CHAPTER 54

Generation Depths from Water Wave Data

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Abstract

It is shown that by determining wave group back tracks for recorded data it is possible to determine whether the waves were generated in deep, intermediate, or shallow water depths. The general movement of a storm can be tracked if a storm moves through water of intermediate depths. For the examples shown reasonable results were obtained by approximating the geometric group velocity by the conventional group velocity.

Introduction

When water waves are generated the depth of the water where the waves begin becomes a characteristic trait of the waves. Through the use of simple refraction concepts it is possible to determine from recorded data whether the waves originated in deep, intermediate, or shallow water depths. If the waves were generated in intermediate water depths the actual water depths can be determined.

It is assumed that initially the wave groups and the individual waves within the groups have the same direction where the waves are generated. In shoaling water, refraction causes the groups and the individual waves to have different directions because of their differing velocities. This difference is used to determine the water depths where the waves are generated, i.e., the generation depths. The method is based on determining back tracks from the measurement site to the source of wave generation.

Breeding (1978, 1986) found that the paths of wave groups are determined

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according to Snell's law with the geometric group velocity. The geometric group speed G is defined

$$G = U\cos\phi \tag{1}$$
$$U = d\omega/dk \tag{2}$$
$$\phi = \theta - \gamma \tag{3}$$

where U is the conventional group speed, ω is the angular frequency, k is the wave number, θ is the wave group direction, and γ is the direction that the individual waves move within the group. The direction γ is given by Snell's law with phase velocity.

This concept can be used to explain a number of physical processes in coastal waters that cannot be explained on the basis of a single refraction law. For example, when both refraction laws are used together it is possible to show (Breeding, 1981b) that the widely occurring sinuous shoreline is part of an equilibrium beach form.

The primary objective in this paper is to determine the generation depths of recorded waves. A second objective is to demonstrate that under certain conditions good results can be obtained if the conventional group velocity is used as an approximation to the geometric group velocity. This approach can be highly desirable for many practical applications since it greatly simplifies the ray equations and the complexity of the computer program for ray tracing.

Wave Group Directions and Trajectories from Wave Data

The directions obtained in a directional-power spectral analysis of wave measurements are those of the individual waves. It is necessary to compute the corresponding wave group directions. If the water depth contours are parallel the wave group directions can be determined through the use of Snell's laws. For the wave group Snell's law can be stated

$$(\sin\theta_m)/G_m = (\sin\theta_g)/G_g \tag{4}$$

where the subscript m denotes the measurement site and the subscript g refers to the place of wave generation. Snell's law for the individual waves is given by

$$(\sin\gamma_m)/v_m = (\sin\gamma_g)/v_g \tag{5}$$

where $v = \omega/k$ is the phase speed. It is assumed that $\theta_g = \gamma_g$. When (4) and (5) are combined it is found that

$$\tan\theta_m = (J\sin 2\gamma_m)/[2(1 - J\sin^2\gamma_m)] \tag{6}$$

$$J = \left(v_g/U_g\right)\left(U_m/v_m\right) \tag{7}$$

The ratio of U_m/v_m is determined by the water depth at the measurement site for a given wave period. So θ_m is a function of γ_m and the ratio of U_g/v_g. For a given wave period the latter ratio depends on the water depth of wave generation. If this depth is known the wave group directions are readily determined. If the water depth where the waves are generated is not known, then this depth and the wave group directions can be determined together by trial and error for a range of wave periods.

The best way to illustrate how the method works is to consider some simulated examples. It will be assumed that water waves have been generated at a distant location where the water depth is 200 m. The directions of the individual waves are presumed to have been determined with high accuracy for a range of wave periods from data obtained at a water depth of 15 m near shore. The water depth contours are parallel. Now assume that the water depth where the waves were generated is unknown. The objective is to work backwards using the measured individual wave directions to determine the corresponding wave group directions and the water depth of wave generation. Equations (6) and (7) are used to calculate wave group directions at the measurement site for a range of wave periods and for trial values of water depths of wave generation. Back tracks produced from the measurement site will determine which trial value of water depth of wave generation is consistent with the wave source. In Figure 1a wave group back tracks are shown for wave periods of 7.7, 8.3, 9.1, 10.0. 11.1, 12.5, and 13.3 s for an assumed depth of wave generation of 50 m. Since the rays are seen to diverge and not go back together to a common source the trial value of generation depth must be incorrect. In Figure 1b the wave group directions at the measurement site were determined on the assumption that the waves were generated in water of 100 m depth. Although the results are better than in the previous example, they are not entirely consistent. The correct answer is obtained in Figure 1c where the assumed depth of wave generation is 200 m. The wave group back tracks are seen to go back together to a common source and to be parallel at the 200 m water depth. The wave group directions determined for this example using (6) and (7) also have to be correct.

Determination of Water Depths of Wave Generation from Field Data

Two field data sets will be used to further illustrate the theory and methodology for determining water depths of wave generation and wave group

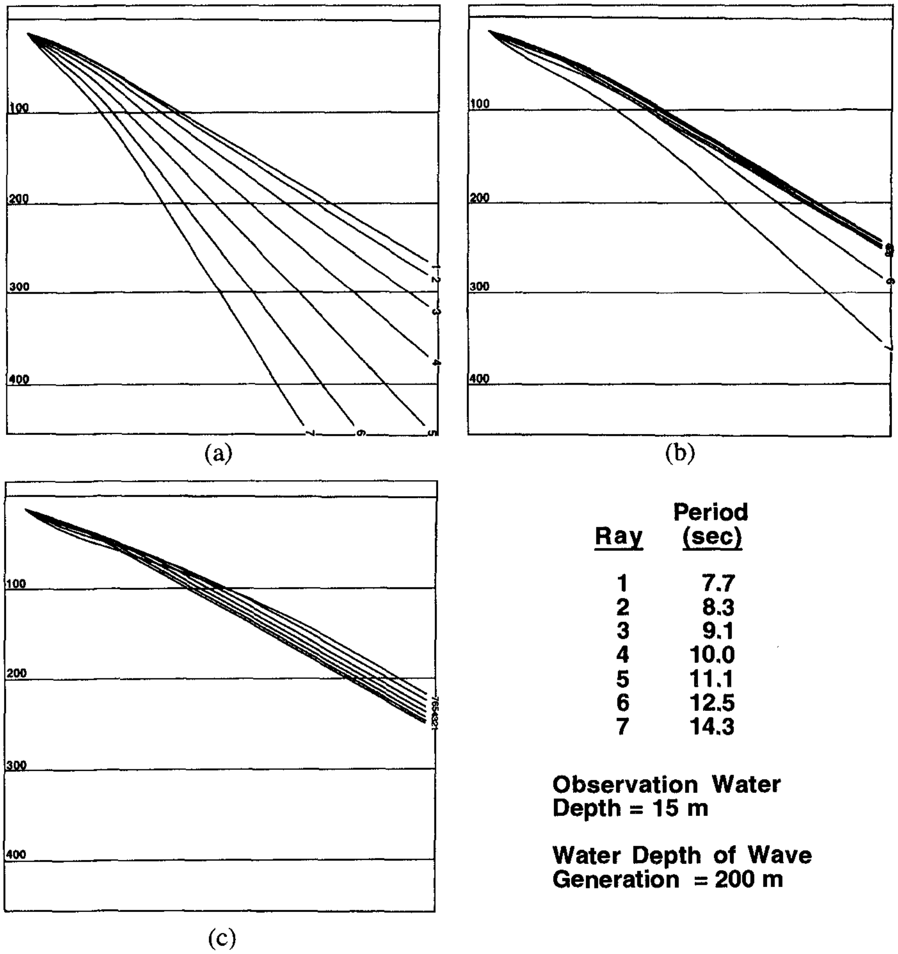

Figure 1. Wave group back tracks for assumed water depths of wave generation of (a) 50 m, (b) 100 m, and (c) 200 m. Soundings in meters.

trajectories. In both cases the waves were measured (Bennett, et al, 1964; Breeding, 1972) with an array of up to six pressure sensors on the sea floor in 31.7 m of water 17.7 km from shore near Panama City, Florida. In the array a pressure sensor was located at each corner of a pentagon of side 35.8 m, and these sensors were located 30.5 m from a sensor at the center of the array. Directional power spectra with 60 degrees of freedom were computed for time series records about 30 minutes in length by Bennett (1972) based on the method of Munk, et al (1963).

The water depth contours for the region of interest are shown in Figure 5. Over much of the region the water depth contours are approximately parallel. Accordingly, in determining wave group back tracks two approximations are made in the calculations. Firstly, the wave group directions at the measurement site are determined on the assumption that the water depth contours are parallel. And secondly, the conventional group velocity is used as an approximation to the geometric group velocity. The latter approximation can be made since Breeding (1981a) has shown for parallel water depth contours that G does not differ from U by more than 14%. The difference is usually much less. When G is replaced by U and the Snell's law equations are combined it is found that

$$\sin\theta_m = J\sin\gamma_m \tag{8}$$

This equation is used as an approximation to (6).

The wave group back tracks for the field data examples are determined using a ray curvature expression for nonparallel water depth contours based on the conventional group velocity **U**. Details are given in Breeding (1972). The approximations noted above greatly simplify the analysis.

Summer Storm

In the first example, waves measured on June 1, 1965 are considered. The directional power spectra computed from the waves are presented in Figure 2. This "3-D" representation is due to Bennett (1968). The bottom of each arrow gives both the frequency and bearing of the waves. Note that the frequency increases in the form of concentric circles moving outward from the center. The direction north (N) represents magnetic north. To obtain true north one adds 3 degrees. The tip of each arrow indicates the log power density of the waves at the depth of measurement. In this example the waves have small amplitudes.

In Figure 2 it can be seen that the directional power spectra near 0.1 Hz varies in a smooth fashion indicating that the waves are due to a common source.

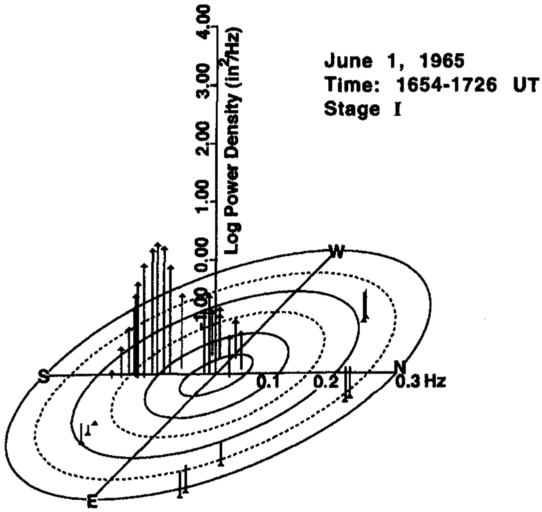

Figure 2. Directional power spectral for summer storm.

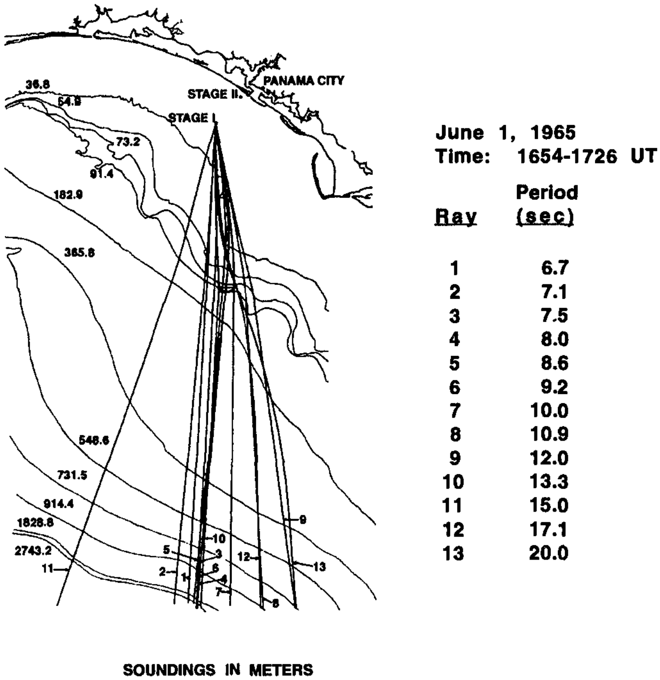

Figure 3. Wave group back tracks for summer storm.

The wave periods which are too short to be affected by refraction indicate the waves come from near south. Back tracks were produced for wave groups assuming that the waves had a deep water source and using (8) to determine the wave group directions at the measurement site. The resulting back tracks, which are shown in Figure 3, are generally consistent with a common source in the Caribbean Sea. All but two of the group rays go back through the Yucatan Channel, and only one misses badly.

Hurricane Betsy

In the second example, an investigation is made of waves measured in September 1965 due to Hurricane Betsy. The storm crossed the tip of South Florida and entered the Gulf of Mexico at about 1500 UT (Greenwich mean time) on September 8. After making a curved path the hurricane entered land at the Mississippi Delta near New Orleans, Louisiana at about 0400 UT on September 10. Over much of the storm path water waves were generated in intermediate water depths.

Figure 4 is an example of the directional power spectra (Bennett, 1972) for September 9 for the time period 1359 - 1430 UT. This is an example of a very energetic source.

The wave group back tracks and water depths of wave generation have to be determined by trial and error for each wave period. The general procedure is as follows. A water depth of wave generation is assumed as a trial value, the initial direction of the back track is calculated, and the ray traced. If the ray does not take a path which crosses water of the depth assumed the depth assumption must be rejected. If the location of the storm is known from meteorological records the range of possible depths is readily restricted. Once a few depths have been tried it is usually easy to decide which values to try next. Further, the answer found for one wave period must be consistent with the results found for other wave periods. After ray trajectories are drawn for the various wave periods a final check is made by comparing travel times computed along the paths to the region of wave generation. The computed travel times should agree with the known movement of the storm.

Equation (8) was used to determine the wave group directions at the measurement site. The corresponding back tracks and the water depths of wave generation are shown in Figure 5. The rays emanate from the measurement site, and they are continued until they reach either shore or a boundary of the water depth grid. Tick marks have been placed on the rays at five hour intervals of travel time. The course of Hurricane Betsy is indicated along the bottom of the illustration. The back tracks appear reasonable when viewed with respect to the hurricane path.

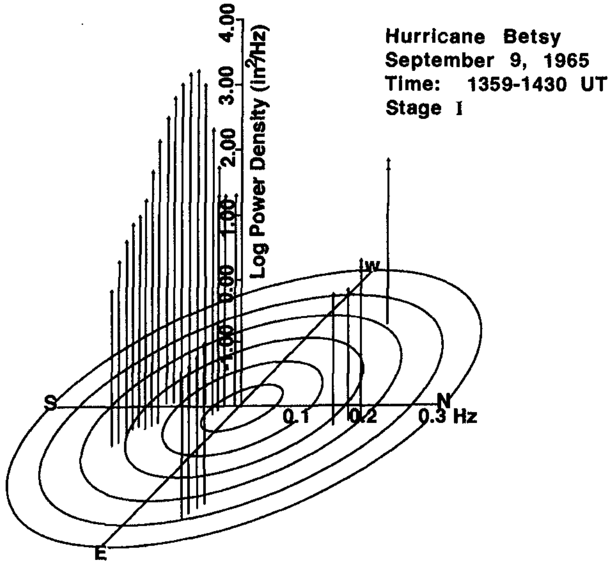

Figure 4. Directional power spectra for Hurricane Betsy.

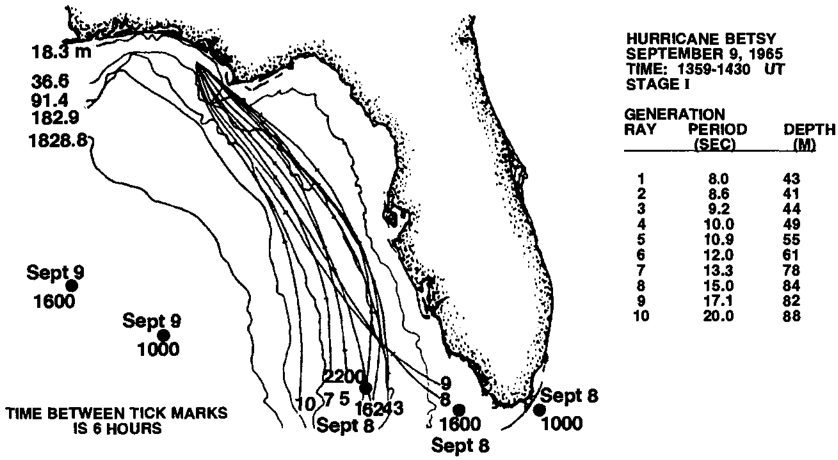

Figure 5. Wave group back tracks for Hurricane Betsy.

Consider, for example, ray number five which has a wave period of 10.9 s. Back tracking along this ray for fifteen hours gives water depths near the determined generation depth of 55 m. Timewise this corresponds to September 8 at 2300 UT. This time and location compare favorably with the record of the storm and its counterclockwise wind pattern.

It can be observed that portions of the wave group trajectories in Figure 5 are nearly parallel with the water depth contours. Although this would not be possible for monochromatic trajectories, it is not unusual for wave group trajectories. For wave group trajectories Breeding (1981a, 1986) has shown that the ray curvature goes to zero for rays parallel as well as perpendicular to the water depth contours.

Water depths of wave generation and wave group back tracks were also determined for the time periods 0800 - 0829 and 1816 - 1847 UT on September 9. The water depths of wave generation for the three wave measurement periods are presented in Table 1. For the first measurement period the waves were generated close to the Florida coast and were in the initial stages of development. For the second and third measurement periods the waves had become quite large. Note that for a given measurement time the water depths of wave generation generally increase with wave period. This result is what one would expect for dispersed gravity water waves due to a storm moving toward deeper water, as was the case. Observe further that for a given wave period the generation depths generally increase with the measurement recording time. This is clearly in agreement with the meteorological record.

Table 1. Water Depths of Wave Generation, 9 September 1965

Period (sec)	Time UT, Generation Depths in m		
	0800 - 0829	1359 - 1430	1816 - 1847
8.0	44	43	depth > 50
8.6	44	41	depth > 58
9.2	46	44	depth > 66
10.0	47	49	67
10.9	47	55	72
12.0	47	61	82
13.3	50	78	93
15.0	50	84	88
17.4	48	82	85
20.0	---	88	111

Monochromatic Trajectories

Monochromatic back tracks are presented in Figure 6 for comparison with the wave group back tracks shown in Figure 3 for the summer storm. The same individual wave directions were used to produce both results. The monochromatic rays were determined using the numerical method for nonparallel water depth contours developed by Wilson (1965). These back tracks are not at all consistent with a common wave source.

In Figure 7 monochromatic back tracks are shown for the same individual wave directions used to determine the wave group back tracks presented in Figure 5 for Hurricane Betsy. The Wilson (1965) program was used to determine the rays. It is seen that the monochromatic rays are in poor agreement with the path of the storm.

Conclusions

If the directions of the individual waves of a group are determined from wave data as a function of wave period, and if some simple refraction concepts are used for parallel water depth contours, it is possible to determine: (1) the wave group directions at the measurement site, (2) the wave group back tracks to their sources, and (3) the depths of water where the waves were generated. A wave example was presented where the apparent source of the waves was in deep water. Waves due to Hurricane Betsy (September 1965) were analyzed, and by determining the wave group back tracks and the corresponding water depths of wave generation, it was possible to track the general movement of the storm as it passed through intermediate water depths. For the cases considered reasonable results were obtained using the conventional group velocity as an approximation to the geometric group velocity. Finally, monochromatic wave back tracks are not consistent with the wave sources for the examples considered.

Acknowledgments

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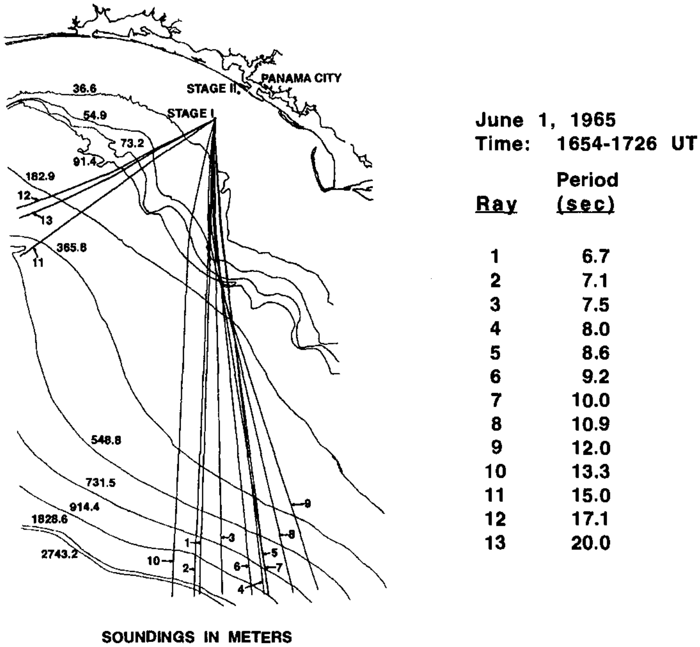

Figure 6. Monochromatic back tracks for summer storm.

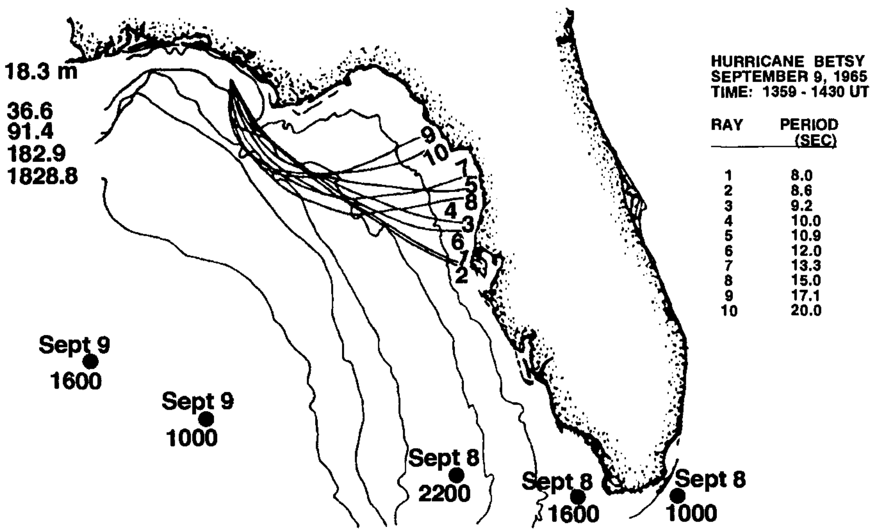

Figure 7. Monochromatic back tracks for Hurricane Betsy.

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