## **CHAPTER 64**

## PROBABILISTIC MODELING OF LONG-TERM WAVE CLIMATE

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## Abstract

Utilizing eight years of wave measurements from the U.S. Army Corps of Engineers Field Research Facility (FRF), this study develops a long-term probabilistic model for the energy-based significant wave height ( $H_{mo}$ ) based on the shifted gamma distribution. The method of moments is used in best-fit model parameter estimation. Shifted gamma distributions are developed for the entire eight years of  $H_{mo}$  data, as well as for the individual years in the data set. The shifted gamma distribution represents the FRF  $H_{mo}$  data for both the total and yearly data sets. Further, the distribution which represents the total, or long-term, data also effectively models the yearly data, indicating that the shifted gamma distribution is a useful engineering tool for predicting long-term wave climate.

## Introduction

In order to obtain the most accurate and reliable information on ocean waves, wave climate needs to be studied over the long-term. Hogben (1990) defines long-term statistics as those which describe sea conditions over time spans of years, and it is in this definition that one discovers the problem; not many data sets exist which have time scales on this order. Isaacson and MacKenzie (1981) note in their review of long-term distributions of ocean waves that design wave estimates based on 100 years of data differ by a factor of 3 when compared to estimates considering only two successive years. Furthermore, Soares (1988) found that predictions of significant wave height varied between 9.02 and 14.11

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meters when based on data sets that were one year long, whereas the significant wave height only varied between 10.49 and 12.79 meters when based on data sets which were 10 years in length. It is clear that an adequate data base is crucial to the value of any long-term wave climate study.

Although the long-term wave climate is a basic element of many coastal engineering issues, few probabilistic models for long-term statistics have been developed. Of those available, e.g., the modified exponential distribution for significant wave height presented in the Shore Protection Manual (1984), estimation of best-fit model parameters has been hampered by a lack of suitable long-term data. There are several means available for acquiring long-term wave data; visual observations, remotely sensed wave data, hindcast results, and in situ measurements. Of these, in situ measurements are regarded as the most accurate and reliable form of wave data. Because of measurement programs initiated in the early 1980's, such in situ data sets are now becoming available. For instance, Teng, Timpe, and Palao (1994) use thirteen years of buoy data from the National Data Buoy Center (NDBC) to estimate design wave heights, and Soares and Henriques (1994) use seven years of data collected from an oil platform in the North Sea to examine model parameter estimation. This study will develop generic, long-term wave climate statistical models utilizing a similar measurement program.

#### **Description of Field Data**

One of the best long-term data sets of nearshore wave climate has been collected by the U.S. Army Corps of Engineers Field Research Facility (FRF), located in Duck, North Carolina. The wave data used in this study were collected from a pressure gage located at a nominal water depth of 8 m. These time series consist of 34 minute records collected once every six hours. The sampling rate of the gages was 2 Hz (U.S. Army, 1993). Both the energy-based significant wave height,  $H_{mo}$ , and the peak wave period,  $T_p$ , are computed from the pressure gage time series and reported in monthly data summaries.  $H_{mo}$  is equal to four times the square root of the area under the wave energy spectrum, and  $T_p$  is the wave period associated with the maximum wave energy in the spectrum. The FRF data used in this study were collected from May 1987 to April 1995 - almost 11,400 entries in all.

Constructing histograms, along with finding some of the basic statistical properties of the data, provides an immediate indication of the range of the data, its most frequently occurring values, and how the data is dispersed about the typical value. A bivariate histogram of  $H_{mo}$  and  $T_p$  (Figure 1) illustrates the entire range of wave heights and periods the FRF experiences. The histograms of  $H_{mo}$  and  $T_p$  are also useful in the visualization of the wave climate at the FRF. Figures 2 and 3 show these histograms respectively.



**Figure 1.** U.S. Army Field Research Facility (FRF) Bivariate histogram of significant wave height  $(H_{mo})$  and peak period  $(T_p)$  for May 1987 - April 1995



Figure 2. Histogram of H<sub>mo</sub> Data from FRF (May 1987 - April 1995)



Figure 3. Histogram of T<sub>p</sub> Data from FRF (May 1987 - April 1995)

## The Shifted Gamma Model

Past research efforts have found several probability distributions which successfully model long-term distributions of wave height. For instance, using between one and seven years of data from 18 sites around the British Isles, Burrows and Salih (1986) concluded that even though the Weibull distribution did not fit the lowermost regions of the significant wave height ( $H_s$ ) histograms as well as the lognormal distribution, it provided a better overall fit. On the other hand, Soares, Lopes, and Costa (1988) took four years of waverider buoy data collected off the coast of Portugal and found that the lognormal distribution provided the best fit for  $H_s$ .

After considering several probability distributions, the present study found that the shifted gamma distribution best represented the FRF  $H_{mo}$  data set (see e.g., Benjamin & Cornell, 1970):

$$pdf(H_{mo}) = \frac{\lambda}{\Gamma(k)} \left[ \lambda (H_{mo} - H_{\star}) \right]^{k-1} e^{-\lambda (H_{mo} - H_{\star})} \qquad H_{mo} \ge H_{\star}$$
(1)

In this model  $H_*$  is a shifting parameter,  $\lambda$  is a scaling parameter, k is a shape parameter, and  $\Gamma(k)$  is the gamma function. The method of moments was used to

determine the best-fit parameters. This method utilizes the mean ( $\overline{H}_{mo}$ ), the standard deviation ( $\sigma$ ), and the skewness (s) of the FRF H<sub>mo</sub> data set to estimate the model parameters. Using this method, equations (2), (3), and (4) are solved simultaneously:

$$\overline{H}_{mo} = H_* + \frac{k}{\lambda} \tag{2}$$

$$\sigma = \frac{\sqrt{k}}{\lambda} \tag{3}$$

$$s = \frac{2}{\sqrt{k}} \tag{4}$$

One property of the shifted gamma model should be noted; i.e. if  $k \le 1$ , then the shifted gamma distribution reduces to an exponential-like function, which does not represent the basic shape of the H<sub>mo</sub> histogram (see Figure 2). Because the shape parameter (k) is determined strictly from the skewness of the data set [refer to equation (4)], s must be less than two. In order to overcome this problem, the H<sub>mo</sub> data set was edited by removing all wave heights above 3.0 meters. As the focus of this study is on long-term wave climate, and not on extreme storm events, the removal of the largest values is not detrimental.

#### H<sub>mo</sub> Data Editing and Statistics

Once  $H_{mo}$  values above 3.0 meters were removed from the FRF data, the edited data could be modeled successfully using the shifted gamma distribution. Table 1 presents some of the basic statistical properties of the raw data, and how these properties fared once wave heights above 3.0 meters were removed. For the entire eight years of FRF  $H_{mo}$  data (11385 observations) a total of 139 observations above 3.0 meters were trimmed. Although the mean and the standard deviation do not change significantly, the skewness undergoes the greatest change. For instance, the mean of the raw and trimmed data sets for the entire eight years are 0.87 and 0.84 meters, respectively. There is a slightly larger change in the standard deviation which drops from 0.58 meters in the raw data set to 0.49 meters in the trimmed set. The skewness undergoes the greatest change. It drops from 2.18 for the raw data to 1.52 for the trimmed data.

Comparisons between the raw and trimmed data statistics for the individual years are similar to those of the entire data set. Out of the eight years of data, the period from 1987-1988 had the fewest observations trimmed (seven observations). Although the mean and the standard deviation do not change significantly, the

skewness drops from 2.44 to 1.75. In contrast, the greatest number of observations trimmed was during 1992-1993, when 39 values were found to be above 3.0 meters. Once again, there is little change in the mean and standard deviation, whereas the skewness value drops from 1.89 to 1.40.

		RawD	ata Set			Trin	med Dat	a Set	
Year (s)	# of Obs.	Mean (m)	Std. dev. (m)	Skewness	# of Olos,	# Trimmed	Mean t (m)	Std. dev. (m)	Slewnes
1987-1995	11385	0.87	0.58	2.18	11246	139	0.84	0.49	1.52
1987-1988	1423	0.82	0.50	2.44	1416	7	0.81	0.45	1.75
1988-1989	1406	0.89	0.58	2.15	1386	20	0.85	0.48	1.33
1989-1990	1423	0.85	0.57	2.40	1410	13	0.82	0.49	1.65
1990-1991	1423	0.88	0.51	1.84	1414	9	0.86	0.47	1.28
1991-1992	1446	0.87	0.62	2.39	1421	25	0.82	0.49	1.55
1992-1993	1440	0.99	0.69	1.89	1401	39	0.92	0.55	1.40
1993-1994	1435	0.80	0.52	2.04	1425	10	0.78	0.48	1.66
1994-1995	1389	0.88	0.60	1.96	13/3	16	0.84	0.53	1.50

Table 1. Basic Statistics of Raw and Trimmed FRF  $H_{mo}$  Data

## **Best-fit Model Parameters**

The best-fit model parameters for the shifted gamma distribution were found using the method of moments as dictated by equations (2), (3), and (4). Table 2 illustrates the values of the shifting parameter ( $H_*$ ), the shape parameter (k), and the scaling parameter ( $\lambda$ ) for the entire eight years of FRF H<sub>mo</sub> data, as well as for the yearly data sets. Table 2 also includes the maximum and minimum values of each model parameter (the bold numbers in each column), and the mean and standard deviation of the model parameters from the eight individual years of data.

The first row in Table 2 contains the values of the best-fit parameters for the entire eight years of  $H_{mo}$  data from the FRF. These values ( $H_*=0.19$  m, k=1.72, and  $\lambda=2.66$  m<sup>-1</sup>) are comparable to the mean values computed from the eight individual years of data. Beginning in the second row of Table 2, the model parameters for the eight individual years of data are presented. The minimum and maximum values of  $H_*$  are 0.13 (1988-89 and 1990-91) and 0.29 meters (1987-1988), respectively. The mean value of the shifting parameter is 0.18 meters and its standard deviation is 0.06 meters. The second column contains the values for the shape parameter, k. It's minimum value occurred during the year 1987-1988 (1.31) and its maximum value occurred during 1990-1991 (2.44). The mean of the shape parameter for the eight years is 1.80 and the standard deviation is 0.40. Finally,

column three of Table 2 contains the values of the scaling parameter,  $\lambda$ . The minimum value for the scaling parameter is 2.49 m<sup>-1</sup>, which is associated with the data from 1989-1990. The maximum value for  $\lambda$  is 3.33 m<sup>-1</sup> (1990-1991). The mean and the standard deviation of  $\lambda$  from the eight individual years of data are 2.72 m<sup>-1</sup> and 0.32 m<sup>-1</sup>, respectively.

It is interesting to note that the maximum values for k and  $\lambda$  and the minimum value for  $H_*$  all occur during the year 1990-1991. For an explanation one need look no further than the skewness values for both the raw and trimmed data during this time period. In both instances, 1.84 for the raw data and 1.28 for the trimmed data, the skewness values are the lowest of all eight years. As the method of moments begins with solving equation (4), it is easy to see how the skewness of the data set can have such an impact on the rest of the model parameters.

Year(s)	H. (m)	<i>k</i>	λ (m <sup>-1</sup> )
1987-1995	0.19	1.72	2.66
1987-1988	0.29	1.31	2.53
1988-1989	0.13	2.25	3.14
1989-1990	0.23	1.47	2.49
1990-1991	0.13	2.44	3.33
1991-1992	0.18	1.66	2.62
1992-1993	0.14	2.02	2.59
1993-1994	0.21	1.46	2.54
1994-1995	0.14	1.78	2.53
Mean	0.18	1.80	2.72
Std. Dev.	0.06	0.40	0.32

Table 2. Best-fit Model Parameters for the Trimmed Data Sets of H<sub>mo</sub>

#### Model Comparison to Long-Term Wave Climate

Figure 5 illustrates the ability of the shifted gamma distribution to model the long-term (entire eight years) FRF  $H_{mo}$  data. In Figure 5 the circles represent the percent occurrences of the data, whereas the shifted gamma model is represented by the solid line. Although the model under-predicts the most frequently occurring wave heights (in the 0.60 meter range) by 3%, it correctly predicts that  $H_{mo}$  values in the 0.40 meter range occur 26% of the time. Overall, the shifted gamma distribution is in good agreement with the data.



Figure 4. FRF H<sub>mo</sub> Model Comparison -- 1987-1995

## **Model Comparisons to Yearly Data Sets**

Figures 5-12 show how the shifted gamma distribution compares to the FRF  $H_{mo}$  data on a yearly basis. Beginning with the year 1987-1988 (Figure 5) and ending with the year 1994-1995 (Figure 12), the shifted gamma distribution shows good agreement with the data. These figures also illustrate how the long-term shifted gamma model (the dashed line) performs when plotted on top of the yearly  $H_{mo}$  histograms. In Figures 6, 8, 9, and 12 the long-term model does as well, or better, than the model developed from only that one year of data. In three of the years (Figures 6, 8, and 12) the long-term model more accurately captures the peak of the distribution. However, there are also several cases where the long-term model fails to provide as accurate a fit as that provided by the yearly model. In

Figures 5, 7, and 11 the long-term model noticeably under-estimates the percent occurrence of the most commonly occurring wave heights. It is interesting to note, however, that these three years of data have the highest skewness values. And for two of the years, 1987-1988 (Figure 5) and 1989-1990 (Figure 7), even the yearly model failed to capture the peak percent occurrence.

Figure 10 presents the only case in which the long-term model noticeably over-predicts the most commonly occurring wave heights. This time period, (1992-1993), is unique from any other because the  $H_{mo}$  histogram lacks a well-defined peak. This year of data had the highest mean and standard deviation values, as well as the third-smallest skewness (see Table 1).

## Conclusions

The problem hampering the development of long-term wave elimate models, i.e. a lack of suitable data, is gradually being overcome as monitoring programs such as that from the FRF continue. Utilizing data sets such as these, best-fit model parameters can be estimated with more confidence than in the past.

Utilizing eight years of data from the FRF (May 1987 to April 1995) and the method of moments, best-fit model parameters were found for the shifted gamma distribution. This probabilistic model appears to represent the FRF  $H_{mo}$  data for the entire eight years of data, as well as on a yearly basis. Furthermore, although it sometimes fell short of capturing the peak percent occurrence, the long-term distribution modeled the yearly Hmo histograms to a reasonable degree. Therefore, the shifted gamma distribution should be considered when developing long-term wave climate models.

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Figure 5. FRF H<sub>mo</sub> Model Comparison -- 1987-1988



Figure 6. FRF H<sub>mo</sub> Model Comparison -- 1988-1989



Figure 7. FRF H<sub>mo</sub> Model Comparison -- 1989-1990



Figure 8. FRF H<sub>mo</sub> Model Comparison -- 1990-1991



Figure 9. FRF H<sub>mo</sub> Model Comparison -- 1991-1992



Figure 10. FRF H<sub>mo</sub> Model Comparison -- 1992-1993



Figure 11. FRF H<sub>mo</sub> Model Comparison -- 1993-1994



Figure 12. FRF H<sub>mo</sub> Model Comparison -- 1994-1995

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