Takigawa et al (1990) showed the differences of the vortex formation between at left side of a ripple and at right under progressive waves, but the shape of the ripples is asymmetric .

The vertical convective effects also exist above symmetric ripples, and the boundary layer flow can be asymmetric under progressive waves.In this report , the boundary layer flow under progressive wave above symetric ripples is numerically simulated , and then the difference of the flow under an oscillatory flow and progressive waves are shown , and finally the mechanism of the asymmetry is also stated.

Methodology of Numerical Simulation

The method of the numerical simulation used is based on the method of Blondeaux et al (1990) or Hamanaka et al (1992) . The basic equations are followings.

$$\omega_t = \{-\Psi_\eta \omega_\xi + \Psi_\xi \omega_\eta + \nu(\omega_{\xi\xi} + \omega_{\eta\eta})\}/J \tag{1}$$

$$\Psi_{\xi\xi} + \Psi_{\eta\eta} = -J\omega \tag{2}$$

where ω is the vorticity, Ψ is the stream function, t is the time, ξ and η are the boundary fitted coordinate system and J is the Jacobian. Sub-scripts denote the differentiation. ξ, η and J are expressed as below.

$$\begin{aligned} x &= \xi + ae^{-\eta}\cos\xi \\ y &= \eta - ae^{-\eta}\sin\xi \end{aligned} \tag{3}$$

$$J = 1 - 2ae^{-\eta}\cos\xi + a^2 e^{-2\eta} \tag{4}$$

where x and y are the horizontal and vertical coordinate in Cartesian coordinate system and a is the ripple height. All valuables are non-dimensionalized by the ripple wave number, $k^* = 2\pi/l^*$ and σ^* is the angular frequency, where l^* is the ripple length.Furthermore, to generate finer meshes near the bottom, the new valuables, ς, is introduced as below.

$$\eta = \eta_T \frac{e^{b\varsigma} - 1}{e^b - 1} \tag{5}$$

where η_T is the non-dimensional height of the calculation region, and b is the coefficient to adjust the mesh size.

The lateral boundary condition is the periodic condition, and at the top of the boundary,

$$\omega = 0 \tag{6}$$

$$\Psi = u_\infty n_r \sinh\frac{\eta_T}{n_r}\sin(\frac{\xi}{n_r} - t) \qquad \text{for progressive waves} \qquad (7)$$

$$\frac{d\Psi}{d\varsigma} = u_\infty \sin(t) \qquad \text{for an oscillatory flow} \qquad (8)$$

where u_∞ is the horizontal velocity amplitude at the top of the boundary layer and n_r is the number of ripples in one wave length.At the bottom, the following boundary conditions are used ;

$$\Psi = 0 \qquad (9)$$

$$\omega = -J^{-1}\left(\frac{\partial\varsigma}{\partial\eta}\right)^2 \frac{2\Psi(\Delta\varsigma)}{\Delta\varsigma^2} \qquad (10)$$

Toue et al (1992) analyze the boundary layer flow by this method. They apply the method to high-Reynolds number flows (Re=3400) .The calculation can not reach the steady condition, and the time-averaged flow can not be symmetric under an oscillatory flow above a symmetric ripple.Exact two dimensional simulation can express the two dimensional high Reynolds-number flow, but not the three dimensional flow that we usually observe.The method, however, can simulate the boundary layer flow exactly as long as the Reynolds number is small, around 600,and is quite useful to understand the difference of the boundary layer.

In this study the calculation condition is limited to the low Reynolds number, but the flow has vortex sheddings. According to Hamanaka et al (1992) , the phenomena are controlled by the non-dimensional ripple height, a, Reynolds number, Re, and Strohal's number, S_t, being expressed as below.

$$\mathrm{Re} = 2\pi\frac{u_\infty}{\nu} \qquad (11)$$

$$S_t = \frac{2\pi}{u_\infty} \qquad (12)$$

where ν is the non-dimensional viscous coefficient.

Nadaoka et al (1988) showed the ratio of the representative horizontal velocity, u, to wave celerity , C, u/C , is another control parameter for progressive waves. If u/C is large, the vertical convective effects is large. u/C is also expressed as

$$u/C = u_\infty / n_r \qquad (13)$$

in our formulation.The calculation condition is listed in Table-1.

Table-1 Condition of Caculation(p-1 and p-2 are for verification)
O.F=Oscillatory Flow,P.W=Progressive Waves

	a	u_∞	b	ν	η_T	Re	St	Ψ_o	nr	u/C	
case p-1	0.00	5.0	3.0	0.05	5.0	628					O. F
case p-2	0.00	0.08	3.0	0.0023	5.0	221		0.37	10	0.01	P.W
case 1−0	0.50	5.0	3.0	0.05	5.0	628	1.26				O. F
case 1−1	0.50	5.0	3.0	0.05	5.0	628	1.26	23.11	10	0.50	P.W
case 1−2	0.50	5.0	3.0	0.05	5.0	628	1.26	19.04	5	1.00	P.W
case 1−3	0.50	5.0	3.0	0.05	5.0	628	1.26	24.11	15	0.33	P.W
case 2	0.80	8.0	3.0	0.05	5.0	1005	0.79				O. F
case 3	0.70	7.0	3.0	0.05	5.0	880	0.90	$\Psi = \Psi_o \sin\left(\frac{\xi}{n_r} - t\right)$			O. F
case 4	0.60	5.0	3.0	0.05	5.0	628	1.26				O. F
case 5	0.75	3.0	3.0	0.03	5.0	628	2.09				O. F

Verification of Method

To verify the the numerical method, the boundary layer flow above a flat bed are calculated. Fig.1 is the velocity distribution under oscillatory flow. The theoretical value is the 1st order solution. The agreement is quite well. Fig.2 shows the distribution of the vorticity under progressive waves. Fig.2 (a) is the analytical value

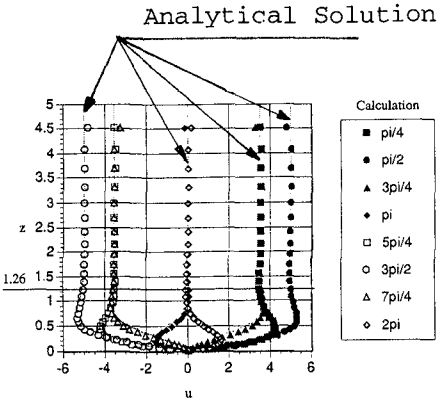


Fig.1 Comparison of Vertical Velocity Profile in Laminar Oscillatory Boundary Layer over Flat Bed between Numerical Simulation and Analytical solution.

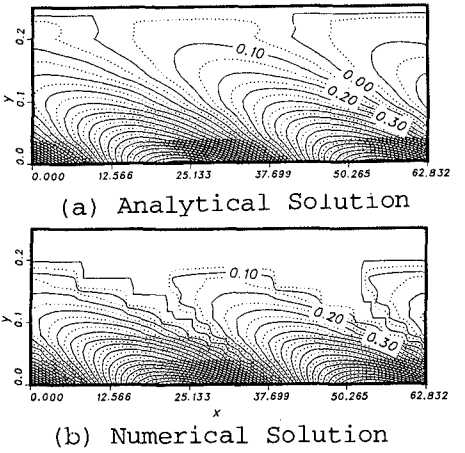


Fig.2 Comparison of Vorticity Profile in Laminar Wave Boundary Layer over Flat Bed between Numerical Simulation and Analytical solution.

, and Fig.2 (b) is the numerical one . Both have quite good agreements.

Time Averaged Vorticity and Stream Function

The time averaged vorticity is shown in Fig.3. A pair of the symmetric vorticity cell can be seen in the case of oscillatory flow, i.e. case1-0, case4 and case5. On the other hand, the profile of the vorticity is asymmetric in case1-1, case1-2 and case1-3 which are the cases of progressive waves.When the flow become high Reynolds number, the calculation could not obtain steady state. Those cases are case2 and case3, and they are not analyzed further.

Asymmetry of the time averaged vorticity means the existence of the residual components of the circulation in a ripple, and the residual components cause steady currents. Fig.4 is the time averaged stream function in case 1-1.The profile of the stream function above all ripples should have been the same for each ripple, but since the calculation has not reached the complete steady condition, there are small differences among the stream functions.In spite of the minor differences, unidirectional and uniform steady currents can be seen near the top boundary.

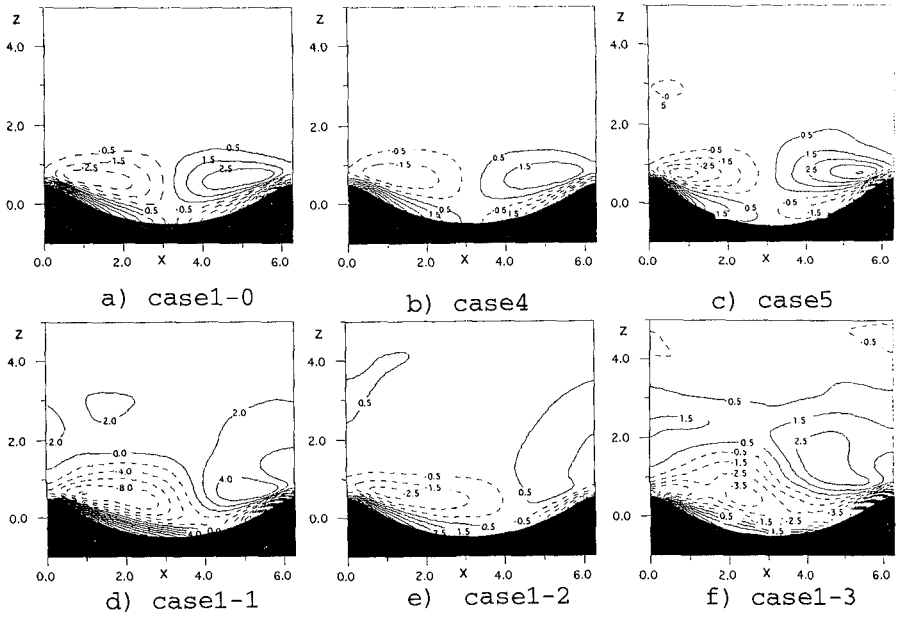


Fig.3 Time Averaged Vorticity(a) case1-0 b) case4 c) case5 d) case1-1 e)case1-2 f) case 1-3)

General Feature of Vorticity Development under Progressive Waves

Fig.5, Fig.6 and Fig.7 are the instant vorticity and the stream function for case1-1,case1-2 and case1-3 respectively in one wave length for $t = \pi/4$. In the figures, waves propagates from the left to the right, and when t=0, the horizontal

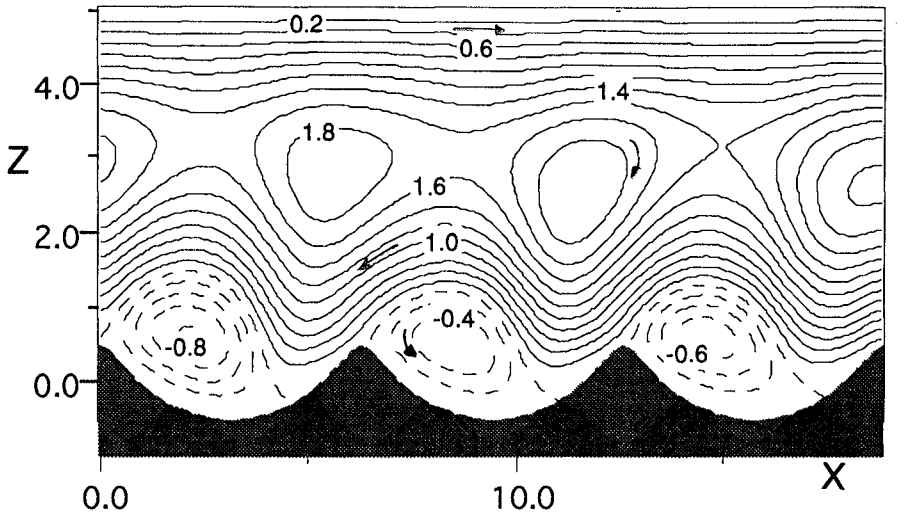


Fig.4 Time Averaged Stream Function for case1-1

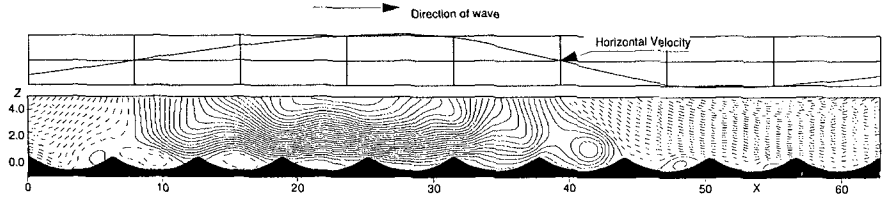
velocity and the vertical velocity on the top boundary are 0 and maximum at the center respectively. Since the calculation has reached almost a steady condition after the third cycle , the figure can be regarded as the time evolution of both vorticity and stream function at every $\pi/5$for case1-1,$2\pi/5$ for case1-2 and $2\pi/15$ for case1-3 when they are looked from the right to the left. It is clear that the dynamics of the vorticity are quite different between the positive vorticity and negative one.

The positive vorticity, denoted by A, is generated at Ripple 9 in Fig.5 where the horizontal velocity is negative.The vorticity A is convected upward by the vertical velocity of waves, and finally disperse during a half period of the waves. On the contrary, the negative vorticity, denoted by B, is generated at Ripple 5 in Fig.5 where the horizontal velocity is positive.The vorticity B is pressed down to the bottom also by the vertical velocity of waves. The another positive vorticity can be found at Ripple 3 on the bottom. That is generated by the velocity of the vortex which consists of vorticity B.The vorticity B and the positive vorticity on the bottom interact and then diminished each other. In other word, the vorticity B is diminished by the bottom friction. The positive vorticity convected upward is at far from the bottom, thus, they can not be affected by the bottom friction so much.

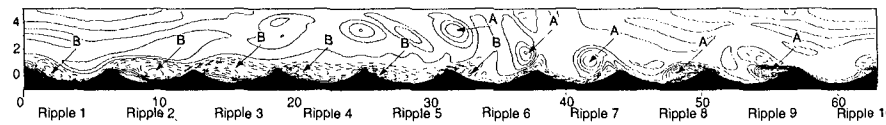
The differences mentioned above is related to the phase of the vorticity developing and the direction of the velocity of the main flow. When the positive vorticity is developing, the horizontal velocity is negative and the vertical velocity being directed upward. On other hand, the negative vorticity is developing, the horizontal velocity is positive and the vertical velocity being directed downward.

The phase when the vorticity stop developing is not clear, but it would be the phase during the velocity decreasing in its amplitude.These processes seem to occur at the phase between ripple 2 and 3 for the positive velocity, and 7 to a8 for the negative velocity.

The similar phenomena can be seen in case 1-2 and 1-3 in Fig.6 and 7. In case 1-2, the convection is stronger than case 1-1 and case 1-3. Thus, the positive vorticity A disperse and the negative vorticity B dissipate more quickly. In case 1-3, the

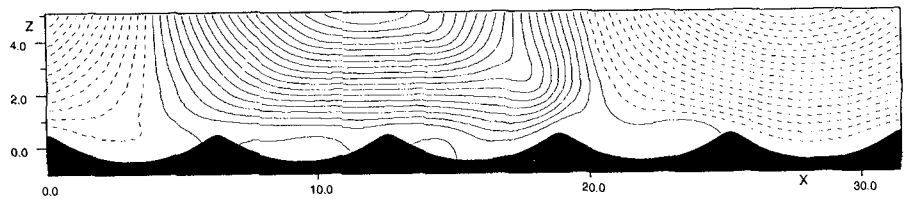


a) Stream Function

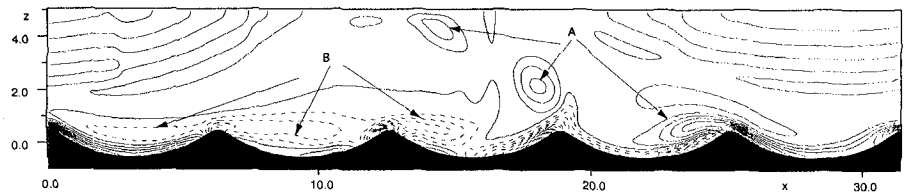


b) Vorticity

Fig.5 Instantaneous Stream Function and Vorticity in One Wave Length at t= π /4 for case1-1 (a) Stream Function $\Delta \psi$ =1.0 b) Vorticity $\Delta \omega$ =2.0 , Top Figure shows the phase of the horizontal velocity on the top boundary)



a) Stream Function



b) Vorticity

Fig.6 Instantaneous Stream Function and Vorticity in One Wave Length at t= π /4 for case1-2 (a) Stream Function $\Delta \psi$ =1.0 b) Vorticity $\Delta \omega$ =2.0 , Top Figure shows the phase of the horizontal velocity on the top boundary)

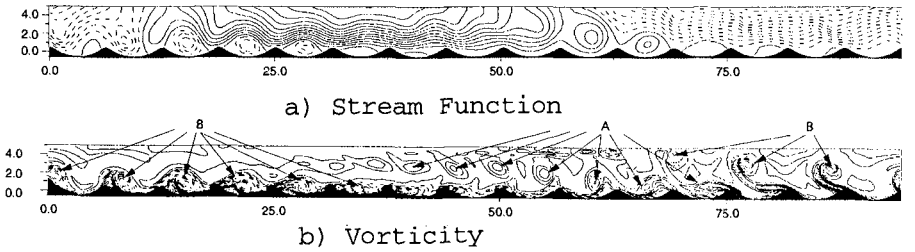
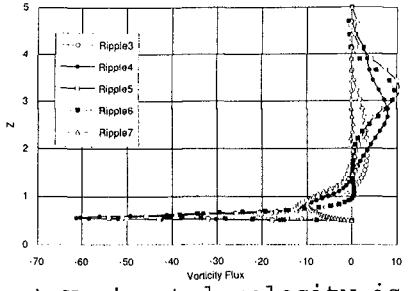


Fig.7 Instantaneous Stream Function and Vorticity in One Wave Length at t= π /4 for case1-3 (a) Stream Function $\Delta\ \psi$ =1.0 b) Vorticity $\Delta\ \omega$ =2.0 , Top Figure shows the phase of the horizontal velocity on the top boundary)

convection effects is week, thus, the negative vorticity B still remain when the positive velocity change the direction. The survived negative vorticity B and the part of the generating positive vorticity interact, and then they goes upward by their own self-propelling velocity.

Vorticity Flux over Ripple Crest

To examine this more quantitatively, the vorticity flux, $u\omega$, in case1-1 that goes across a ripple crest is examined next.In Fig.8, the vertical profiles of $u\omega$ at the left crests of ripples are shown. Fig.8 (a) is the vorticity flux when the velocity is positive, and (b) is for negative velocity. From Fig.8 (a) , when the horizontal velocity is positive, the negative vorticity enters near the bottom and the positive vorticity enters near the top boundary. On the contrary, in the case of the negative velocity, the positive vorticity goes into near the bottom. By comparing the strength of the vorticity flux between the positive and the negative, it is clear that $u\omega$ which goes across a ripple crest is also quite asymetric.



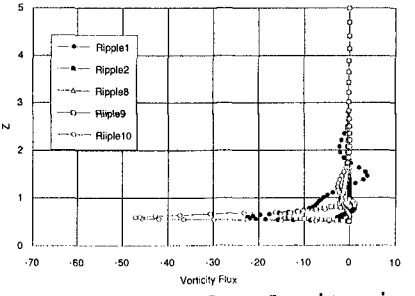


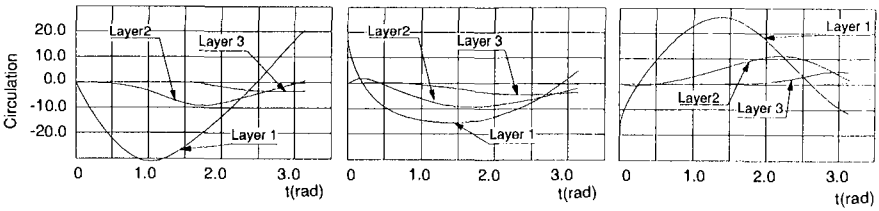
Fig.8 Vertical Distribution of Vorticity Flux at Ripple Crests (a) The horizontal velocity is positive, b) The horizontal velocity is negative.)

Circulation in A Ripple

The time evolution of the circulation that a ripple obtain for case1-1 is shown in Fig.9 from t=0 to t=π of the first wave cycle. The time evolution of the circulation for case 1-0 is also shown in Fig.9. The reason why the results of the first cycle is examined is to eliminate the effect of the vortex that is formed at a half cycle before and the effects of the steady currents. The circulation are divided into four layers which are shown in Fig.10 to understand how the vorticity which is generated at the bottom transfer to upword. The circulation of Fig.9 (b) is the value for Ripple 6 in Fig.5, and that of Fig.9 (c) is the value for Ripple 1. Fig.9 (a) is the circulation under an oscillatory flow, (b) and (c) are the circulation under progressing waves. In Fig.9 (a) and (b) , the horizontal velocity is positive, and the horizontal velocity is negative in Fig.9 (c) .

In Fig.9 (b) and (c) , the circulation in the lowest layer (Layer 1) has some value from the beginning , because even though the main flow is exactly zero at t=0 at the crest, the velocity is not zero above the other part of the ripple.The circulation in Layer 1 develop first since the source of the vorticity exists on the bottom. The value of circulation monotonously decreases in (b) and that monotonously increase in (c) .The value has a peak before $\pi/2$, since the opposite sign of vorticity is generated on the bottom beneath a vortex. This opposite sign of vorticity is caused by the velocity of the vortex that has oppsite sign to the main flow. Before the peak of the circulation in Layer 1, the circulation in Layer 2 and Layer 3 start to develop, because the vorticity near the bottom disperses and the vortex at a ripple begins to form. The characteristics of the circulation mentioned above are common in Fig.9 (a) to (c) .

The circulations of (b) and (c) in Layer 1 are the same up to around t = 0.5, though the sign is different. After this stage, the differences between (b) and (c) can be seen. The circulation of (c) develops more than that of (b) , because the negative vorticity remains near the bottom, thus, the positive vorticity beneath the negative vorticity develops. In Fig.9 (c) , the positive vorticity is convected upward



a) Under Oscillatory Flow(case1-0)

b) Under Progressive Waves; Horizontal Velocity is positive(case 1-1)

c) Under Progressive Waves; Horizontal Velocity is negative(case 1-1)

Fig.9 Evolution of Circulation in a Ripple (a) under Oscillatory Flow b)under Progressive Waves and Positive Horizontal Velocity c) under Progressive Waves and Negative Horizontal Velocity)

, thus, the negative vorticity on the bottom beneath the vortex is small. Also comparing the circulation in Layer 2 and 3 between (b) and (c) , the circulation in (b) has a peak earlier than (c) has a peak, and the peak vaule of (b) is smaller than that of (c) . This means that the positive vorticity goes upward more easily than the negative vorticity as mentined before.

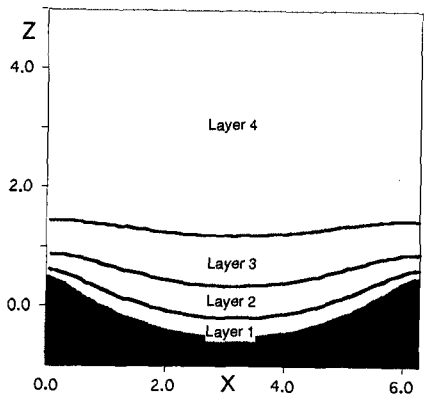


Fig.10 Explanation of Layers

Difference of Phase in Separation

From the difference of the time development of the circulation among Fig.9 (a) , (b) and (c) , there might be a phase difference in vorticity separation . Fig.11 is the vorticity distribution at $t = \pi/4$ of Ripple 5 and 1 in case1-1 in the first cycle.When the velocity is negative, the vorticity forms along the bottom topography, but the vorticity starts to separate when the velocity is positive.The separation time (phase) of the vorticity is earliest when the velocity is positive under progressive waves (see the right side of the bottom figure of Fig.11) . The separation time under oscillatory flow is next and that is latest when the velocity is negative under progressive waves.This is caused by the difference of the attack angle of the velocity against the ripple crest.In the vortex evolution, i.e.,t=0 or $\pi/2$, the positive velocity of the progressive waves attack with a steep angle, and the angle becomes milder in oscillatory flow ,and much milder for negative velocity. This is also effects of the vertical velocity.

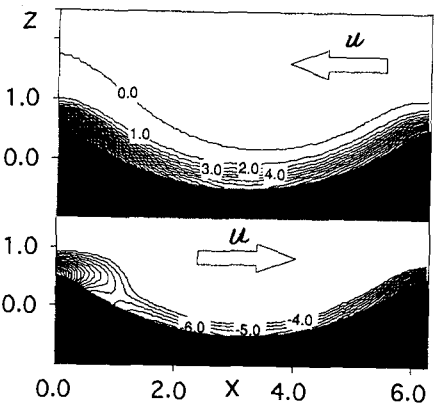


Fig.11 Instantaneous Vorticity Distribution at t= π /4 of The First Wave Cycle in case1-1(Top figure is for Ripple 5 where the horizontal velocity is negative and the bottom is for Ripple 1 where the horizontal velocity is positive.

Concluding Remarks

Boundary layer flow above ripples under both an oscillatory flow and progressive waves is examined by two dimensional numerical simulation. The simulation shows the differences between the flow under an oscillatory flow and progressive waves. The boundary layer flow under an oscillatory flow is symmetric, on the other hand, the flow under progressive waves is asymmetric. The mechanism of the asymmetric flow is explained by the vertical convection of waves. Furthermore, there are the differences of the phase of the vortex separation.

The Reynolds number of the flow is small and the relation of main flow and the dimension of ripples are neglected in this simulation because this calculation is to examine the mechanism of the asymmetric flow. It is necessary to calculate the high Reynolds number flow by including the appropriate turbulent model for further quantitative discussion.

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