WAVE ENERGY DISSIPATION IN KELP VEGETATION

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ABSTRACT

A laboratory experiment was carried out to investigate wave energy dissipation in a coastal kelp field of *Laminaria hyperborea* with a purpose of finding the damping rate. Parameters measured were wave heights at 8 locations along the direction of wave propagation, forces on artificial kelp plants, and velocities in between the plants in 4 different water depths. It was found that in 4, 6, 8 and 10 meter water depths, the damping coefficient was 0.0094, 0.0032, 0.0014 and 0.0005m^{-1} respectively. Results show that the damping rate varies with population density, size of plant, water [1]depth and wave period.

1. INTRODUCTION

Through a variety of processes, waves lose energy. Among the commonly known processes are wave breaking, bottom friction, reflection and interaction with porous beds and soft muds. Waves propagating through vegetation also lose energy due the work done on the vegetation.

Following Ippen (1966), the wave amplitude attenuation can be evaluated through the steady-state conservation of wave energy flux equation:

$$\nabla \cdot \left(E \vec{C}_g \right) = -E_D \quad \text{.......} (1.1)$$

where $E = \frac{1}{2}\rho g a^2$ is the wave energy density, ρ is the density of water, C_g is the group velocity and E_D is the energy dissipation rate. The local wave amplitude a is found to decay exponentially (see for example, Asano et al., 1992) as

$$a(x) = a_o e^{-k_i x} \quad \text{.......} (1.2)$$

where k_i is the damping rate (also known as the damping coefficient) of the wave height with distance and $a = a_o$ at $x = 0$.

In a study on the interaction between ocean waves and a kelp farm, Dalrymple et al. (1982) arrived at a wave height attenuation formula of the form:

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$$\frac{a(x)}{a_o}=\left(\frac{1}{1+\alpha_d x}\right) \quad \text{.......(1.3)}$$

where

$$\alpha_d=\frac{2C_D}{3\pi}\left(\frac{d_k}{b}\right)\left(\frac{a_o}{b}\right)\left[\sinh^3 kd+3\sinh kd\right]\left[\frac{4k}{3\sinh kD(\sinh 2kD+2kD)}\right] \quad \text{.......(1.4)}$$

in which only the real part of the wave number k is used. C_D is the drag coefficient assumed constant over depth, d is the height of plant above the bottom, d_k is the plant diameter, and b is the spacing between plants. This approximation, however, takes plants as rigid cylinders; an assumption which results in assigning different values to the drag coefficient to cover the ignorance of the plant motion. Dalrymple et al. (1982) applied Eq.(1.4) for waves propagating through *Macrocystis pyrifera* kelp forest using known values of $C_D = 1.0$, $d_k = 0.3$ meters, $b = 1.5$ meters and $D = 15$ meters. They found that incident wave heights of 6.1 meters with 10-20 seconds periods could be reduced by 50% over a distance of 800 meters. This is equivalent to $\alpha_d = 0.0013m^{-1}$ and applying the exponential law of attenuation, $k_i = 0.0009\ m^{-1}$.

Recently, Kobayashi et al. (1991) developed an analytical model, hereafter known as the Kobayashi model, to describe a 2D problem of small amplitude waves propagating over submerged or subaerial vegetation. They assumed the effect of the vegetation to be expressible in terms of the drag resistance against the fluid motion. The vertical component of the drag resistance and the proximity effects of the surrounding strips on C_D were neglected. Although an analytical solution was obtained, the calibrated drag coefficient C_D varied widely due to the fact that the swaying motion of an individual vegetation stand had been neglected.

Later, Asano et al. (1992) improved the Kobayashi model by including the interaction between the wave and the vegetation motion. Whereas the Kobayashi gave the calibrated values of C_D to the order of 0.1, the model of Asano et al. produced C_D of the order unity and the vertical force was neglected while the horizontal drag force was assumed to be dominant and linearized as

$$F_x=\rho D_k u_2 \quad \text{.......(1.5)}$$

where D_k is a linear force coefficient determined by the least squares method, u_2 is the horizontal particle velocity in the vegetation zone. Solution of the linear model gives the dispersion equation as

$$\omega^2=gk\frac{k\tanh kh+\alpha_k\tanh\alpha_k d}{k+\alpha_k\tanh\alpha_k d\tanh kh} \quad \text{.......(1.6)}$$

in which ω is the angular frequency, d is the vegetation height, h is the water depth above vegetation and

$$\alpha_k^2=\frac{\omega k^2}{\omega+iD_k} \quad \text{.......(1.7)}$$

In the case of weak damping, $\omega >> D_k$, separation of the real and imaginary parts of the dispersion equation yields the following

$$\omega^2 = gk_r \tanh k_r(h+d) \quad (1.8)$$

which implies that the presence of vegetation does not change the real wave number k_r as long as $\varepsilon_k \ll 1$. The damping rate is then approximated by

$$k_i = \varepsilon_k k_r \frac{2k_r d + \sinh 2k_r d}{2k_r(h+d) + \sinh 2k_r(h+d)} \quad (1.9)$$

where $\varepsilon_k = \frac{D_k}{2\omega}$ ...(1.10)

On the other hand, applying the energy conservation equation, Eq.(1.1), and invoking Eq.(1.2), the Kobayashi model gives the damping rate as

$$k_i = \frac{D_k}{C_g}\left(\frac{2k_r d + \sinh 2k_r d}{2\sinh 2k_r(h+d)}\right) \quad (1.11)$$

A more recent theoretical model based on limited experimental data is that of Wang and Tørum (1994) in which the dispersion equation is given by

$$\omega^2 = \frac{\tanh kh - \frac{i}{\alpha f_x}\tanh k_s h_s}{1 - \frac{i}{\alpha f_x}\tanh k_s h_s \tan kh} \quad (1.12)$$

where $\alpha = \sqrt{\left|\frac{f_z}{f_x}\right|}$ is the force ratio ...(1.13)

and $k_s = \alpha k$; f_x and f_z are linearized horizontal and vertical force coefficients respectively, h_s = height of plant. The complex wave number is solved by iteration for known parameters and an assumed value of $\alpha = 0.8$.

Results from this model show that the damping coefficient increases with increasing wave period until it reaches an asymptotic values of just below *0.0015*m^{-1}, *0.0020*m^{-1} and *0.0025* m^{-1}, for densities of 8, 12 and 16 *plants/*m^2.

In this paper three different ways of finding the damping rate are used and compared. The first way is to get an analytical expression of the energy dissipation rate E_D using the linearized dissipative force and then apply Eq. (1.1). The second way, is to get a numerical value of the dissipation rate from time series of the horizontal velocities measured in between the plants and the corresponding horizontal force on the plants and apply Eq.(1.1) again. The third way, is to solve the complex dispersion equation for the complex wave number k by iteration as propsed by Wang and Tørum, (1994).

2. THE DAMPING RATE

2.1 Energy dissipation and the damping rate

Energy dissipation considered to be real and positive takes place mainly in the vegetation zone and partly in the boundary layer at the interface and near the solid bottom. For a given volume of the vegetation zone, energy dissipation during a time period T is given by (e.g Madsen, 1974)

$$E_D = \frac{1}{T}\int_t^{t+T} dt \int_V \vec{F}\cdot\vec{U} dV \quad \text{.......... (2.1)}$$

where $\vec{F} = \vec{F}(x,y,z,t) = (F_x, 0, F_z)$ is the dissipative force vector in the vegetation zone and $\vec{U} = (u_2, 0, w_2)$ is the complex velocity vector for the 2D case. Defining the force and velocity vectors as (**i**, **j**, **k**) in the (x, y, z) directions as

$$\left.\begin{aligned} \mathbf{F} &= F_x\,\mathbf{i} + F_z\,\mathbf{k} \\ \mathbf{U} &= u\,\mathbf{i} + w\,\mathbf{k} \end{aligned}\right\} \quad \text{.......... (2.2)}$$

and substituting into Eq.(2.1) we get

$$\left.\begin{aligned} E_D &= \frac{1}{T}\int_t^{t+T} dt \int_{-(h+d)}^{-(h+l_k)} (F_x.u_2 + F_z.w_2)dz \\ &= \frac{1}{T}\int_t^{t+T} dt \int_{-(h+d)}^{-(h+l_k)} F_x.u_2(1+\alpha^2(\frac{w_2}{u_2}))dz \end{aligned}\right\} \quad \text{.......... (2.3)}$$

The second term under the integral in Eq.(2.3) is found to be at least of the order α^2 and if we assume that $w_2/u_2 = O(\alpha^2) << 1$, which is generally true in coastal waters (Mei, 1982), the time averaged energy dissipation then becomes

$$E_D = \frac{1}{T}\int_t^{t+T} dt \int_{-(h+d)}^{-(h+l_k)} F_x.u_2 dz + O(\alpha^4) \quad \text{.......... (2.4)}$$

The dissipative force is linearized and expressed as

$$F_x = \rho f_{dx} u_2 \quad \text{.......... (2.5)}$$

where f_{dx} is a linear drag force coefficient (Dubi, 1995) given as

$$f_{dx} = \frac{4n_k\alpha_r F_\lambda k_s|1-A_m|\cosh^2 k_s h_f}{\rho(2k_s d + \sinh(2k_s d))} \quad \text{.......... (2.6)}$$

where n_k is the number of plants per unit horizontal area, α_r is a force reduction factor accounting for group shielding and is found experimentally as a fitting coefficient equal to 0.08. F_λ is an empirical force coefficient evaluated at the canopy level, i.e. at

$z = -(h + l_k)$ and $h_f = (d-l_k)$ where l_k is half length of the kelp frond. A_m is the velocity amplification factor obtained from the solution for the vegetation motion. The complex horizontal velocity in the vegetation zone is given in Dubi and Tørum (1994) as

$$u_2 = \frac{iCk}{\omega f_x} \cosh(k_s(h+d+z)) \exp[i(kx-\omega t)] \quad \text{.......... 2.7)}$$

where

$$C = \frac{ga}{\cosh(k_s d)\cosh(kh)\left[1-\dfrac{i}{\alpha f_x}\tanh(k_s d)\tanh(kh)\right]} \quad \text{..........(2.8)}$$

Substituting Eq.(2.7) into Eq.(2.4) we get the linearized form of energy dissipation, after time averaging, as

$$E_D = \frac{\rho f_{dx}}{2}\int_{-(h+d)}^{-(h+l_k)} |u_2|^2 dz = \frac{\rho f_{dx}|C|^2|k|^2}{2\omega^2|f_x|^2}\frac{\sinh(2k_s h_f)+2k_s h_f}{4k_s} \quad \text{..........(2.9)}$$

in which only the real part is considered. Substituting Eq.(2.9) into Eq.(1.1) for the 2D case we have

$$\frac{\partial}{\partial x}(EC_g) = \frac{1}{2}\rho g C_g \frac{\partial a^2}{\partial x} = -Ba^2 \quad \text{..........(2.10)}$$

in which C_g is assumed to be constant due to constant water depth and

$$B = \frac{\rho g^2 |\alpha k|^2 f_{dx}}{2\omega^2 F^2 |\alpha fx - iG|^2}\left(\frac{\sinh(2k_s h_f + 2k_s h_f}{4k_s}\right) \quad \text{..........(2.11)}$$

where $F = \cosh k_s d \cosh kh$ and $G = \tanh k_s d \tanh kh$. The solution of Eq.(2.10) is

$$\frac{a}{a_0} = e^{-\frac{B'}{2}x} \quad \text{..........(2.12)}$$

where $B' = B/½\rho g C_g$. Equating Eq.(2.12) and Eq.(1.2) we find the damping rate to be

$$k_i = \frac{g|\alpha k|^2 f_{dx}}{2\omega^2 F^2 |\alpha f_x - iG|^2 C_g}\left(\frac{\sinh(2k_s h_f + 2k_s h_f}{4k_s}\right) \quad \text{..........(2.13)}$$

in which we shall adopt $|k| \cong k_r$ derived from the solution of the dispersion equation.

2.2 Solution of the complex dispersion equation and the damping rate.

The dispersion equation contains complex unknown variables k and f_x. The angular frequency ω is real and assumed to be known. The solution for the complex dispersion equation is found by iteration when Eq.(1.12) is rewritten as:

$$F(k,f_x) = \omega^2(1 - \frac{i}{\alpha f_x}\tanh(k_s d)\tanh(kh)) - gk(\tanh(kh) - \frac{i}{\alpha f_x}\tanh(k_s d)) = 0 \quad \text{.....(2.14)}$$

The iteration scheme is carried out as follows:

(i) As a starting approximation we apply Eq.(2.14) for very small damping force coefficient, that is, for $f_x \to -i$, leading to an approximate solution for the wave number k_o for given α, ω, h and d. Then $k_1^{(0)}$ = real(k_o)

(ii) With an initial guess $k_2^{(o)}$ for the imaginary part of k, we solve Eq.(2.14) with $k^{(o)}=k_1^{(o)}+ik_2^{(o)}$ as initial value input . The solution gives $k_r = Re(k)$ and $k_i = Im(k)$. The convergence criterion for the case is

$$\left|\frac{k^{(n)} - k^{(n-1)}}{k^{(n)}}\right| \le e(k) \quad \text{.....(2.15)}$$

with $e(k)$ being an arbitrary small number which in our case is set equal to 10^{-5}. Results of the damping coefficient computed from the dispersion equation shall be compared with those obtained from the energy dissipation equation.

2.3 The damping rate from measured force and horizontal particle velocity

The damping rate can also be evaluated by making use of the measured force and horizontal velocity time series. Integrating the product of these two quantities and time averaging gives the energy dissipated per unit time as

$$E_D = \frac{1}{AT}\int_0^T F_m U_m \cos^2 \omega t \quad dt = \frac{1}{2}\frac{F_m U_m}{A} \quad \text{.....(2.16)}$$

where F_m is the total measured force amplitude and U_m is the measured horizontal velocity amplitude. To get energy dissipation per unit horizontal surface area we have divided by $A = 8\ m^2$, which is the horizontal surface area of the shear plate used in the experiment. Inserting Eq.(2.16) into Eq. (1.1) we find that

$$k_i = \frac{1}{A}\frac{F_m U_m}{\rho g C_g a_o^2} \quad \text{.....(2.17)}$$

in which ρ, g , C_g and maintain the same definition as in previous formulae, converted to full scale. For irregular waves we shall take significant force and significant velocity.

3. LABORATORY EXPERIMENT

The experiment was carried out in a 33 m long, 1 m wide and 1.6 m high wave tank as shown in Figure 1. The width of the channel was partitioned to give a width of 0.5m. Five thousand models (scale 1:10) of *Laminaria hyperborea* plants were fixed on the bottom over a span of 9.3 meters. This represented a density of about 0.12 plants per horizontal square centimeter in the laboratory or 12 plants/square meter in the field. The setup consisted of eight capacitance wave gauges to measure surface elevations, one shear plate which was fixed flush with the bottom, to measure the horizontal force and a mini-current meter which was inserted in between the plants 4 centimeters above the shear plate to measure horizontal particle velocities. One of the wave gauges was placed above the shear plate. The first wave gauge, taken to be located at $x = 0$ and the last taken to be located at $x = 7$m, were fixed about 1.2 meters inward from the inner and outer boundaries respectively.

We started with 6m water depth and tested with regular waves with periods ranging from 4-14 seconds, full scale. With the same water depth, irregular waves with peak periods ranging from 4-14 seconds were tested. For each wave period, several wave heights were tested.

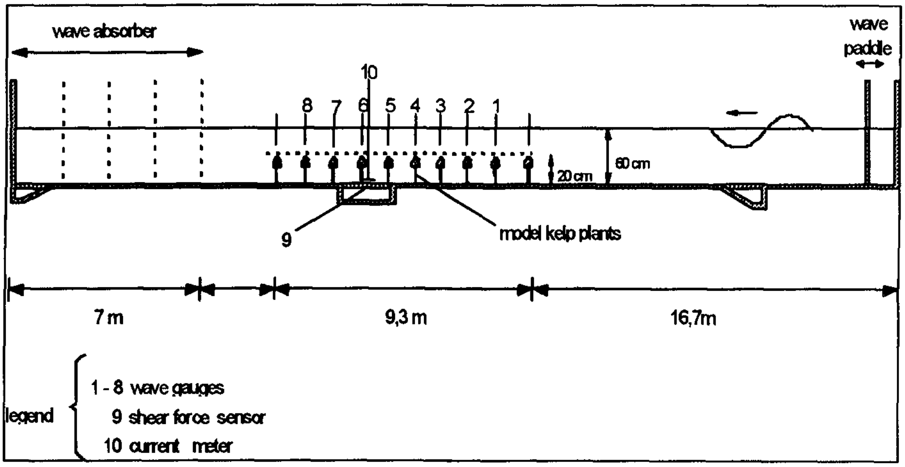

***Figure 1** Laboratory experiment layout*

4. RESULTS

(1) The exponential decay model given by Eq.(1.2) was fitted to measured data by regreesion analysis based on the least squares method. Figure 1 shows an overview of the exponential decay of the wave heights with respect to distance from the origin. Figure 3 shows theoretical variation of the damping rate with wave period T obtained from Eq.(2.14) and Eq.(2.13) for total water depth D = 6m using the following kelp

plant properties: total kelp height $d = 2$m, frond half length $l_k = 0.5$m, density of kelp $\rho = 1120$ kg/m^3, mass of frond and stipe = 0.75 kg/m, linearized spring constant k_o = 20N/m, linearized (empirical) force coefficient F_λ = 30N/(m/s), added mass for both frond and stipe $C_a = 1$, number of plants per horizontal square meter n_k = 12plants/m^2. For the given parameters the damping rate using Eq.(2.13) increases with the wave period for small values of T to a maximum and then decreases gently for larger values of T to an almost asymptotic value. On the other hand, the damping rate from the dispersion equation decreases rapidly for larger values of T. In the same figure, we also compare the damping rates derived from Eq.(1.11) given by Asano et al. (1992) and the solution of the dispersion equation given by Wang and Tørum (1994).

(2) Figure 4 shows a comparison of theoretical values obtained from Eq.(2.13) with those obtained experimentally for irregular waves in a water depth of 4m. Theoretical models by Asano et al. (1992) and Wang and Tørum (1994) are also compared to the measured data points.

(3) Figures 5 through 8 show a comparison of theoretical values of the energy dissipation equation with measured values from regular and irregular waves in water depths of 6, 8 and 10 meters. From these figures we observe several things:

First, the experimental damping coefficient varies more widely for shorter peak periods and almost converges (narrower scatter) for longer peak periods.

Second, for longer peak periods the theoretical damping coefficient is almost independent of the peak period despite a gradual decrease.

Third, when we compare the damping rates in 4 and 6 meters water depths, we find larger values for the shallower water. We may conclude that the shallower the water, the higher the damping coefficient. This observation is also in agreement with field results by Mork (1995).

Fourth, The theoretical model by Asano et al. (1992) is good only for the prediction of the damping rate for small damping. For large damping, the model grossly overestimates the damping rate. The model by Wang and Tørum (1994) underestimates the damping rate for small waves and overestimates it for large damping. For the whole range and especially for practical wave periods (6-14 seconds), the present model is very good.

Fifth, the damping rate obtained from the experiment for regular waves shows a very wide scatter. However, the theoretically predicted values appear to fit within the range of scatter as shown in Figure 6.

Sixth, as the water depth increases, the energy dissipation equation (Eq.(2.13.)) tends to over-estimate the damping coefficient, while the dispersion equation (Eq.(2.14)) comes closer and closer to the measured values. The reason for this discrepancy is not clear. However, since the empirical force coefficient F_λ was evaluated at 6m water depth, in the absence of force data for other water depths, it may be that this coefficient is variable with water depth.

(4) Figures 9 and 10 show the general functional relationship between the damping coefficient and total water depth and the number of plants per square meter respectively. For all wave periods $k_i = 0.0658\exp(-0.5*D)$.

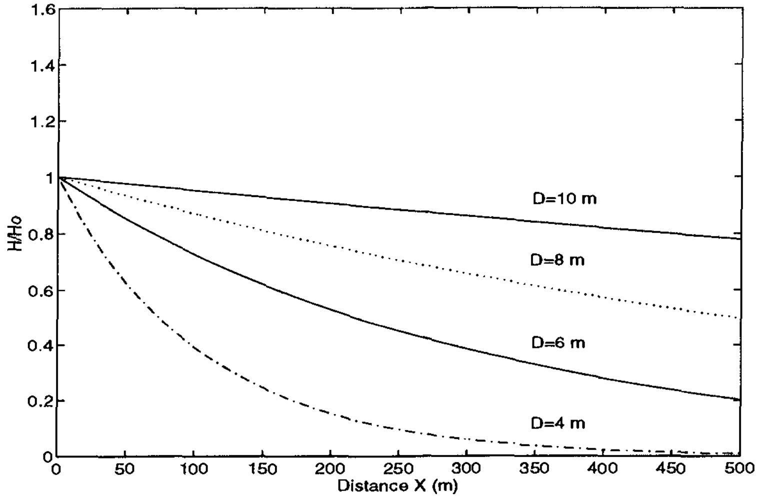

***Figure 2** Comparison of exponential decay for different total water depths.*

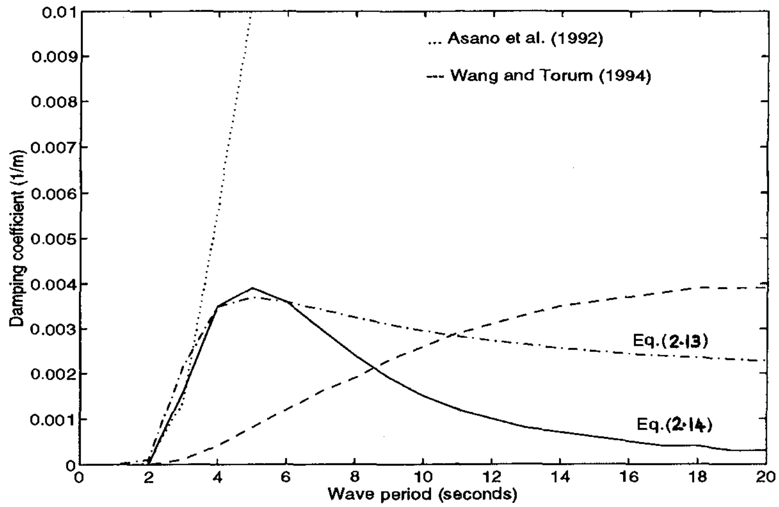

***Figure 3** Comparison of theoretical models of Asano et al. (1992), Wang and Tørum (1994) and the present model (Eq.(2.13) and Eq(2.14)). Total water depth = 6 m.*

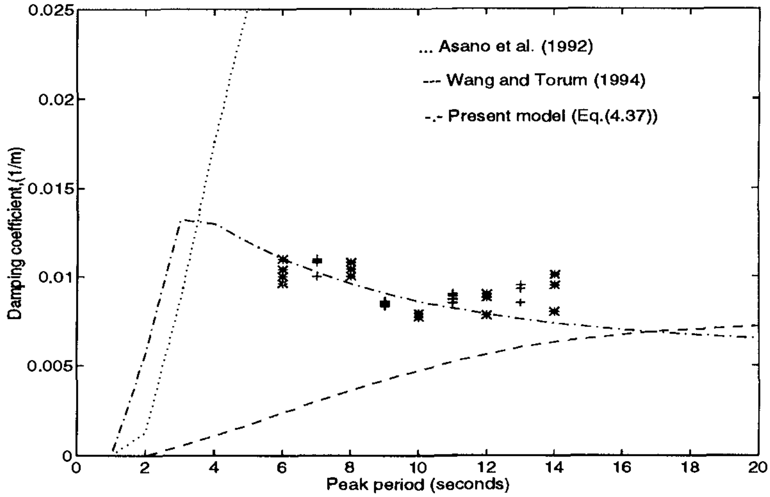

Figure 4 *Comparison of theoretical values of previous models and the present model (Eq. (2.13) with measured values (*+). Total water depth = 4 m*

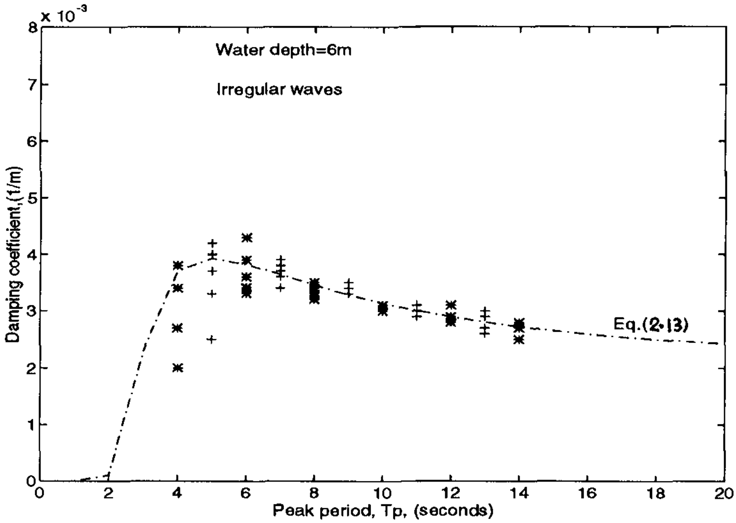

Figure 5 *Comparison of theoretical values of Eq. (2.13) with measured values (*+). with irregular waves. Total water depth = 6 m*

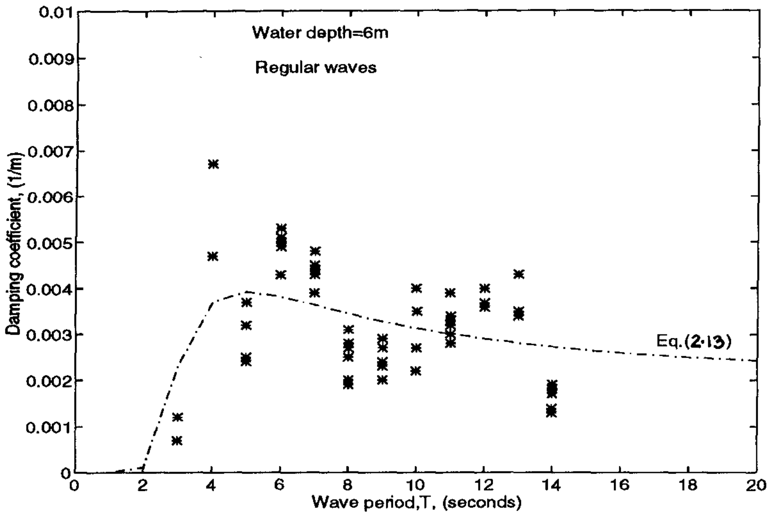

Figure 6 *Comparison of theoretical values of Eq. (2.13) with measured values with regular waves (*+). Total water depth = 6 m*

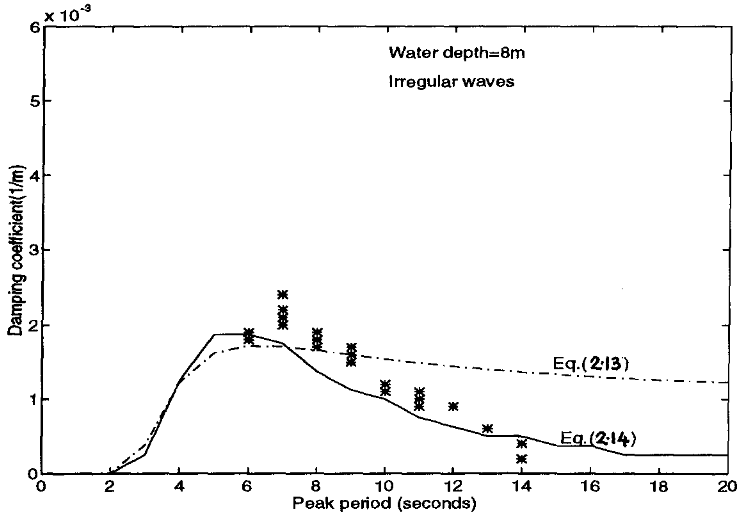

Figure 7 *Comparison of theoretical values of Eq.(2.13) and Eq. (2.14) with measured values (*) with irregular waves. Total water depth = 8 m*

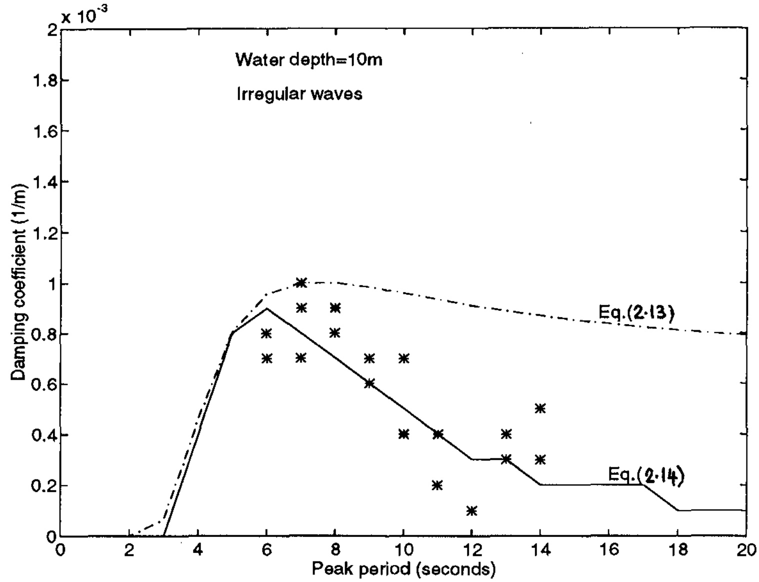

***Figure 8** Comparison of theoretical values of Eq.(2.13) and Eq. (2.14) with measured values (*) with irregular waves. Total water depth = 10 m*

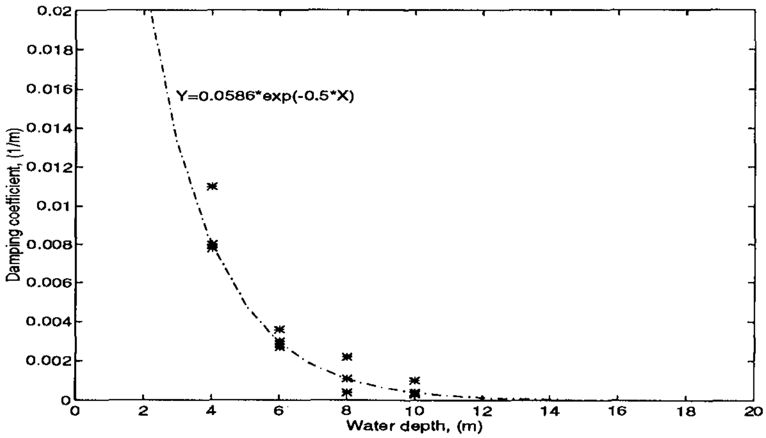

***Figure 9** General functional relationship between damping coefficient and total water depth $D \geq d$, $T=7$, 10 and 14s. For all wave periods $k_i = 0.0658 \exp(-0.5*D)$*

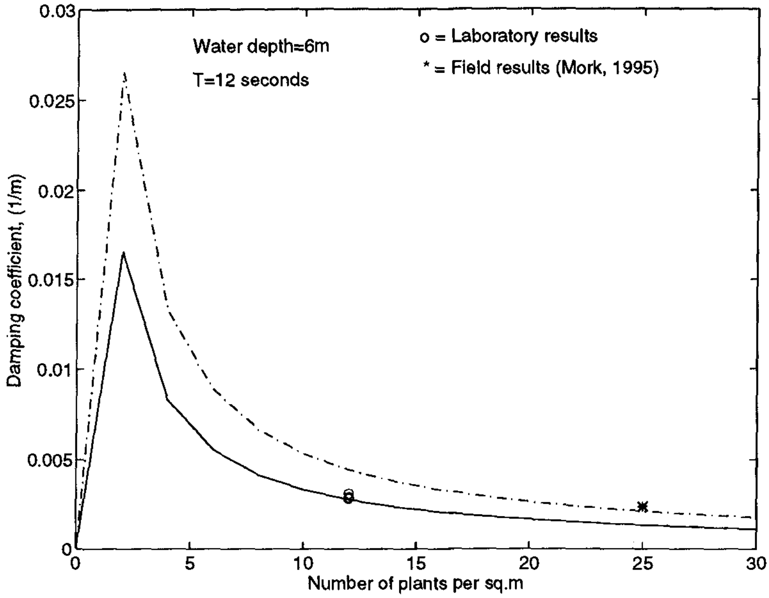

Figure 10 *Variation of damping coefficient with number of plants per square meter. Total water depth = 6 m,* ——— *d =2m,* -·-·-·-· *d=3.2m*

CONCLUSION

(1) As waves propagate in kelp fields, their wave heights are reduced significantly and wave lengths are shortened. Both theoretical and experimental studies indicate that the damping coefficient is governed by the wave period. It increases with wave period for shorter periods, reaches a maximum and then decreases gradually with the wave period for longer periods (Figures 5 through 8)

(2)The damping coefficient is also governed by the water depth and population density of plants. Figures 9 shows that the damping coefficient decreases with increasing water depth. Figure 10 shows that it increases with increasing population density until it reaches a maximum, after which it decreases sharply with increasing density.

(3) Basing on the theoretical and experimental results, wave height attenuation is more substantial in shallow water than in deep water. In fact it is almost negligible in water depths greater than 10 meters. For a population density of 12 *plants/m*^{2} , significant heights of irregular waves will be reduced by say 50% over a distance of 74, 216, 495 and 1390 meters in 4, 6, 8 and 10 meters mean water depth respectively.

(4) The linear assumption of the wave force in relation to the particle velocities leads to an exponential decay (damping) coefficient of the wave height with respect to distance.

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