

CHAPTER 198

Design wave height related to structure lifetime

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Abstract

The determination of the design wave height (often given as the significant wave height) is usually based on statistical analysis of long-term extreme wave height measurement or hindcast. The result of such extreme wave height analysis is often given as the design wave height corresponding to a chosen return period. Sometimes confidence band of the design wave height is also given in order to include various sources of uncertainties.

In this paper the First Order Reliability Method (FORM) is used to determine the design wave height corresponding to a certain exceedence probability within the structure lifetime. This includes the statistical vagrancy of nature, sample variability and the uncertainty due to measurement or hindcast error. Moreover, based on the discussion on the statistical vagrancy of nature, a formula for the calculation of encounter probability is presented.

1 Introduction

The determination of the design wave height (often given as the significant wave height) is usually based on statistical analysis of long-term extreme wave height measurement or hindcast. The sources of uncertainty contributing to the uncertainty of the design wave height are (Burcharth 1992):

- 1) Statistical vagrancy of nature, i.e. the extreme wave height X is a random variable.
- 2) Sample variability due to limited sample size.
- 3) Error related to measurement, visual observation or hindcast.
- 4) Choice of distribution as a representative of the unknown true long-term distribution
- 5) Variability of algorithms (choice of threshold, fitting method etc.)
- 6) Climatological changes

The sources 1, 2 and 3 and their influence on the design wave height will be discussed in this paper.

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We use an example to demonstrate how the design wave height is determined. The data consist of 17 most severe storms in a period of 20 years for a deep water location. The Gumbel distribution curve in Fig.1 is obtained by fitting these 17 wave data to a Gumbel distribution.

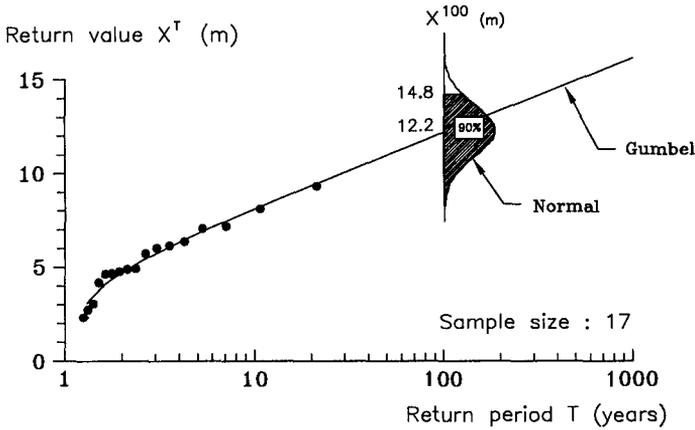


Fig.1. Design wave height.

If the design level for the design wave height is a return period of 100 years, i.e. $T = 100$, the design wave height is $x^{100} = 12.2 \text{ m}$, which means that on average this 12.2 m design wave height will be exceeded once in every 100 years.

The design wave height can be better described by the use of encounter probability, i.e. the probability that the design wave height will be exceeded within the structure lifetime. For example if the structure lifetime L is 25 years, the encounter probability of the design wave height x^{100} is

$$p = 1 - \exp\left(-\frac{L}{T}\right) = 22\% \tag{1}$$

Eq (1) is derived as eq (11) in section 2.

This means that the 12.2 m design wave height will be exceeded with 22% probability within a structure lifetime of 25 years.

If the sample variability is included, the design wave height x^{100} becomes a random variable. The distribution of the design wave height x^{100} , which is usually assumed to follow the normal distribution, can be obtained by numerical simulation, cf. Fig.1. If the upper bound with 90% confidence is taken as the design level, the design wave height is 14.8 m. What is the exceedence probability of the 14.8 m design wave height within the structure lifetime ? It cannot be calculated straight away but it might be guessed that it is

$$p = 1 - 0.9 (1 - 0.22) = 30\%$$

In this paper the First Order Reliability Method (FORM) is used to determine a design wave height corresponding to a certain exceedence probability within a specified structure lifetime. This includes the statistical vagrancy of nature, sample variability and the uncertainty due to measurement/hindcast error. Moreover, based on the discussion on the statistical vagrancy of nature, a formula for the calculation of encounter probability is presented.

2 Design wave height related to the statistical vagrancy of nature: Encounter probability

Even if we had an infinite quantity of historic true wave data and knew the related distribution precisely, there would still be uncertainty as to the largest wave which will occur during any period of time - simply due to the statistical vagrancy of nature. In this case the design wave height related to structure lifetime is characterized by the encounter probability, i.e. the probability that the design wave height will be exceeded within the structure lifetime.

Assume that the number of the extreme events is N within the structure lifetime L . X^1 denotes the maximum value in these N independent trials. Then the distribution function of X^1 is

$$F_{X^1}(x) = P(X^1 < x) = (F_X(x))^N \quad (2)$$

Note that F_{X^1} can be interpreted as the non-occurrence of the event ($X > x$) in any of N independent trials.

Assuming that the number of the extreme events $N = \lambda L$, where the sample intensity λ is

$$\lambda = \frac{\text{number of extreme events}}{\text{number of years of observation}} \quad (3)$$

From the definition of the return period T

$$T = \frac{1}{\lambda (1 - F_X(x))} \quad (4)$$

we get from eq (2)

$$F_{X^1}(x) = (F_X(x))^{\lambda L} = \left(1 - \frac{1}{\lambda T} \right)^{\lambda L} \quad (5)$$

The encounter probability of x , i.e. the probability that x will be exceeded within the structure lifetime L , is

$$p = 1 - F_{X^1}(x) = 1 - \left(1 - \frac{1}{\lambda T} \right)^{\lambda L} \quad (6)$$

For the case where $\lambda = 1$, eq (6) becomes

$$p = 1 - \left(1 - \frac{1}{T}\right)^L \quad (7)$$

However, the number of the extreme events within the structure lifetime N is also a random variable. N is usually assumed to follow the Poisson distribution

$$P(N = n) = \frac{(\lambda L)^n}{n!} \exp(-\lambda L) \quad n = 0, 1, 2, \dots \quad (8)$$

The probability of the event ($X^1 < x$ or $\bar{X} < x$) within the structure lifetime is

$$\begin{aligned} F_{X^1}(x) &= P(X^1 < x) = \sum_{n=0}^{\infty} [P(N = n) F_{X^1}(x, n)] \\ &= \sum_{n=0}^{\infty} \left[\frac{(\lambda L)^n}{n!} \exp(-\lambda L) (F_X(x))^n \right] \\ &= \sum_{n=0}^{\infty} \left[\frac{(\lambda L F_X(x))^n}{n!} \exp(-\lambda L) \right] \\ &= \exp(-\lambda L) \sum_{n=0}^{\infty} \left[\frac{(\lambda L F_X(x))^n}{n!} \right] \\ &= \exp(-\lambda L) \exp(\lambda L F_X(x)) \\ &= \exp[\lambda L (F_X(x) - 1)] \end{aligned} \quad (9)$$

Inserting eq (4) into eq (9) is obtained

$$F_{X^1}(x) = \exp\left(-\frac{L}{T}\right) \quad (10)$$

The encounter probability of x within the structure lifetime is

$$p = 1 - F_{X^1}(x) = 1 - \exp\left(-\frac{L}{T}\right) \quad (11)$$

Eq (11) is not only simpler than eq (6), but has stronger theoretical background as well because it treats N as a random variable.

3 Design wave height related to the statistical vagrancy of nature, sample variability and measurement/hindcast error

To exemplify the discussion, it is assumed that the extreme wave height follows the Gumbel distribution

$$F = F_X(x) = P(X < x) = \exp\left(-\exp\left(-\left(\frac{x-B}{A}\right)\right)\right) \quad (12)$$

where X is the extreme wave height which is a random variable, x a realization of X , A and B the distribution parameters.

Due to the sample variability and measurement/hindcast error, the distribution parameter A and B become random variables, and the maximum wave height within the structure lifetime, X^1 , becomes a conditional random variable $X^1|_{A,B}$. The probability of $X^1 > x_0$ within the structure lifetime is

$$P(X^1|_{A,B} \geq x_0) = P(x_0 - X^1|_{A,B} \leq 0) \quad (13)$$

Now consider the failure function

$$g(x^1, a, b) = x_0 - X^1|_{A,B} \begin{cases} < 0 & \text{failure} \\ = 0 & \text{limit state} \\ > 0 & \text{no failure} \end{cases} \quad (14)$$

It can be seen that the failure probability of the failure function is actually the exceedence probability of the design wave height x_0 within the structure lifetime.

By the use of the Rosenblatt transformation, the Hasofer and Lind reliability index β for the failure function can be estimated by the First Order Reliability Theory (FORM). The failure probability, i.e. the probability of $X^1 > x_0$ within the structure lifetime, is calculated by

$$P(X^1|_{A,B} \geq x_0) \approx \Phi(-\beta) \quad (15)$$

where Φ is the standard normal distribution. The procedure for the calculation of β is detailed in the Appendix.

4 Numerical simulation of σ_A and σ_B

The only unknown in the calculation of β is the distribution of A and B .

Due to the sample variability, i.e. the influence of limited number of data, the distribution parameters A and B , estimated from a sample, are subject to an uncertainty.

Wave data set contains measurement/hindcast error. Measurement error is from malfunction and non-linearity of instruments, such as accelerometer and pressure cell, while hindcast error occurs when the sea-level atmospheric pressure fields are converted to wind data and further to wave data. The accuracy of such conversion depends on the quality of the pressure data and on the technique which is used to synthesize the data into the continues wave field. Burcharth (1986) gives an overview on the variational coefficient C (standard deviation over mean value) of measurement/hindcast error.

In order to account for the sample variability and measurement/hindcast error, A and B are assumed to follow the normal distribution. The mean values μ_A

and μ_B are obtained by fitting the data to the distribution by one of the fitting methods, such as maximum likelihood method or the least square method. The standard deviations σ_A and σ_B are obtained by numerical simulations, taking into account the sample variability and the hindcast error, as explained as follows:

A sample with size N is fitted to the Gumbel distribution

$$F_X(x) = P(X < x) = \exp\left(-\exp\left(-\left(\frac{x-B}{A}\right)\right)\right) \quad (16)$$

The obtained distribution parameters A_{true} and B_{true} are assumed to be the true values. Numerical simulation is applied to get the standard deviations of the estimators A and B , taking into account the sample variability corresponding to the sample size N . The procedure is as follows:

- 1) Generate randomly a number between 0 and 1. Let the non-exceedence probability F equal the number. the single extreme data x is obtained by

$$x = F_X^{-1}(F) = A_{\text{true}} [-\ln(-\ln F)] + B_{\text{true}} \quad (17)$$

- 2) Repeat step 1) N times. Thus we obtain a sample belonging to the distribution of eq (16) and the sample size is N .
- 3) Fit the sample to the Gumbel distribution and get the new estimated distribution parameters A and B .
- 4) Repeat steps 2) and 3), say, 10,000 times. Thus we get 10,000 values of A and B .
- 5) Calculate the standard deviations σ_A and σ_B .

In order to include the measurement/hindcast error it is assumed that the hindcast error follows a normal distribution. The following step can be added after step 1).

- 1*) Generate randomly a number between 0 and 1. Let the non-exceedence probability F be equal to the number. The modified extreme data x_{modified} is obtained by

$$x_{\text{modified}} = x + C x \Phi^{-1}(F) \quad (18)$$

where Φ is the standard normal distribution and C is the coefficient of variation of the measurement/hindcast error. C ranges usually from 0.05 to 0.2 as suggested by Burcharth (1986).

5 Examples

The deep water wave data presented in Fig.1 is used as an example to demonstrate the determination of the design wave height and the influence of sample variability and measurement/hindcast error.

The data set consists of 17 significant wave heights corresponding to the 17 most severe storms in a period of 20 years, i.e. $\lambda = 17/20$. By fitting a Gumbel distribution to the extreme data we obtain the distribution parameters $A = 1.73$ and $B = 4.53$, cf. Fig.1.

If only the statistical vagrancy of the nature is considered, i.e. A and B are exact values, the wave height corresponding to any return period can be found from the graph. The 100 year return period significant wave height is 12.2 m, which by use of eq (11) is found to correspond to 22% exceedence probability within 25 year structure lifetime.

Sample variability

Taking into account the sample variability, the distribution parameters A and B become random variables. Their distributions shown in Fig.2 are obtained by the Monto-Carlo simulation as explained in section 4.

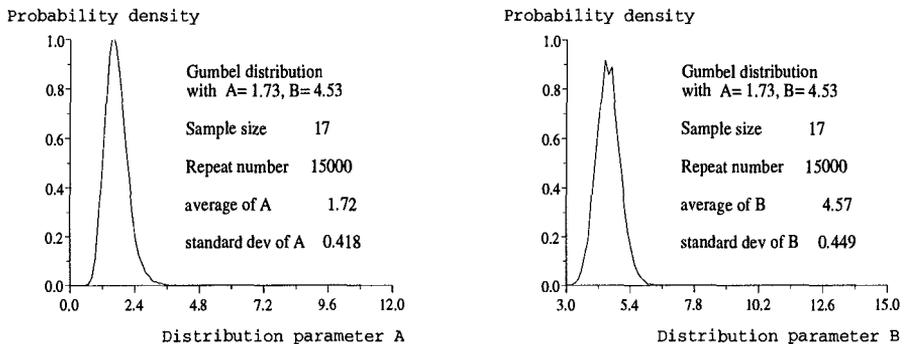


Fig.2. Distribution of the Gumbel parameters A and B by sample variability ($N=17$).

The probability density and the non-exceedence probability of the maximum significant wave height within any structure lifetime can be estimated by FORM. Fig.3 shows the results for a structure lifetime of 25 years. The figure includes for comparison also graphs where the sample variability is omitted. These graphs are obtained by eq (11).

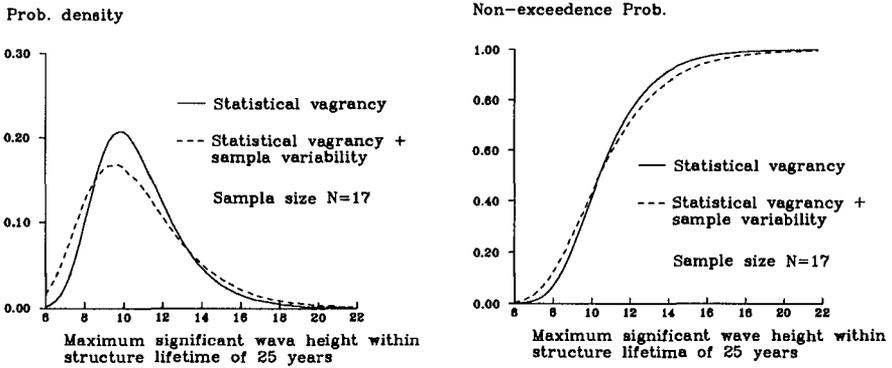


Fig.3. Distribution of maximum significant wave height (sample size $N=17$).

If the design level is the significant wave height corresponding to 22% exceedence probability within 25 years ($T = 100$ years), it can be seen from Fig.3 that the design wave height with consideration of the sample variability is 12.7 m, which is a little larger than the value without the consideration of the sample variability (12.2 m). It can also be seen that the design wave height of 14.8 m (upper bound with 90% confidence, cf. Fig.1) corresponds to 9% exceedence probability within 25 years, not 30% as guessed in Section 1.

In the case of a bigger sample size, e.g. $N = 100$, there is almost no difference between the design wave height with and without sample variability, cf. Fig.4. For comparison the same λ value is applied.

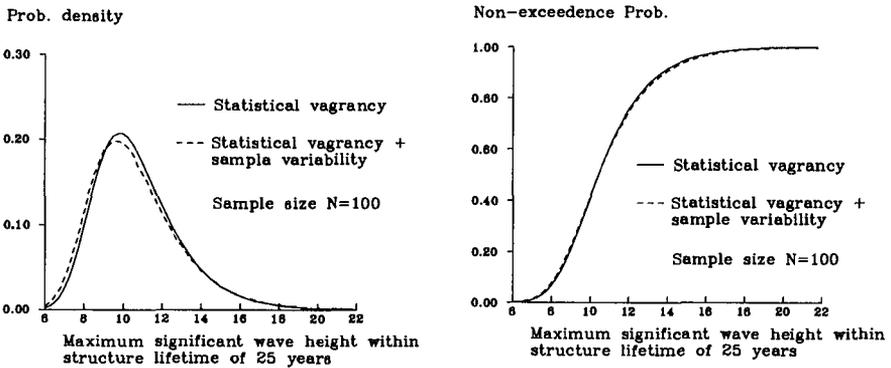


Fig.4. Distribution of maximum significant wave height (sample size $N=100$).

Table 1 shows the design wave height corresponding to different sample size. Sample size ∞ means that there is no sample variability. Keep in mind that a

typical sample size is about 20, it can be said that sample variability has some limited influence on the design wave height.

Table 1. Significant wave height corresponding to $p = 22\%$ within $L = 25$ years

sample size	10	17	50	100	∞
H_s (m)	13.0	12.7	12.4	12.3	12.2
relative difference	6.5%	4%	1.6%	0.8%	0

Measurement/hindcast error

The same procedure can be applied to further include the measurement/hindcast error.

The variational coefficient of the extreme data listed in Table 2 is taken from Burcharth (1986). Data based on visual observation from ships should in general not be used for determination of design wave height because ships avoid poor weather on purpose. With the advances in measuring techniques and numerical models, generally the C value has been reduced to app. 0.1 or less.

Table 2. Coefficient of variation for significant wave height (Burcharth 1986).

Methods of determination	Coefficient of variational
Accelerometer buoy	
Pressure cell	0.05 - 0.1
Vertical radar	
Horizontal radar	0.15
Hindcast, SPM method	0.15 - 0.2
Hindcast, numerical	0.1 - 0.2
Visual observation	0.2

In Fig.5 the coefficient of variation C of the extreme data due to measure-

ment/hindcast error is assumed a typical value of 0.1.

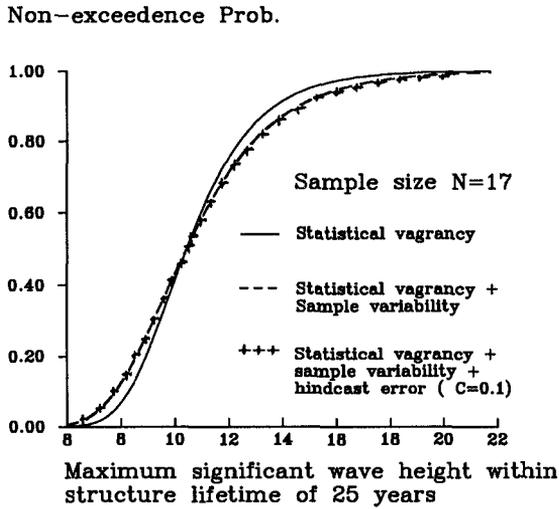


Fig.5. Distribution of maximum significant wave height ($N = 17, C = 0.1$).

In Table 3 is given values extracted from Fig.5 corresponding to an exceedence probability of $p = 22\%$.

Table 3. Significant wave height corresponding to $p = 22\%$ within $L = 25$ years

Case	H_s (m)	Remarks
Statistical vagrancy	12.2 m	
Statistical vagrancy + sample variability	12.71 m	sample size $N = 17$
Statistical vagrancy + sample variability + hindcast error	12.75 m	sample size $N = 17$ variational Coeff. $C = 0.1$

It can be seen from Fig.5 and Table 3 that the influence of measurement/hindcast error on the design wave height is very small.

6 Conclusions

The paper concentrates on the exceedence probability of the design wave height within the structure lifetime (encounter probability).

If only the statistical vagrancy of the nature is included, a new and simple encounter probability formula is derived which takes into account the randomness of the extreme events within the structure lifetime.

If other uncertainties should be considered, the paper shows that the reliability theory can be applied to determine the encounter probability of the design wave height. A practical example shows that normally sample variability has little influence on the design wave height, while the influence of measurement/hindcast errors is almost negligible.

7 References

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Appendix: Estimation of reliability index β

The followings explain the procedure for the calculation of β .

From eq (9) is obtained

$$\begin{aligned}
 F_{X^1}(x^1) &= \exp[\lambda L (F_X(x^1) - 1)] \\
 &= \exp\left[\lambda L \left(\exp\left(-\exp\left(-\left(\frac{x^1 - B}{A}\right)\right)\right) - 1\right)\right] \tag{19}
 \end{aligned}$$

which can be rewritten as

$$x^1 = A \left[-\ln \left(-\ln \left(1 + \frac{\ln F_{X^1}(x^1)}{\lambda L} \right) \right) \right] + B \tag{20}$$

X^1 can be converted to the standard normal distributed random variable U^1 by

$$\Phi(u_1) = F_{X^1}(x^1) \tag{21}$$

Inserting eq (21) into eq (20) is obtained

$$x^1 = A \left[-\ln \left(-\ln \left(1 + \frac{\ln \Phi(u_1)}{\lambda L} \right) \right) \right] + B \tag{22}$$

The failure function becomes

$$g(u_1, a, b) = x_0 - A \left[-\ln \left(-\ln \left(1 + \frac{\ln \Phi(u_1)}{\lambda L} \right) \right) \right] - B \tag{23}$$

The normal random variables A and B are converted into the standard normal distributed random variables U_2 and U_3 respectively

$$\frac{A - \mu_A}{\sigma_A} = u_2 \quad \frac{B - \mu_B}{\sigma_B} = u_3 \tag{24}$$

Insert eq (24) into eq (23) is obtained

$$\begin{aligned}
 g(u_1, u_2, u_3) &= x_0 - (\mu_A + \sigma_A u_2) \left[-\ln \left(-\ln \left(1 + \frac{\ln \Phi(u_1)}{\lambda L} \right) \right) \right] \\
 &\quad - (\mu_B + \sigma_B u_3) \tag{25}
 \end{aligned}$$

The differentiations of the failure function are

$$\begin{aligned}
 a_1 &= \frac{\partial g}{\partial u_1} = \frac{(\mu_A + \sigma_A u_2) \phi(u_1)}{\ln \left(1 + \frac{\ln \Phi(u_1)}{\lambda L} \right) \left(1 + \frac{\ln \Phi(u_1)}{\lambda L} \right) \lambda L \Phi(u_1)} \\
 a_2 &= \frac{\partial g}{\partial u_2} = -\sigma_A \left[-\ln \left(-\ln \left(1 + \frac{\ln \Phi(u_1)}{\lambda L} \right) \right) \right] \\
 a_3 &= \frac{\partial g}{\partial u_3} = -\sigma_B \tag{26}
 \end{aligned}$$

where ϕ is the density function of the standard normal distribution.

The iterative procedure for calculation of β is

- 1) Select trial values : $u^* = (u_1^*, u_2^*, u_3^*)$.
- 2) Insert u^* into eq (26) and get (a_1, a_2, a_3) .
- 3) Determine a better estimate of u^* by

$$u_i^* = a_i \frac{\sum_{i=1}^3 (a_i u_i^*) - g|_{u^*}}{\sum_{i=1}^3 a_i^2}$$

- 4) Repeat steps 2) and 3) to achieve convergence.
- 5) Calculate β by

$$\beta = \left(\sum_{i=1}^3 (u_i^*)^2 \right)^{\frac{1}{2}}$$