CHAPTER 69

TRANSITION OF STOCHASTIC CHARACTERISTICS OF WAVES IN THE NEARSHORE ZONE

David J. Robillard[1] and Michel K. Ochi[2]

Abstract

The purpose of this paper is twofold. The first is to clarify the transition of stochastic characteristics of waves; in particular, the transition from a Gaussian to non-Gaussian random process in the nearshore zone as a function of water depth and sea severity, and the second is to develop a method to estimate the non-Gaussian properties of waves at a specified water depth in the nearshore zone from knowledge of the sea severity in deep water. The probability density function applicable to non-Gaussian waves in finite water depth is applied to more than 1,000 samples of wave data obtained by the Coastal Engineering Research Center, U.S. Army, during the ARSLOE Project. From analysis of the data, the limiting water depth is defined below which wind-generated waves can no longer be considered Gaussian for a given sea severity. Furthermore, by presenting the parameters of the probability density function as a function of water depth and sea severity, it is possible to estimate the sea condition at a specified water depth in the nearshore zone from knowledge of sea severity in deep water.

Introduction

It has been known that the probability distribution of the displacement of waves in the deep ocean obeys the normal probability distribution and hence waves are considered to be a Gaussian random process. As waves propagate from deep to shallow water, however, a significant change is evident in the wave profile and this results in waves in shallow water being usually categorized as a non-Gaussian random process. It is noted, however, that transformation of wave characteristics

[1] Lieutenant, US Navy, Engineering Officer, Naval Coastal Systems Station, Panama City Beach, Florida, 32407, USA.
[2] Professor, Department of Coastal & Oceanographic Engineering, University of Florida, Gainesville, Florida, 32611, USA.

from a Gaussian to non-Gaussian random process may not always occur when waves propagate from deep to shallow water. If the sea severity is very mild, waves in shallow water are still a Gaussian random process. In other words, the non-Gaussian characteristics of waves in finite water depth depend on the water depth and sea severity.

One of the purposes of this study is to clarify the transition of wave characteristics from a Gaussian to a non-Gaussian random process as a function of water depth and sea severity. Another purpose is to develop a method to estimate the non-Gaussian characteristics at a specified water depth in the nearshore zone from knowledge of the sea severity in deep water, which is an extremely significant subject in practice.

Throughout the present study, the probability density function applicable to non-Gaussian waves in finite water depths is applied and analyses are carried out on more than 1,000 wave records obtained by the Coastal Engineering Research Center, U.S. Army, during the ARSLOE Project at Duck, North Carolina. The probability density function was developed based on the concept that stochastic characteristics of waves in finite water depth may be considered to be the same as the output of a nonlinear system (Ochi and Ahn 1994).

A broad range of sea conditions and water depths are examined to ensure the probability density function well represents the histograms of wave displacement data over various conditions. With the applicability verified, the probability density function is used to define criteria for the boundary where the wave field can no longer be considered Gaussian. Furthermore, analyses are carried out on parameters involved in the probability density function to present them as a function of water depth and sea severity. From the results of these analyses it is possible to estimate the sea condition, including the probability density function, at a specified water depth in the nearshore zone from knowledge of the sea severity (significant wave height) in deep water, although applicability of the results may be limited to a water depth of 25 meters where wave data were acquired and analyzed.

Probability Distribution Representing Non-Gaussian Waves

For evaluating the transformation of Gaussian to non-Gaussian characteristics of nearshore waves, the following probability density function developed based on the concept of the stochastic response of a nonlinear mechanical system (Ochi and Ahn 1994) is considered throughout the analysis in this paper:

$$f(y) = \frac{1}{\sqrt{2\pi}\,\sigma_*} \exp\left\{ -\frac{1}{2\left(\gamma a \sigma_*\right)^2} \left(1 - \gamma a \mu_* - e^{-\gamma a y}\right)^2 - \gamma a y \right\} \qquad (1)$$

where y = wave displacement from the mean, $\gamma = 12.8$ for $y \geq 0$ and 3.00 for $y < 0$.

The probability density function carries three parameters, a, μ_* and σ_* . In particular, the parameter 'a' controls the nonlinearity; the severity of nonlinear characteristics increases with increase in the magnitude of 'a'. The magnitude of 'a' becomes zero for Gaussian random waves.

It may be well to show that the above probability density function provides a good representation of the histogram of wave displacement constructed from data obtained in deep as well as in shallow water during a storm. Comparisons between the non-Gaussian probability density function given in Eq.(1) (solid line), the Gaussian distribution (broken line) and the histogram constructed from data obtained during the ARSLOE Project are shown in Figures 1(a) through 1(e). Included also in each figure is a portion of the wave data obtained at each location. The water depth, D, rms-value, σ, evaluated from data and the ratio between them, σ/D are tabulated in Table 1. The water depth is that at the time of measurement including the effect of tide as well as storm surge.

As can be seen in Figure 1(a), the histogram of wave displacement obtained at a water depth of 24.7 meters obeys the Gaussian probability distribution, while all other histograms obtained at shallower water depths deviate from the normal distribution but are well represented by the non-Gaussian probability distribution given in Eq.(1). Table 1 shows that the magnitude of σ/D increases as waves propagate from deep to shallow water, and hence the ratio σ/D may be used as a criterion indicating the limiting condition for the transformation from Gaussian to non-Gaussian random waves. Although only five examples of comparisons between the non-Gaussian distribution and histogram are shown here, about 100 comparisons for various water depths and sea severities all show excellent agreement.

Since Eq.(1) represents well the probability distribution of displacement of waves in the nearshore area, over 1,000 wave data obtained during a storm encountered at the ARSLOE Project (October 23-25, 1980) are reduced for evaluating parameters of the probability distribution function. Then, further analysis is carried out to represent these parameters as a function of water depth and sea severity.

Limiting Water Depth and Sea Severity of Gaussian Waves

The non-Gaussian property of waves in the nearshore zone depends on water depth and sea severity. Analysis of wave records obtained in shallow water shows clearly that they are still a Gaussian random process if sea severity is mild. Table 1 indicates that the transition from Gaussian to non-Gaussian waves occurs for σ/D values between 0.042 and 0.086. From analysis of about 100 samples of wave data with σ/D between 0.045 to 0.080, it is found that σ/D = 0.060 is the limiting condition below which waves can be considered as a Gaussian random process. As an example of the analysis results, Figure 2(a) shows a comparison between the Gaussian (broken line) and non-Gaussian (solid line) probability density functions of wave displacement data for σ/D = 0.071. In this case, the χ^2-test with level of significance of 0.05 shows that the measured data cannot be considered to

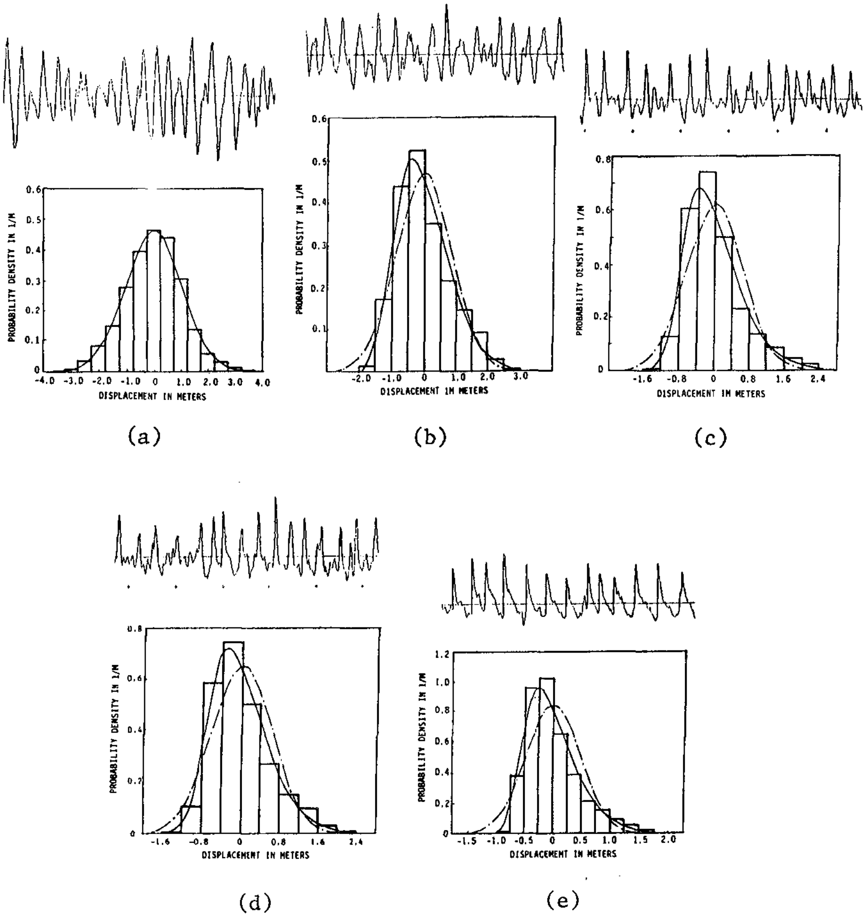

Figure 1 Comparison between probability density function and histogram of wave displacement

Table 1 Water depth and rms-value of data shown in Figure 1

WAVE DATA SHOWN IN FIG. 1	WATER DEPTH D (m)	RMS-VALUE σ (m)	σ/D
(a)	24.70	1.034	0.042
(b)	10.04	0.858	0.086
(c)	6.52	0.650	0.100
(d)	4.66	0.612	0.131
(e)	2.23	0.480	0.215

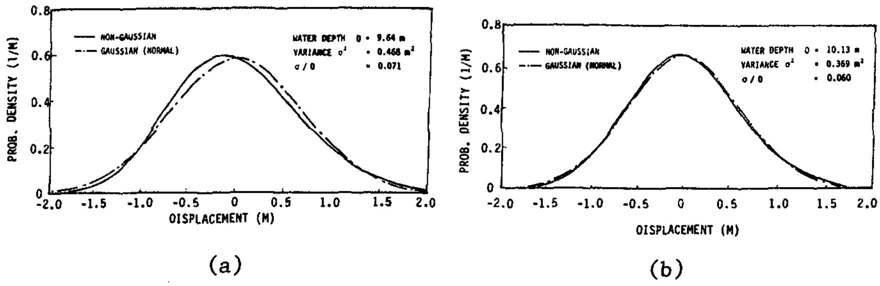

Figure 2 Comparison between Gaussian (broken line) and non-Gaussian (solid line) probability density functions of wave displacement data

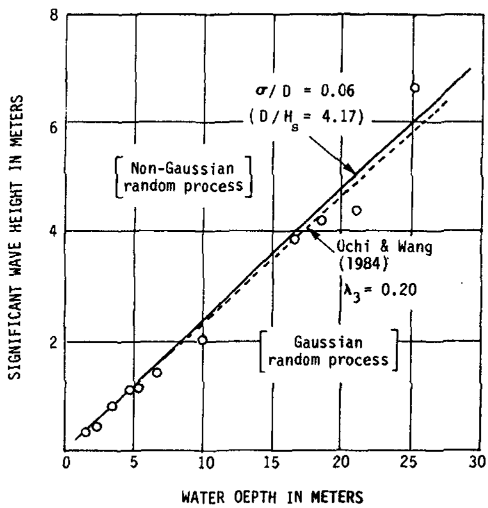

Figure 3
Limiting water depth for non-Gaussian waves as a function of significant wave height

be a Gaussian random process. On the other hand, Figure 2(b) shows a comparison for $\sigma/D = 0.060$. As seen, the non-Gaussian probability distribution computed from data is virtually the normal probability distribution. It was found that all probability distributions of wave displacement obey the normal probability distribution and/or pass the χ^2-test (using a normal distribution as the hypothesized distribution) when the ratio σ/D is less than 0.06. Assuming that waves are narrow-banded, $\sigma/D = 0.06$ is equivalent to $D/H_s = 4.17$, where H_s = significant wave height.

Based on the findings presented above, Figure 3 shows the limiting water depth below which wind-generated waves are considered to be a non-Gaussian random process for a specified significant wave height. The criterion $\sigma/D = 0.06$ for the limit of Gaussian waves is given as the solid line in the figure. It is noted that the parameter 'a' of the probability density function given in Eq.(1) is zero for

Gaussian waves, which can certainly be used as another criterion for evaluating the limiting Gaussian random process. Hence, the sea severities (significant wave heights) for which the computed a-values of the density function is zero or near zero are evaluated at various water depths and plotted as the open circles in the figure. Finally, a criterion for limiting Gaussian random waves was developed from analysis using the Gram-Charlier series probability distribution with skewness $\lambda_3 = 0.2$ (Ochi and Wang 1984). This criterion is given as the dotted line in the figure.

The results obtained from the above mentioned three different analyses for evaluating limiting conditions of Gaussian waves as a function of significant wave height, H_s, and water depth show good agreements; therefore, it may be concluded that the water depth in a given sea severity for which $\sigma/D = 0.06$ (or $D = 4.17\ H_s$) is the limiting depth below which waves may be considered to be a non-Gaussian random process. This criterion, however, is valid only for water depths not exceeding 25 meters where the data were acquired and analyzed.

Parameters of the probability Distribution

In this section, the functional relationship between parameters of the non-Gaussian probability density function given in Eq.(1) will be discussed and presented as a function of water depth and sea severity. Since the three parameters, a, μ_* and σ_* carry different dimensions, these parameters are expressed in dimensionless forms, $a\sigma$, μ_*/σ and σ_*/σ, respectively, where σ is the rms-value of waves at a specified location.

Figure 4 shows the functional relationship between σ_*/σ and $a\sigma$. In this figure, the relationship obtained from data at three locations (calm water depth of 1.35 m, 5.5 m and 8.8 m) are given, but data obtained at other water depths fall within the upper and lower bounds drawn in the figure. The vertical scale range is very small; from 1.00 to 1.05. Therefore, the difference between the formula given in the figure and the upper and lower bounds of data is within one per cent. That is, we can evaluate σ_*/σ as a function of $a\sigma$ within one per cent as follows:

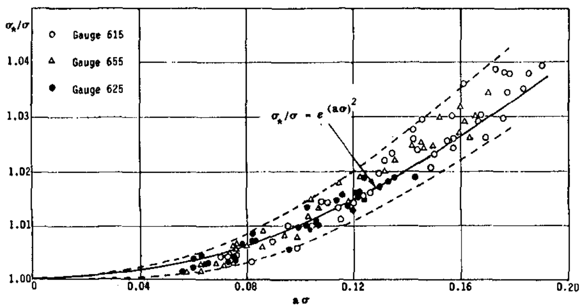

Figure 4 Functional relationship between σ_*/σ and $a\sigma$

$$\sigma_* / \sigma = \exp\left\{(a\sigma)^2\right\} \tag{2}$$

Figure 5 shows the functional relationship between μ_* / σ and $a\sigma$. Again, the figure shows the results obtained at the three locations shown in Figure 5, but all data obtained at other water depths fall within the range of scatter shown in the figure. By drawing the average line, μ_* / σ can be presented as a function of $a\sigma$ as follows:

$$\mu_* / \sigma = -1.55(a\sigma)^{1.20} \tag{3}$$

Figure 6 shows the dimensionless parameter $a\sigma$ as a function of the sea severity-water depth ratio σ/D. As seen, $a\sigma$ is zero for $\sigma/D = 0.06$ as mentioned

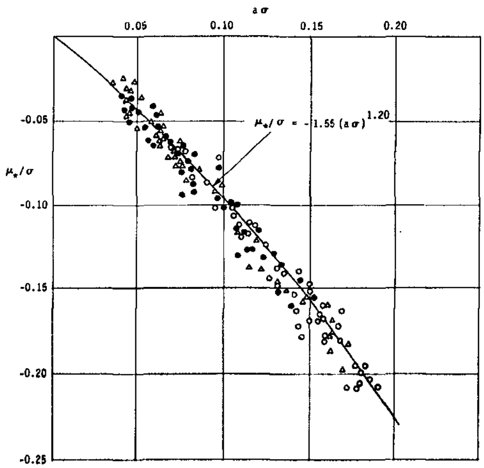

Figure 5
Functional relationship between μ_*/σ and $a\sigma$

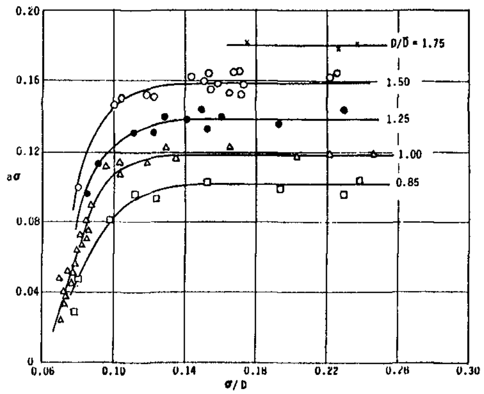

Figure 6
Relationship between $a\sigma$ and σ/D as a function of dimensionless water depth $D/\bar{D}$

earlier, and $a\sigma$ depends to a great extent on the water depth. In order to present the results as a function of water depth, the dimensionless water depth $D/\overline{D}$ is used where $\overline{D}$ is the calm water depth at each location where data were obtained. The water depth in the shallow water area, in general, increases to 1.5 (or greater) times the calm water level. Points for $D/\overline{D}$ greater than 1.5 and σ/D greater than 0.12 in the figure are those in shallow water, on the order of less than 4 meters depth in calm water. As seen in the figure, for a specified $D/\overline{D}$, the value of $a\sigma$ increases rapidly for σ/D between 0.06 and 0.12 beyond which it remains constant. That is, for σ/D greater than 0.12, the parameter $a\sigma$, representing the severity of the nonlinear property of waves may be represented approximately by

$$a\sigma = 0.032 + 0.085\left(D / \overline{D}\right) \qquad (4)$$

<u>Estimation of Sea Condition in the Nearshore Zone</u>

It may be of considerable interest to estimate the sea condition in the nearshore zone from knowledge of the sea severity (significant wave height) in the offshore area. For this, it is assumed that the sea condition remains constant in the offshore area where the water depth is greater than 4.17 H_s; the relationship obtained in Figure 3. This water depth is denoted by D_0 in Figure 7. Next, let the variance of non-Gaussian random waves at an arbitrary location in the nearshore zone be σ^2 where the water depth is D. Figure 8 presents the sea severity-water depth ratio, σ/D, evaluated from analysis of data obtained at various locations in the nearshore zone as a function of dimensionless water depth D/D_0. The solid circles represent high tide condition, while the open circles are for low tide. The triangular marks are indicative of the water depth during the storm being almost equal to the still water level at the various locations. By drawing the average line in the figure, the functional relationship between σ/D and D/D_0 can be obtained from the figure as

$$\sigma / D = 0.06\left(D / D_0\right)^{-0.58} \qquad (5)$$

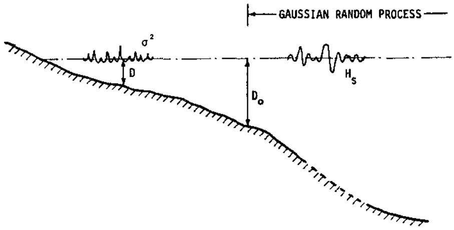

Figure 7 Sketch indicating the estimated transition of sea severity due to water depth variation in the nearshore zone

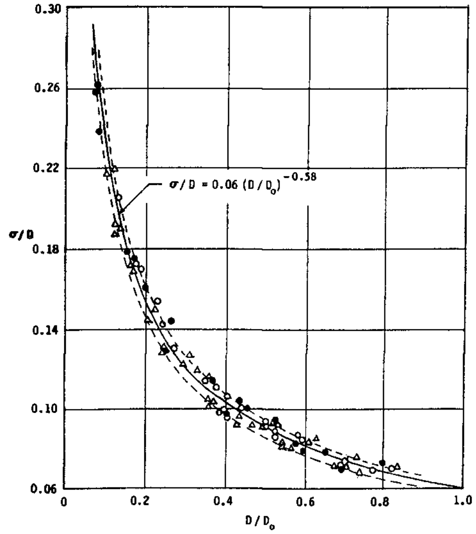

Figure 8 Relationship between σ/D and dimensionless water depth D/D_0

The two lines in Figure 8 are $\pm$ 5 % lines of the mathematical formulation which cover all data. Thus, upon estimating the water depth D at a specific location including the effect of tide and storm surge, the sea condition in the nearshore zone can be predicted by Eq.(5) from knowledge of the significant wave height in deep water. The parameters of the probability distribution of non-Gaussian waves applicable to that location can then be estimated from Eqs.(2) through (4) with information on the water depth in calm water.

As an example, Figure 9 shows the application of this approach for predicting the sea condition in the nearshore zone from knowledge of significant wave height in the offshore area, the sea condition at Duck, North Carolina at noon on October 25th, 1980 is estimated. The records show the tide was high at this time. The horizontal scale in Figure 9 is the distance from the shoreline given in logarithmic scale. The significant wave height at a distance 12 Km offshore was 4.28 meters where the water depth was 24.7 meters. It is assumed that the sea severity remains constant in areas deeper than D_0 = 17.5 m, which is equal to 4.17 times the significant wave height and that waves are non-Gaussian in areas shallower than D_0. The values of σ/D are estimated at various water depths from Figure 8, and compared with measured data which are given by the square-marks in the figure. The variances, σ^2, are then computed and compared with measured data given by the open circles. Furthermore, the dimensionless parameters $a\sigma$ are estimated at various water depths from Figure 6 (or by Eq. 4 for $\sigma/D > 0.12$) and are plotted in the figure. Good agreement between estimated and computed values from measured data is obtained in this example.

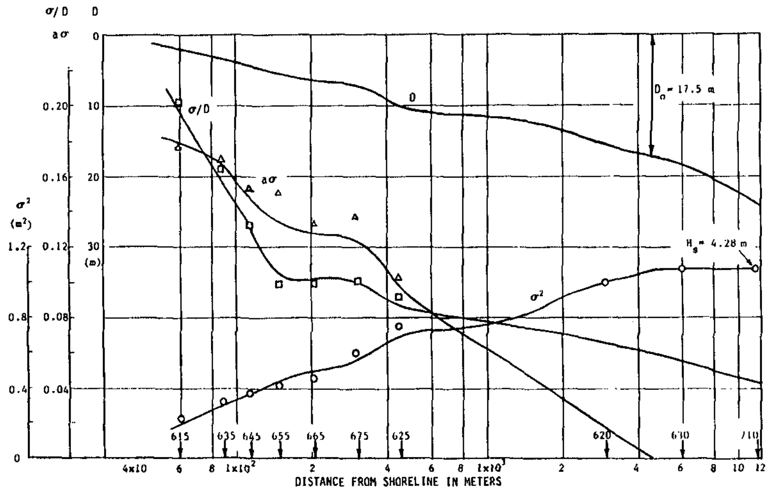

Figure 9 Comparison between estimated σ/D, σ^2, aσ and those computed from measured data at various locations in the nearshore zone

Conclusions

This paper presents the results of a study to clarify the transition characteristics of waves from a Gaussian to non-Gaussian random process in the nearshore zone and presents them as a function of water depth and sea severity. From analysis of more than 1,000 samples of wave data obtained by the Coastal Engineering Research Center, U.S. Army, during the ARSLOE Project, it is found that for a given sea severity a water depth of 4.17 times the significant wave height is the limiting depth for which waves may be considered to be a non-Gaussian random process. Furthermore, the parameters of the probability density function representing non-Gaussian waves are presented as a function of water depth and sea severity. By applying the results of this analysis, a method is developed to estimate the sea condition at a specified water depth in the nearshore zone from knowledge of sea severity (significant wave height) in deep water.

Acknowledgments

The authors express their appreciation to Dr. Ahn for his valuable contribution in evaluating the limiting water depth associated with non-Gaussian waves. They are also grateful to Ms. Laura Dickinson for typing the manuscript.

References

Ochi, M.K. and Wang, D.W.C. (1984), Non-Gaussian Characteristics of Coastal Waves, Proc. 19th Coastal Eng. Conf., Vol.1, pp.516-531.

Ochi, M.K. and Ahn, K. (1994), Non-Gaussian Probability Distribution of Coastal Waves, Proc. 24th Coastal Eng. Conf., Vol.1, pp.482-496.