CHAPTER 51

COMPARISON OF DIRECTIONAL WAVE DATA QUALITY FROM TWO DIFFERENT MONITORING SYSTEMS

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ABSTRACT

A simple criterion was derived for evaluation and comparison of directional wave data quality from two underwater measuring systems – a point gage system consisting of a pressure transducer and a bi-axial current meter (PUV gage), and a slope array consisting of four pressure transducers. By using this criterion to the measured field data, it was demonstrated that directional wave data analyzed from PUV gage contain absolutely better quality than those from pressure array gages. Further examination of the pressure array data alone showed that the resolved directional wave quality was worser for long waves than short waves. However, the flaw of directional wave data quality from pressure array gages can be mended by forcing simple linear corrections on the analyzed directional data with a maximal tolerance to the criterion introduced in the present study.

1. INTRODUCTION

Directional wave data have been used widely in many coastal planning, designing, and operating projects. The data can be obtained from several sources: (1) measured directly in the coastal water, (2) measured at ocean and carried in numerically to the coastal area, (3) hindcast data, and (4) imitated data by numerical simulation. With no doubt, the measured directional data should be more representable to the real sea waves than the data from other sources. However, the usefulness of the measured directional data relies primarily on the quality and accuracy of the data being collected and analyzed.

Three basic types of measuring systems have been utilized today in finding directional wave information. They are: (1) a point gage system which will either measure the temporal changes of water surface slope in two horizontal principle directions, e.g., a pitch/roll buoy, or those of underwater horizontal wave orbital

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velocity vectors, e.g., a biaxial current meter, (2) a slope array which measures the spacial changes of water surface elevations in several cross directions, and (3) casting images by remote sensing. In general, the first two systems are more applicable in the offshore area while the last one is useful in the nearshore area. In terms of the analysis, a directional wave spectrum is usually estimated based on a truncated Fourier series solved from the measured data. Nonetheless, this estimated directional spectrum presents only a limited directional information since a true spectrum shall include an infinite long Fourier series.

Comparison of directional wave data collected from different measuring systems is essential in order to know if the data quality is influenced by a particular instrumental system. Carrying out this comparison, however, can be difficult without clear criteria or standards to evaluate the data. A recent study by Corson and McKinney (1991), who compared the analyzed directional spectra from three different monitoring systems including a PUV gage, a slope array, and an ocean buoy, has addressed the difficulty of comparing the measured spectra without knowing the true spectrum.

The present study evaluates and compares the quality of directional wave data analyzed from two underwater measuring systems – a point gage consisting of a pressure transducer and a bi-axial current meter (PUV gage), and a slope array consisting of four pressure transducers. Evaluating directional wave data quality was carried out by examining the Fourier coefficients which are used to estimating a directional spectrum rather than a simple inspection of the spectrum itself.

2. BACKGROUND THEORY

A general but unique expression of directional spectrum, $E(f,\phi)$, is

$$E(f,\phi) = E(f)H(f,\phi), \quad (f = \text{frequency}, \ \phi = \text{direction})$$

where $E(f)$, the one-dimensional frequency spectrum, and $H(f,\phi)$, a directional distribution function, satisfy the following conditions:

$$E(f) = \int_0^{2\pi} E(f,\phi)\mathrm{d}\phi; \quad \int_0^{2\pi} H(f,\phi)\mathrm{d}\phi = 1, \quad \text{and} \quad H(f,\phi) \geq 0.$$

In terms of a Fourier Series,

$$H(f,\phi) = \frac{1}{\pi}[\frac{1}{2} + \sum_{n=1}^{\infty} a_n(f)\cos n\phi + \sum_{n=1}^{\infty} b_n(f)\sin n\phi],$$

where $$a_n(f) = \int_0^{2\pi} H(f,\phi)\cos n\phi\mathrm{d}\phi, \quad b_n(f) = \int_0^{2\pi} H(f,\phi)\sin n\phi\mathrm{d}\phi$$

define the dimensionless, frequency-dependent Fourier coefficients.

It can be easily proved that $|a_n(f)| \leq 1$ and $|b_n(f)| \leq 1$ by using the facts of $|\cos n\phi| \leq 1$, $|\sin n\phi| \leq 1$, and integrating the quantities of $H(f,\phi)\cos n\phi$ and $H(f,\phi)\sin n\phi$, respectively, over the entire directional domain of $|\phi| \leq \pi$. However, a more rigid constraint on $a_n(f)$ and $b_n(f)$ also exits as a consequence of that $a_n(f)$ is related to $b_n(f)$ through the extant of $H(f,\phi)$. Since $(\cos n\phi \pm \sin n\phi)^2 = 1 \pm \sin 2n\phi \leq 2$, or $|\cos n\phi \pm \sin n\phi| \leq \sqrt{2}$, it can be shown by first multiplying $H(f,\phi)$ to both sides and then integrating over entire ϕ domain that

$$\text{(i)} \qquad |a_n(f) \pm b_n(f)| \leq \sqrt{2}.$$

Now, for any real numbers of a and b, there is a binomial inequality:

$$-(a-b)^2 \leq 4\,ab \leq (a+b)^2.$$

Let $a = a_n(f)$, $b = b_n(f)$, and use the result from (i), it is further shown that

$$\text{(ii)} \qquad |a_n(f) b_n(f)| \leq \frac{1}{2}.$$

Combining the results in (i) and (ii), and using the fact that $a_n(f)$ and $b_n(f)$ are resemblant in functional form, yields

$$\text{(iii)} \qquad a_n^2(f) + b_n^2(f) \leq 1.$$

3. EVALUATION OF a_n(f) AND b_n(f) COEFFICIENTS

The methods used in evaluating $a_n(f)$ and $b_n(f)$ coefficients are different for the data collected by the PUV gage system and pressure array gages. Since these methods have been well developed and documented in the past, only the results from these methods are summarized here.

3.1 Submerged PUV Gage(1 pressure transducer and 1 biaxial currentmeter)Data

Based on a standard stochastic approach, only the first 2 pairs of $a_n(f)$ and $b_n(f)$ can be determined from the PUV data (Cartwright, 1963; Long, 1980):

$$a_1(f) = \frac{R_{pu}(f)}{\sqrt{R_{pp}(f)[R_{uu}(f) + R_{vv}(f)]}}, \quad b_1(f) = \frac{R_{pv}(f)}{\sqrt{R_{pp}(f)[R_{uu}(f) + R_{vv}(f)]}},$$

$$a_2(f) = \frac{R_{uu}(f) - R_{vv}(f)}{R_{uu}(f) + R_{vv}(f)}, \qquad b_2(f) = \frac{2\,R_{uv}(f)}{R_{uu}(f) + R_{vv}(f)},$$

where $R_{xy}(f)$ is the measured cospectrum of the random variables of $X(t)$ and $Y(t)$. The subcripts p, u, and v are corresponding to the measurements of underwater dynamic pressure, and two orthogonal horizontal wave orbital velocity components, respectively.

3.2 Submerged Pressure Transducer Array Data

By means of a standard stochastic approach, the cross spectrum of simultaneous measurements of water surface elevation from two horizontally spaced gages can be expressed as (Borgman, 1969)

$$E_{1,2}(f) = R_{1,2}(f) + iQ_{1,2}(f) = \int_0^{2\pi} E(f,\phi)\exp[ikD_{1,2}\cos(\phi-\beta_{1,2})]\mathrm{d}\phi$$
$$= E(f)\{J_o(kD_{1,2}) + 2\sum_{n=1}^{\infty}[a_n(f)\cos n\beta_{1,2} + b_n(f)\sin n\beta_{1,2}](i)^n J_n(kD_{1,2})\},$$

where $i=\sqrt{-1}$ denotes the imaginary unit, k is the wave number, $Q_{1,2}(f)$ is the quadrature spectrum presenting the imaginary part of $E_{1,2}$, $D_{1,2}$ is the distance between two gages 1 and 2, $\beta_{1,2}$ is the angle of the vector from Gages 1 to 2, and

$$J_n(z) = \frac{1}{\pi(i)^n}\int_0^{\pi}\exp[iz\cos\phi]\cos n\phi \mathrm{d}\phi$$

is the Bessel function of the first kind of order n. For N gages, a total of $2(N,2) = N(N-1)$ equations are available for evaluation of $a_n(f)$ and $b_n(f)$ coefficients. These equations are

$$\frac{R_{jm}}{E(f)} = J_o(kD_{jm}) + 2\sum_{n=1}^{M}(-1)^n J_{2n}(kD_{jm})[a_{2n}(f)\cos 2n\beta_{jm} + b_{2n}(f)\sin 2n\beta_{jm}],$$
$$\frac{Q_{jm}}{E(f)} = 2\sum_{n=1}^{M}(-1)^{n-1} J_{2n-1}(kD_{jm})[a_{2n-1}\cos(2n-1)\beta_{jm} + b_{2n-1}\sin(2n-1)\beta_{jm}],$$

where $j,m = 1,2,\cdots N$, $j \neq m$ and $M = N(N-1)/4$. Here, M is the total number of pairs or harmonics of $a_n(f)$ and $b_n(f)$ existing in the above equations. This method of evaluating $a_n(f)$ and $b_n(f)$ is also valid on the source data from submerged pressure measurements instead of water surface elevations by calculating the cross spectrum and one-dimensional frequency spectrum from the measured pressure data.

In practice, it is preferable to solve $a_n(f)$ and $b_n(f)$ less than M harmonics. This is because that solving a finite number of $a_n(f)$ and $b_n(f)$ from a set of cross spectrum equations can be contaminated by the possible existence of directional wavelets higher than M harmonics. The contamination is deemed to be more severe to $a_n(f)$ and $b_n(f)$ with higher harmonics than lower harmonics being solved from the cross spectrum equations. Now, to solve for a less number of $a_n(f)$ and $b_n(f)$ than M harmonics, a least squared method can be used. This is the case of solving $\{x\}$ in the matrix system of $[A]_{m\times n}\cdot\{x\}_{n\times 1} = \{y\}_{m\times 1}$, with $m > n$ (more equations than unknowns). Utilization of a least squared method, which minimizes the quantity of $|[A]\{x\} - \{y\}|^2$, leads to the solution:

$$\{x\} = ([A]^T[A])^{-1}\cdot([A]^T\{y\}), \text{ where } [A]^T = \text{the transpose of } [A].$$

Table 1: Coastal wave monitoring stations (1995).

Station	Depth(m)	Gage Type	Operating Period
Cape Canaveral	8.5	PUV	Oct.,Nov.
		P-array	Feb.-Apr.
Palm Beach	3.5	PUV	Apr.-Jun.
Miami	6.5	PUV	May-Jun.
		P-array	Jan.,Feb.,Apr.
			Nov.-Dec.

4. FIELD EXPERIMENTS

Wave data for use in the present study were measured from a number of PUV gages and pressure transducer arrays deployed in three coastal stations at Cape Canaveral, Palm Beach, and Miami Beach, Florida, in 1995. The data were collected four times daily, each containing a 20-minutes of measurements with a sampling rate of 1 Hz. The employed PUV gage is a submerged point gage system consisting of a pressure transducer and a bi-axial current meter. The slope array consists of four pressure transducers fixed by an aluminum tripod anchored on the sea floor. The geometry of the slope array is an equilateral triangle with one pressure transducer located at the center and three others at each corner of the triangle. This slope array setup is known as the star array geometry using four pressure transducers. The distances between the center and side transducers, and between any two side transducers, are 2m and 3.4m, respectively (Figure 1). Figure 2 shows the locations of wave monitoring stations and Table 1 lists the water depth, type of gage, and operating period of the stations.

5. COMPARISON OF DIRECTIONAL WAVE QUALITY

Directional wave data quality from both PUV and slope array data measurements was evaluated by examining $a_n(f)$ and $b_n(f)$ which are used in the estimation of directional spectrum rather than a simple inspection of the spectrum itself. The comparison of directional data quality is carried out for $a_1(f)$, $b_1(f)$, $a_2(f)$, and $b_2(f)$, based on the criterion of $a_n^2(f) + b_n^2(f) \leq 1$ shown earlier in Condition (iii). Table 2 presents a summary of the result of evaluation and comparison of these data. It is seen that directional data obtained from a PUV gage have absolutely better quality than those from a slope array. The worser quality of the latter is caused by the aliasing of higher directional wave modes to a finite number of $a_n(f)$ and $b_n(f)$ solved based on the slope array data. Figure 3 shows the histograms of directional data quality, based on the data satisfying the criterions of $a_1^2(f) + b_1^2(f) \leq 1$ and $a_2^2(f) + b_2^2(f) \leq 1$ over the frequency domain of $0 \leq f \leq 0.32$ Hz, for all the slope array data collected from Cape Canaveral and

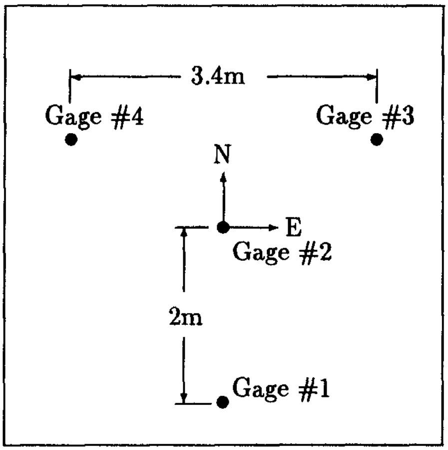


Figure 1: The star array geometry of 4 pressure gages.

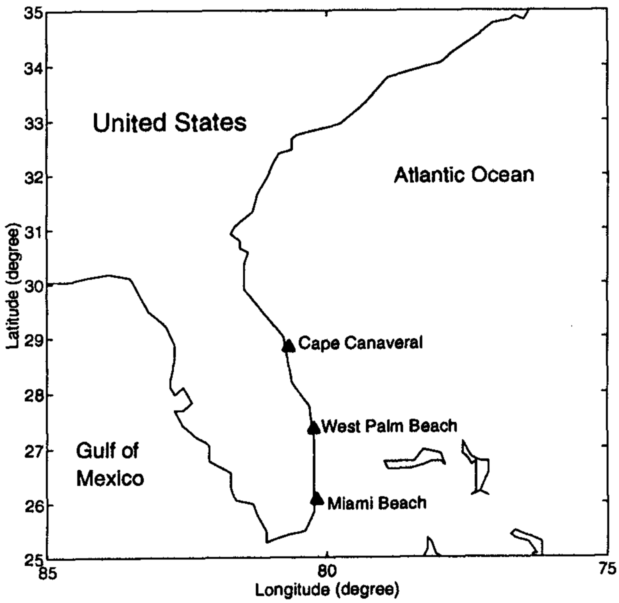


Figure 2: Location of field wave monitoring stations.

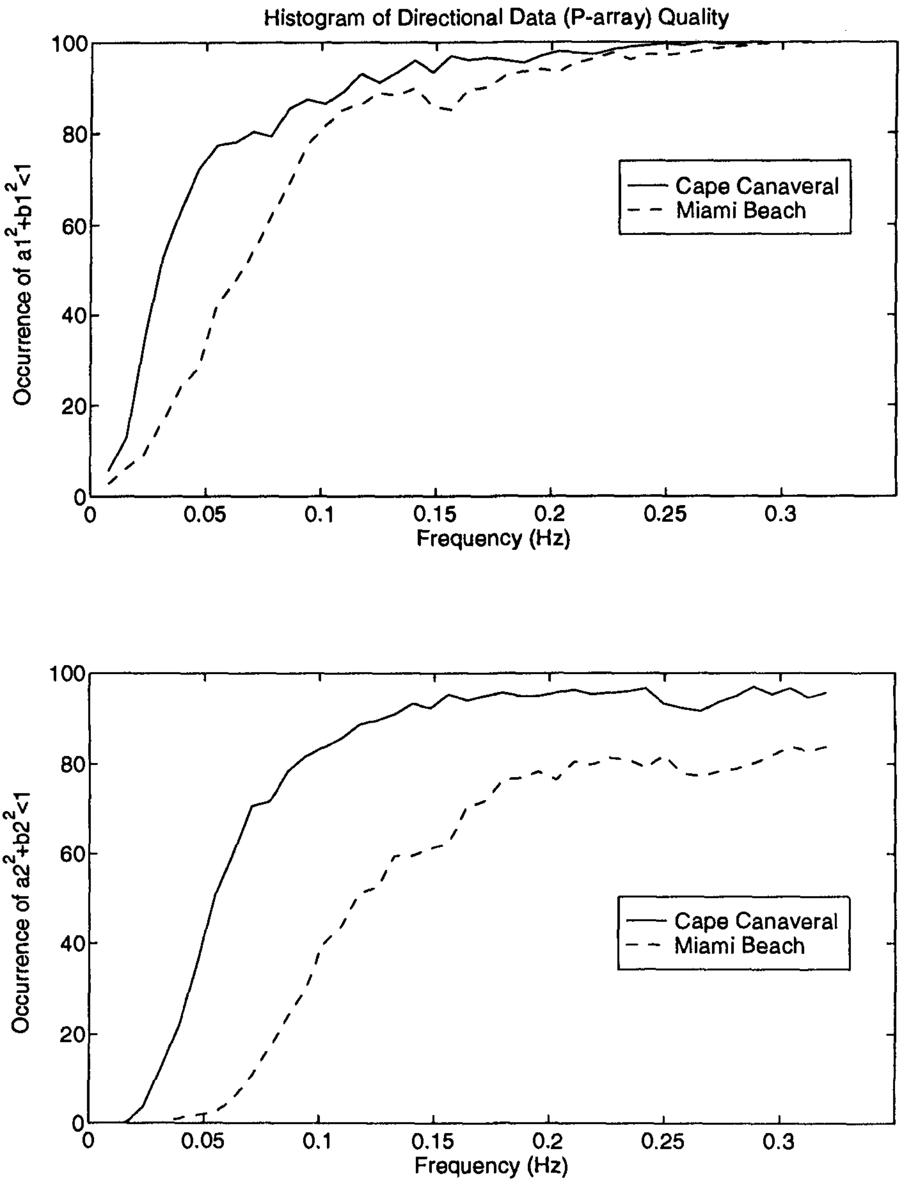


Figure 3: Histograms of directional wave data quality versus frequency.

Table 2: Comparison of directional data quality based on $a_n^2(f) + b_n^2(f) \leq 1$, $n = 1, 2$.

Month (1995)	Percent of Data Passing Criterion				
	Cape Canaveral		Palm Beach	Miami Beach	
	P-Array	PUV	PUV	P-Array	PUV
Jan.				58 %	
Feb.	87 %			52 %	
Mar.	88 %				
Apr.	86 %		100 %	83 %	
May			100 %		100 %
Jun.			100 %		100 %
Oct.		100 %			
Nov.		100 %		64 %	
Dec.				51 %	
Summary	87 %	100 %	100 %	62 %	100 %

Miami Beach stations. The analyzed results of slope array data show that $a_2(f)$ and $b_2(f)$ have much worser quality than $a_1(f)$ and $b_1(f)$. The results further indicate that all of these coefficients, regardless of $a_1(f)$, $b_1(f)$ or $a_2(f)$, $b_2(f)$, show overall worser quality for longer waves than shorter waves. Therefore, directional wave data quality measured from slope array gages is expected to be worser in shallow water than in deep water.

In fact, the quality of directional data measured from a slope array depends also on the total number of pressure gages employed and displacement geometry of the gages in the array. However, the degree of improvement of directional wave quality from the use of more gages in a slope array will always be affected by the finite number of $a_n(f)$ and $b_n(f)$ solved from the data.

Although the slope array measurements are seen to yield less satisfactory directional data quality, the analyzed data can be modified according to the criterion of $a_n^2(f)+b_n^2(f) \leq 1$ when the criterion is violated. A simple modification is proposed in the present study to multiply a common factor of γ to both $a_n(f)$ and $b_n(f)$ such that

$$\text{(iv)} \qquad (a_n)' = \beta\, a_n, \;\; (b_n)' = \beta\, b_n, \quad \beta = \frac{\gamma}{\sqrt{a_n^2 + b_n^2}},$$

where γ has a magnitude between 0 and 1. Again, without knowing the true directional spectrum, it is not possible to find γ. However, it is clear that γ approaching to 1 indicates a narrower directional band while γ approaching to 0 implies a broader directional distribution. Therefore, when Condition (iii) is violated, the sea is likely to have a quite narrower distribution of directional waves and the correction factor of γ shall have a value close to its upper limit of 1. A constant value of $\gamma = 0.99$ has been adopted in the present study for modifying the less perfect directional data quality from the pressure transducer arrays.

An example is presented here comparing the directional spectra analyzed for the data collected at midnight of 95/04/12 by a PUV gage from West Palm Beach station and a pressure transducer array system from Cape Canaveral station. At this particular time, waves may have reached to an equilibrated state under relatively strong winds with consistent magnitude and direction within the interval of $\pm$ 12 hrs (Figure 4). Figures 5 and 6 present the computed one-dimensional frequency spectra and Fourier coefficients of $a_1(f)$, $b_1(f)$, $a_2(f)$, $b_2(f)$ from the measured data. It is seen that all of these Fourier coefficients computed from PUV gage data satisfy Criterion (iii) whereas those from pressure transducer array can violate the same criterion. In order to compute the directional spectrum, the coefficients of $a_1(f)$, $b_1(f)$, $a_2(f)$, $b_2(f)$ determined based on the pressure transducer array data were modified according to the equations presented in (iv) if they violate Condition (iii). The computation of directional spectrum is, based on a Maximum Entropy approach (Kim *et al.*, 1993),

$$E(f,\phi) = E(f) \cdot \exp[-\lambda_o - \lambda_1 \cos\phi - \lambda_2 \sin\phi - \lambda_3 \cos 2\phi - \lambda_4 \sin 2\phi],$$

where λ_i, the Lagrange's multipliers, can be approximated by

$$\begin{aligned}
\lambda_1 &= 2a_1a_2 + 2b_1b_2 - 2a_1(1 + a_1^2 + b_1^2 + a_2^2 + b_2^2), \\
\lambda_2 &= 2a_1b_2 - 2b_1a_2 - 2b_1(1 + a_1^2 + b_1^2 + a_2^2 + b_2^2), \\
\lambda_3 &= a_1^2 - b_1^2 - 2a_2(1 + a_1^2 + b_1^2 + a_2^2 + b_2^2), \\
\lambda_4 &= 2a_1b_1 - 2b_2(1 + a_1^2 + b_1^2 + a_2^2 + b_2^2),
\end{aligned}$$

and

$$\lambda_o = \ln[\int_{-\pi}^{\pi} \exp(-\lambda_1 \cos\phi - \lambda_2 \sin\phi - \lambda_3 \cos 2\phi - \lambda_4 \sin 2\phi) d\phi].$$

Figure 7 compares the directional spectra computed based on the PUV and pressure transducer array data. The computed spectra show that the associated wave systems are mainly moving westward against the coastal shore. The comparison shows that the two computed spectra are very similar in size and shape expect that the one measured from Cape Canaveral station has the spectral tail twisted more towards NW direction than the spectrum from West Palm Beach station. This twisting in spectral direction is due to the effect of wave refraction in shallow water. The refraction effect is stronger for the spectrum measured at West Palm Beach station where the water depth is much shallower than Miami station.

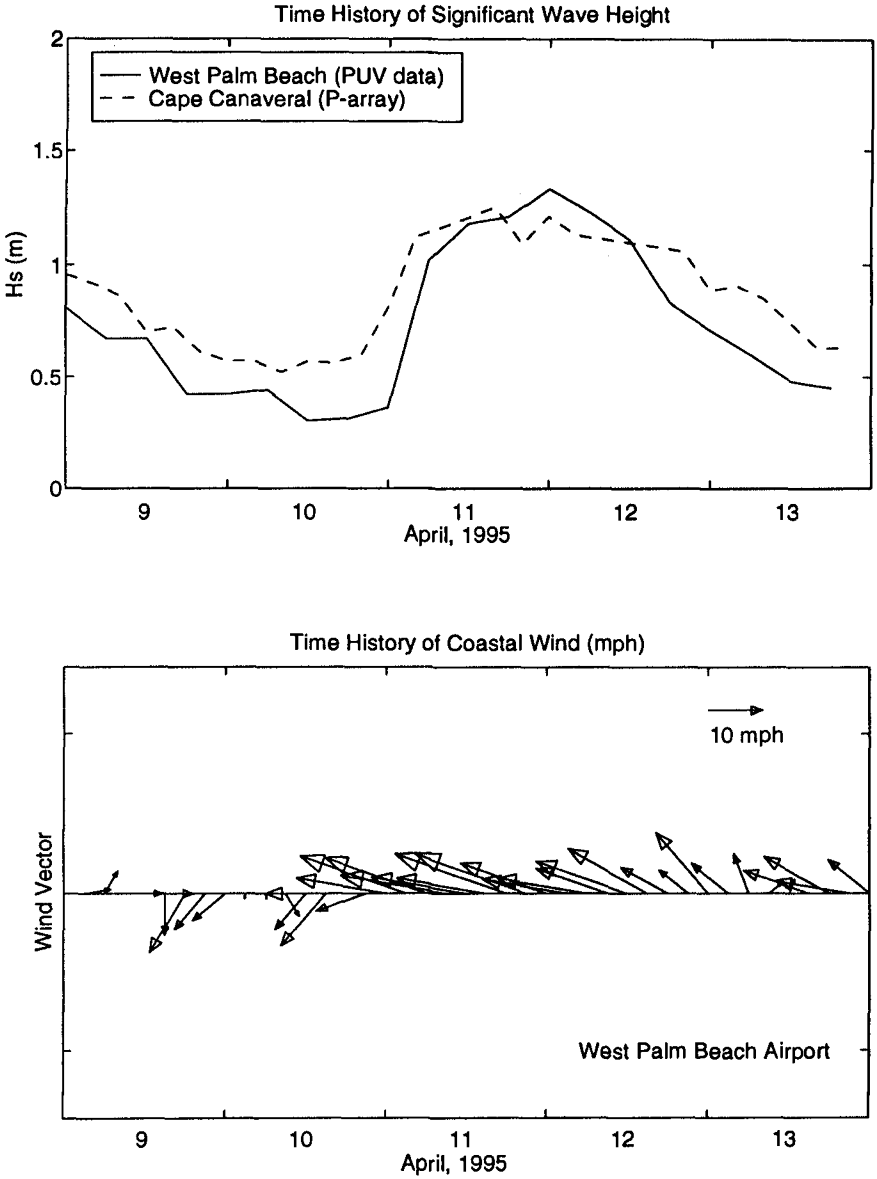


Figure 4: Measured significant wave heights and surface winds.

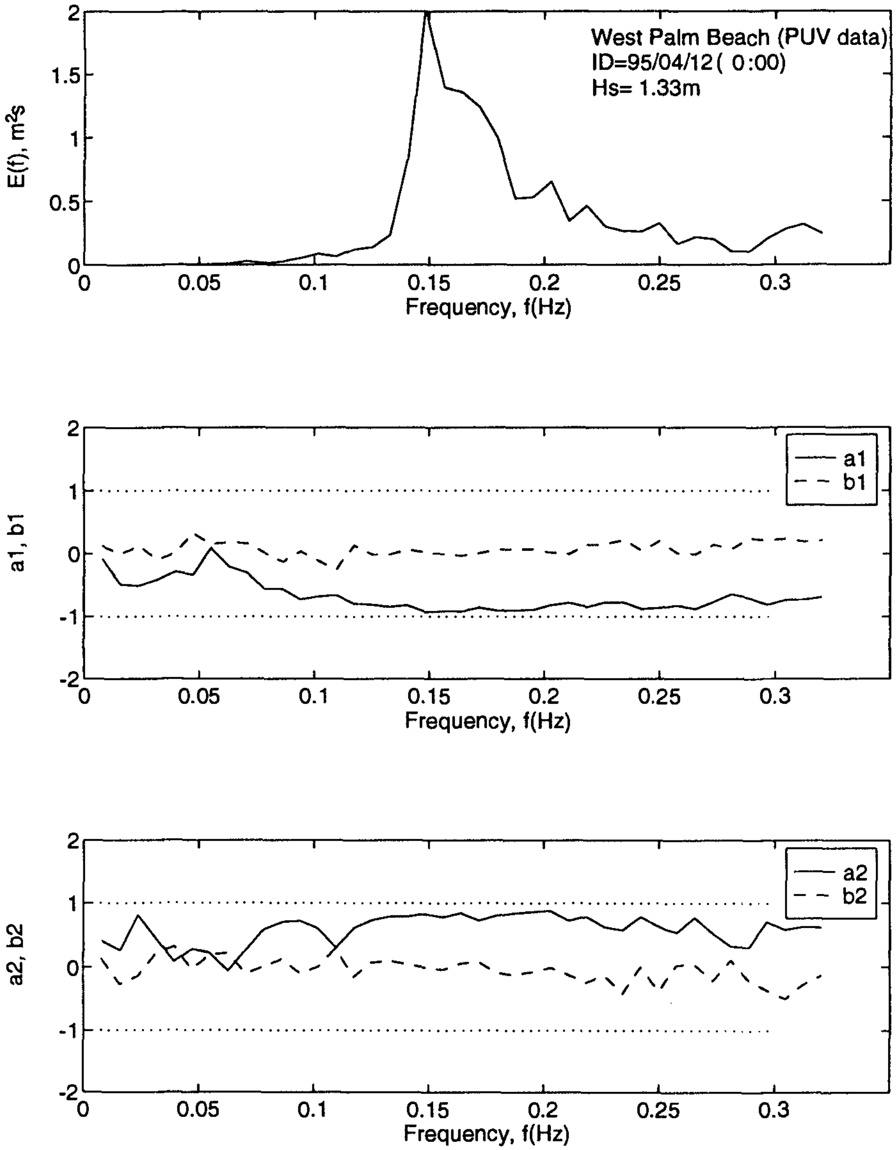


Figure 5: Wave information analyzed from West Palm Beach gage data.

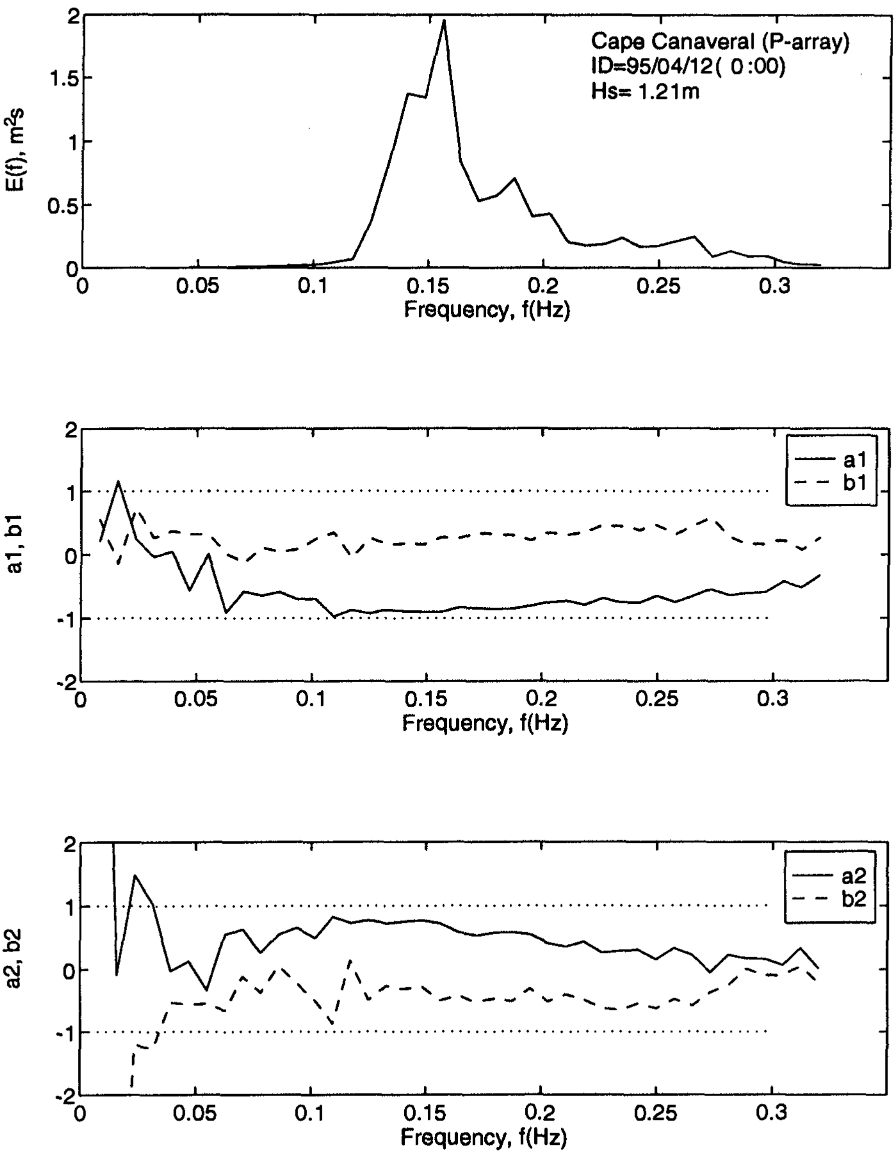
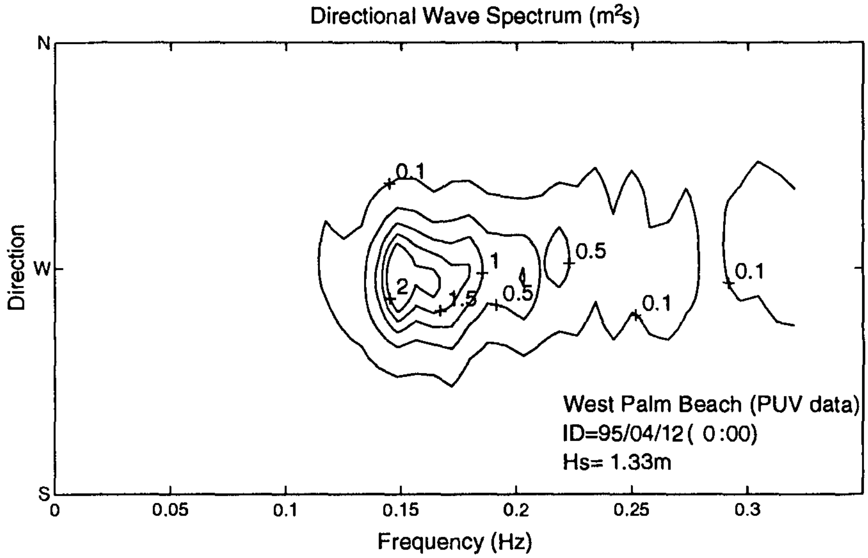


Figure 6: Wave information analyzed from Cape Canaveral gage data.



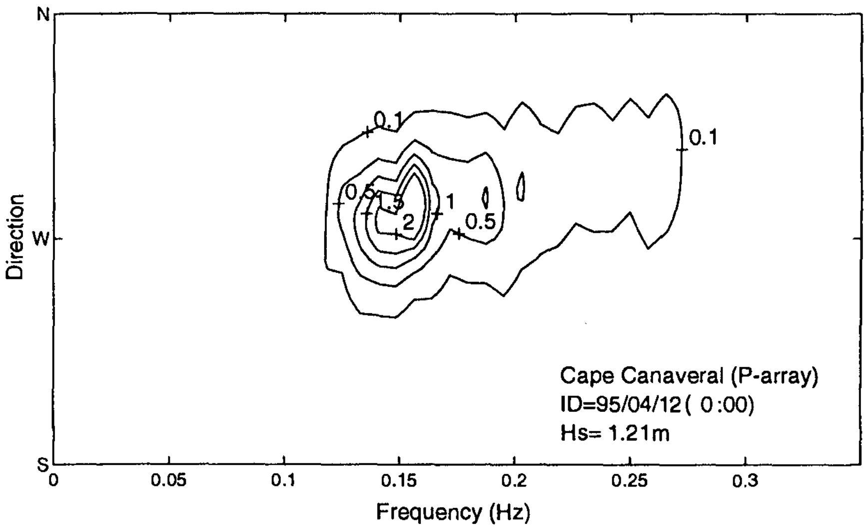


Figure 7: Comparison of analyzed directional wave spectra.

6. CONCLUSION

The conclusion drawn from this study is summarized as follows:

(1) Directional sea wave data need to satisfy the condition:

$$a_n^2(f) + b_n^2(f) \leq 1.$$

(2) The analyzed directional wave data based on PUV measurements show absolutely better quality than those based on P-array data (100% vs 75%), according to the criterion presented in (iii).

(3) The analyzed pressure gage array data show that the coefficients of a_2, b_2 have much worser quality than a_1, b_1. Nevertheless, all of these coefficients show worser quality for longer waves in the range of shallow water condition.

(4) Corrections to the slope array directional wave data from

$$(a_n)' = \beta\, a_n, \quad (b_n)' = \beta\, b_n, \quad \text{with} \quad \beta = \frac{\gamma}{\sqrt{a_n^2 + b_n^2}}, \quad \gamma = 0.99$$

have shown to yield directional spectra similar to those from PUV gage data.

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