CHAPTER 40

Kinematic Predictions in Large Shallow Water Waves.

Susan Smith[1] & Christopher Swan[2].

Abstract.

The present paper concerns the description of extreme waves in shallow water, and in particular contrasts the success of two recently developed wave theories (Sobey, 1992 and Baldock and Swan, 1994) to model the maximum water particle velocities arising beneath a highly nonlinear and transient (or unsteady) wave event. To achieve these comparisons exact numerical calculations were undertaken using the time-stepping procedure outlined by Dold and Peregrine (1984). Comparisons between this numerical "data" and the kinematics models confirm the importance of both the non-linearity and the unsteadiness, and further suggest that the nature of the wave-wave interactions has important implications for the accuracy of the kinematics predictions. In particular, the local Fourier series solution (Sobey, 1992) is shown to be unable to model the global wave frequency-difference terms, and therefore tends to over-predict the fluid velocities beneath the still water level. In contrast, the double Fourier series solution (Baldock and Swan, 1994) explicitly incorporates both the nonlinearity and the unsteadiness of the wave, and typically provides a good description of the flow field beneath the still water level. However, this solution is limited in terms of the total number of Fourier components included, and consequently the largest velocities arising close to the water surface are typically under-predicted in the largest wave events. Nevertheless, both kinematics models provide a significant improvement over the existing design solutions.

Introduction.

Within both coastal and offshore engineering the identification of an appropriate wave climate represents a fundamental input into the design process. If,

[1]Research student & [2]lecturer, Department of Civil Engineering, Imperial College, London, SW7 2BU. United Kingdom.

as is usually the case, design calculations are required to assess the applied fluid loading, the stability and transport of bed material, or the potential for over topping, a second and equally important aspect of the design involves the prediction of the water particle kinematics. In respect of this latter point current design procedures are either based upon a non-linear steady wave theory, or an unsteady linear theory. In the first of these cases the wave model incorporates the non-linearity of the wave motion, but neglects the unsteadiness or the transient nature of the sea state. Indeed, within this category the most commonly applied model is based upon a fifth-order Stokes' solution (Fenton, 1985). In this case a large wave is characterised by a representative wave height (H) and a wave period (T), and it is assumed that the wave forms part of a regular wave train which propagates without change of form. Alternatively, a higher-order stream function solution (Dean, 1965) may also be applied in which the wave profile is either specified in terms of H and T, or the series solution is matched to a measured surface profile, $\eta(t)$, using some form of constrained minimisation. However, in both applications of the stream function solution the wave is assumed to propagate without change of form, and thus the dispersive properties are neglected.

In contrast, a linear random wave theory recognises the fact that a realistic sea state comprises a large number of wave components, many of which may be described as free waves and thus travel at a phase velocity (c) which is only dependent upon the wave period (T) and the water depth (d). Solutions of this type fall into the second category (noted above) in that they partially incorporate the unsteadiness of the sea state, but are restricted in the sense that they neglect the nonlinearity. Indeed, in solutions of this type all terms of order a^2k^2 and above (where a is the wave amplitude and k is the wave number) are neglected. As a result, such solutions make no attempt to model the nonlinear wave harmonics which include both the higher-order Stokes' terms and the nonlinear wave-wave interactions arising between two or more frequency components (Longuet-Higgins and Stewart, 1960).

Unfortunately, it is now widely recognised that the largest waves within a sea state are both nonlinear and unsteady. As such, they do not arise as part of a regular wave train, but occur as individual events within a random, or irregular, sea state. If one considers a typical broad-banded spectrum in deep water, frequency dispersion provides a plausible explanation for the evolution of the largest wave events. In this case the extreme events are produced by constructive interference when a large number of wave crests arise at one point in space and time. Indeed, recent field measurements undertaken at the Tern Platform (Rozario et al, 1993) have confirmed the validity of this argument, and have demonstrated that the most probable shape of the largest wave is closely related to the auto correlation function of the underlying wave spectra (Tromans et al, 1991).

However in intermediate and shallow water depths the evolution of the largest waves is perhaps more complicated. For example, although frequency dispersion provides an effective method of focusing wave energy in all but the shallowest water depths (kd< 0.4), large waves may also be related to changes in the local bathometry (Peregrine, 1983); the interaction with uniform and depth-varying currents (Swan, 1990, 1992); or the occurrence of side-band instabilities (Benjamin and Feir, 1969). Although these effects are entirely separate, they will each produce extreme events which are both highly non-linear and unsteady. As a result, it is clear that the present design solutions do not model the fundamental characteristics of the largest wave events. Indeed, previous work presented by Baldock and Swan (1994) and Baldock, Swan and Taylor (1996) has considered large deep water waves, and has demonstrated that the underlying kinematics can only be predicted if both the nonlinearity and the unsteadiness is taken into account. However, the relative importance of these effects in shallow water remains unclear. For example, recent experimental observations (Baldock and Swan, 1996) have shown that the transfer of energy to the nonlinear harmonics is strongly dependent upon the water depth. However, these results are primarily concerned with intermediate water depths, and only contrast the measured data with one fully nonlinear unsteady wave model. The present paper will address this point, and will consider the extent to which recently developed wave theories are able to model extreme shallow water waves. In particular, exact numerical calculations based on the time-stepping procedure outlined by Dold and Peregrine (1984) will be used to contrast the kinematic models presented by Sobey (1992) and Baldock and Swan (1994).

Kinematics Models.

In recent years the possible inadequacy of the existing design solutions has been recognised, and in particular two alternative wave models have been proposed. In both cases these models are based upon a time-history of the water surface elevation measured at one spatial location, $\eta(t)$, and are thus appropriate to design calculations. In particular, they represent a significant improvement over the existing design solutions in that they attempt to incorporate both the nonlinearity and the unsteadiness of the wave motion. The first model, proposed by Sobey (1992) is based upon a local Fourier series approximation originally outlined by Fenton (1992). Within this solution the time-history of the water surface elevation, $\eta(t)$, is sub-divided into a large number of local windows within which the surface profile is assumed to be locally steady. This does not imply that the entire wave form propagates without change of form, merely that a small section of the wave profile is locally steady during the time interval that it passes the measuring point. If (x, z) represent the usual Cartesian co-ordinates in which z=0 corresponds to the still water level, and z=-d represents the bottom boundary, a velocity potential ϕ may be defined within each window such that:

$$\phi(x,z,y) = C_E x + \sum_{j=1}^{J} A_j \frac{\cosh(jk(d+z))}{\sinh(jkd)} \sin(jkx - j\omega t) \tag{1}$$

where C_E is the co-flowing Eulerian current, ω is the fundamental wave frequency (or $2\pi\backslash T$, where T is the corresponding wave period) and k is the fundamental wave number (or $2\pi\backslash\lambda$, where λ is the wave length). The constant coefficients A_j (for j = 1 to J) thus represent the amplitudes of the individual harmonics which are determined by minimising the errors in the free surface boundary conditions. Within this solution the input parameters correspond to the upper limit of the series expansion (or the truncation order J), the local window length (τ), and the time interval between adjacent data describing the surface profile (Δt). Although the minimisation procedure is somewhat involved, and the outcome strongly dependent upon the input parameters, the scheme is computationally efficient, and suitable for use on a standard personal computer.

In contrast, the wave solution proposed by Baldock and Swan (1994) explicitly includes both time and space dependence. This solution is based upon a double Fourier series solution first proposed by Lambrakos (1981), and adopts a velocity potential defined by:

$$\phi(x,z,t) = \sum_{m=1}^{M}\sum_{n=1}^{N} \cosh k(z+d)\left[A_{nm}\cos(k_n x - \omega_m t) + B_{nm}\sin(k_n x - \omega_m t)\right] \tag{2}$$

In this case the proposed solution incorporates a total of M wave numbers each with M frequency components (where typically N = M), such that a total of 2NM wave harmonics are included within the overall solution. If T_1 defines some large fundamental period, over which the solution is assumed periodic, the fundamental wave frequency is defined by $\omega_1 = 2\pi\backslash T_1$, and the corresponding wave number (k_1) is calculated using the linear dispersion equation. Having identified these values the solution matrix (k_n, ω_m) is assembled from wave numbers and frequency components which are integer multiples of the fundamental:

$$k_n = nk_1, \quad \omega_m = m\omega_1 \tag{3}$$

The final solution is again based upon a time-history of the water surface elevation measured at one spatial location (i.e. ξ (t) at x = 0). In effect the unknown coefficients A_{nm} and B_{nm}, together with the unknown surface profiles at all other spatial locations (i.e. η (t) at $x \neq 0$), are determined by a minimisation of the error in the nonlinear free surface boundary conditions arising at all points in space and time. Provided the fundamental period is sufficiently large relative to the dominant (or peak) wave period, an appropriate mix of free waves and bound waves are generated. As a result, the final wave solution deforms in both space and time, and has to date been shown to provide a good description of the wave-induced water particle kinematics arising in deep water (Baldock and Swan, 1994).

To assess the effectiveness of these solutions in intermediate and shallow water depths, we have sought to compare the predicted water particle velocities with exact numerical calculations based upon the time-stepping procedure outlined by Dold and Peregrine (1984). Previous studies (Johannessen and Swan, 1996) have shown that these and other time-stepping procedures (notably Longuet-Higgins and Cokelet (1976), Fenton and Rienecker (1980) and Craig and Sulem (1993)) are in near-perfect agreement with laboratory data, and thus provide an excellent benchmark with which to test potential design solutions. At this stage it should be noted that these time-stepping solutions, although exact, are not appropriate for typical design calculations since they are based upon initial conditions which require a spatial representation of both the water surface elevation and the velocity potential at the water surface (i.e. $\eta(x)$ and $\phi(x)$ at $z=\eta$). This information is seldom available in a laboratory study, and never available from field data.

Test Conditions.

To assess the effectiveness of the wave solutions three test cases were considered in which an extreme wave group was produced in a water depth of d=0.7m, 0.3m, and 0.15m respectively. In each case the underlying frequency spectrum was broad-banded (at a laboratory scale), and consists of a total of 29 frequency components which were each of equal amplitude and equally spaced within the period range $0.6 \leq T \leq 1.4$s. Expressed in terms of an energy spectrum within the frequency domain this distribution of wave components has a peak frequency at approximately 0.71Hz, and decays according to ω^{-4}. In each of the three test cases the initial conditions used to commence the time-stepping procedure were such that the wave energy was fully dispersed within the spatial domain. On this basis a linear wave theory was used to provide both the surface elevation $\eta(x)$ and the magnitude of the velocity potential at the surface ($\phi(x)$ at $z=\eta(x)$). However, the relative phasing of the wave components was such that after a large number of time-steps (typically larger than 100) frequency dispersion produced a focusing of wave energy such that a large highly nonlinear wave event was generated at one point in space and time. The numerically predicted horizontal velocities, $u(z)$, arising beneath this large wave event were chosen as an appropriate yard-stick with which to judge the success of the kinematics models discussed previously.

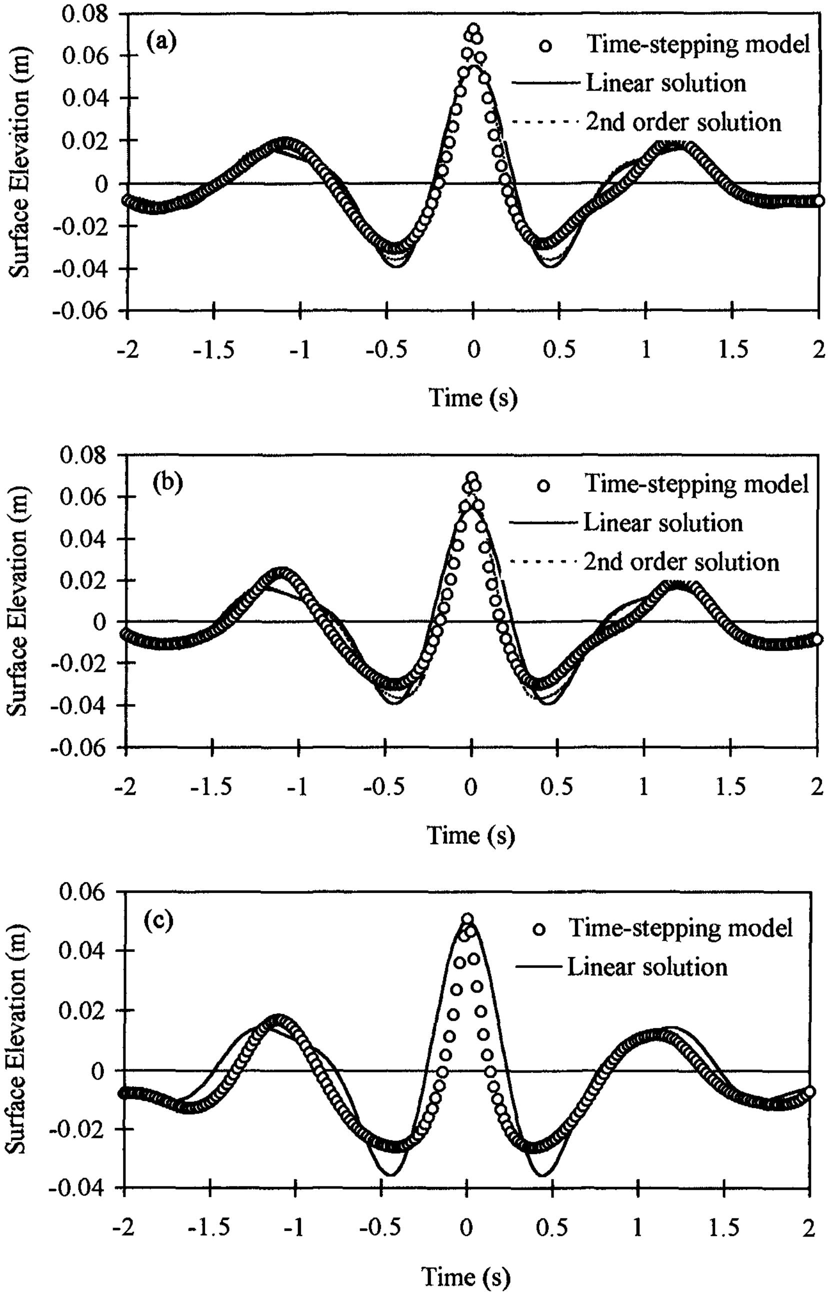
Discussion of Results.

Figures 1a-1c concern the broad-banded spectrum ($0.6s \geq T \geq 1.4s$) in the three different water depths (d = 0.7m, 0.3m, and 0.15m respectively), and describes the time-history of the water surface elevation, $\eta(t)$, predicted at the spatial location at which the maximum crest elevation arises. In each of these cases the exact numerical calculations (following Dold and Peregrine, 1984) are compared to both a linear wave solution and a second-order approximation based upon the sum of the interactions identified by the Longuet-Higgins and Stewart (1960). These latter two models are based upon the twenty nine free waves which were initially implemented within the time-stepping procedure, and are thus very different to either a linear random wave theory or a hybrid second-order solution (Stansberg, 1993) based upon a Fourier analysis of the measured water surface elevation. To facilitate comparisons of this type the time base on the horizontal axis has been shifted such that (for each wave solution) the maximum crest elevation arises at t=0. Comparisons of this type, and in particular the difference between the exact numerical calculations and the linearly predicted results, demonstrate the importance of the nonlinear wave-wave interactions. For example, in deep water conditions (d=0.7m) figure 1a suggests that a linear solution underestimates the maximum crest elevation by approximately 25%; while the inclusion of the second-order terms only accounts for approximately 30% of the total nonlinear effects. These results are consistent with deep water data presented by Baldock and Swan (1994).

However, comparisons with figures 1b and 1c clearly suggest that the nature of the nonlinear contribution differs significantly with the water depth. For example, in figure 1b (corresponding to a water depth of 0.3m) the nonlinear increase in the maximum crest elevation is reduced to approximately 20%; while the second-order interactions account for approximately 50% of this increase. Furthermore, if one considers the shallow water depth (d=0.15m in figure 1c) there is almost no nonlinear increase in the maximum crest elevation when compared to a linear theory based upon the component free waves. However, the maximum crest elevation (relative to linear theory) does not alone provide an appropriate measure of the nonlinearity of a wave form. Indeed, if one considers the steepness of the wave profiles outlined in figures 1a-1c, there is almost no change depending on the water depth.

An explanation for this effect lies in the nature of the nonlinear wave-wave interactions and, as will be indicated below, these also have implications for the ability of the nonlinear wave models to accurately predict the water particle kinematics. If one considers the second-order solution originally outlined by Longuet-Higgins and Stewart (1960), the nonlinear wave-wave interactions may be subdivided into two categories. Firstly, there are the so-called frequency-sum terms which correspond to a transfer of energy into the short wave or high frequency components. These may be interpreted as representing local nonlinear wave-wave interactions, and are responsible for increases in the local energy density as well as the increase in the maximum crest elevation. In contrast, the second category of interactions are represented by the so-called frequency-difference terms. These correspond to a transfer of energy into the long wave, or low frequency, wave components. These interactions may be interpreted as representing the global changes which are, in particular, responsible for the set-down beneath the wave group.

Before considering the relative magnitude of these varying nonlinear wave-wave interactions, we will first contrast the ability of the nonlinear kinematics models discussed previously (namely Sobey, 1992 and Baldock and Swan, 1994) to

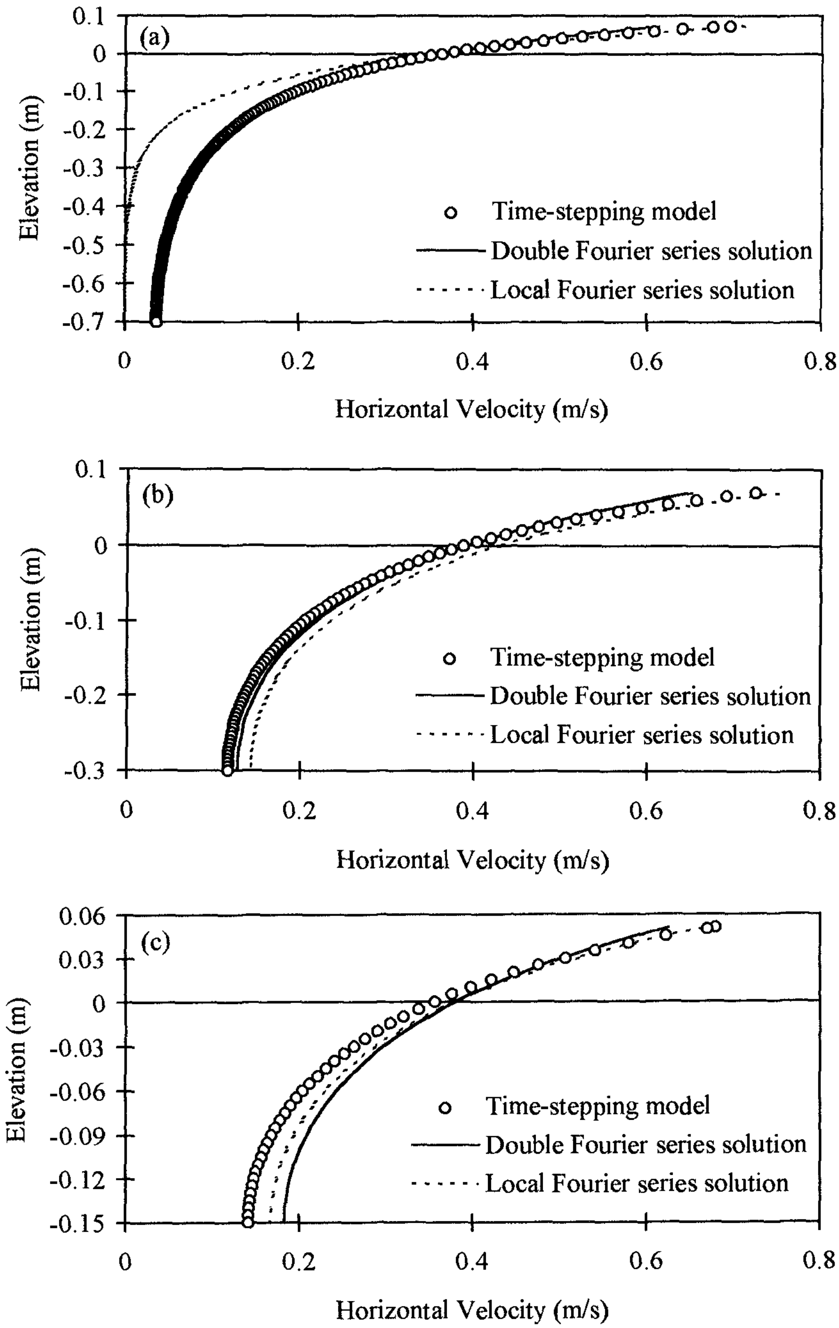


Figures 1a-1c. Time-histories of the water surface elevation, $\eta(t)$.
(a) Deep water: d=0.7m (b) intermediate water: d=0.3m (c) shallow water: d=0.15m.

reproduce the exact numerical calculations provided by Dold and Peregrine (1984). Figures 2a-2c again relate to the broad-banded spectrum in three different water depths, and describe the depth variation in the maximum horizontal velocity, u(z), arising beneath the largest wave crest (i.e. t=0 in figures 1a-1c). Since the kinematics models are both based upon a least squares fit to the predicted nonlinear surface profile, it may be assumed that in all cases the surface profile, $\eta(t)$, is well represented by these models. Indeed, if this were not the case the underlying kinematics could not be confidently predicted. However, despite near-perfect agreement with the surface profile, the predicted kinematics show considerable variation. For example, in the deep water case (d=0.7m on figure 2a) the double Fourier series solution (Baldock and Swan, 1994) provides a very good description of the nonlinear kinematics up to the still water level, but slightly underestimates the maximum velocities within the vicinity of the water surface. In contrast, the local Fourier series solution (Sobey, 1992) provides an excellent description of the kinematics within the wave crest, but wildly underestimates the horizontal velocities beneath the still water level.

The reasons for these discrepancies are clear. Firstly, the double Fourier series solution is based upon a fifteenth order approximation (or N=M=15 in equation 2), and thus incorporate a total of 225 harmonics. However, since the peak of the input spectrum arises at N=M=3, the accuracy of the solution is approximately equivalent to a fifth-order Stokes' solution. Since the wave events under consideration are extremely steep (within 5% of their breaking limit), it is not at all surprising that this level of approximation underestimates the near-surface crest kinematics. Unfortunately, the nature of this solution is such that small increases in the order of the approximation (i.e. N=M=18) provides little by way of increased accuracy, but has a large effect upon the computational requirements. Indeed, significant increases in the order of the approximation become practically unrealistic. In contrast, the local Fourier series solution is only fitted to a small section of the wave profile in any one window, and in the vicinity of the largest crest the curvature of the surface profile is such that the solution locates a large proportion of the wave energy in the high frequency range. As a result, it obtains a good description of the surface profile and the near-surface kinematics, but the predicted velocities decay too rapidly with depth (as is consistent with high frequency wave components), and the solution thus underestimates the kinematics at all points beneath the still water level.

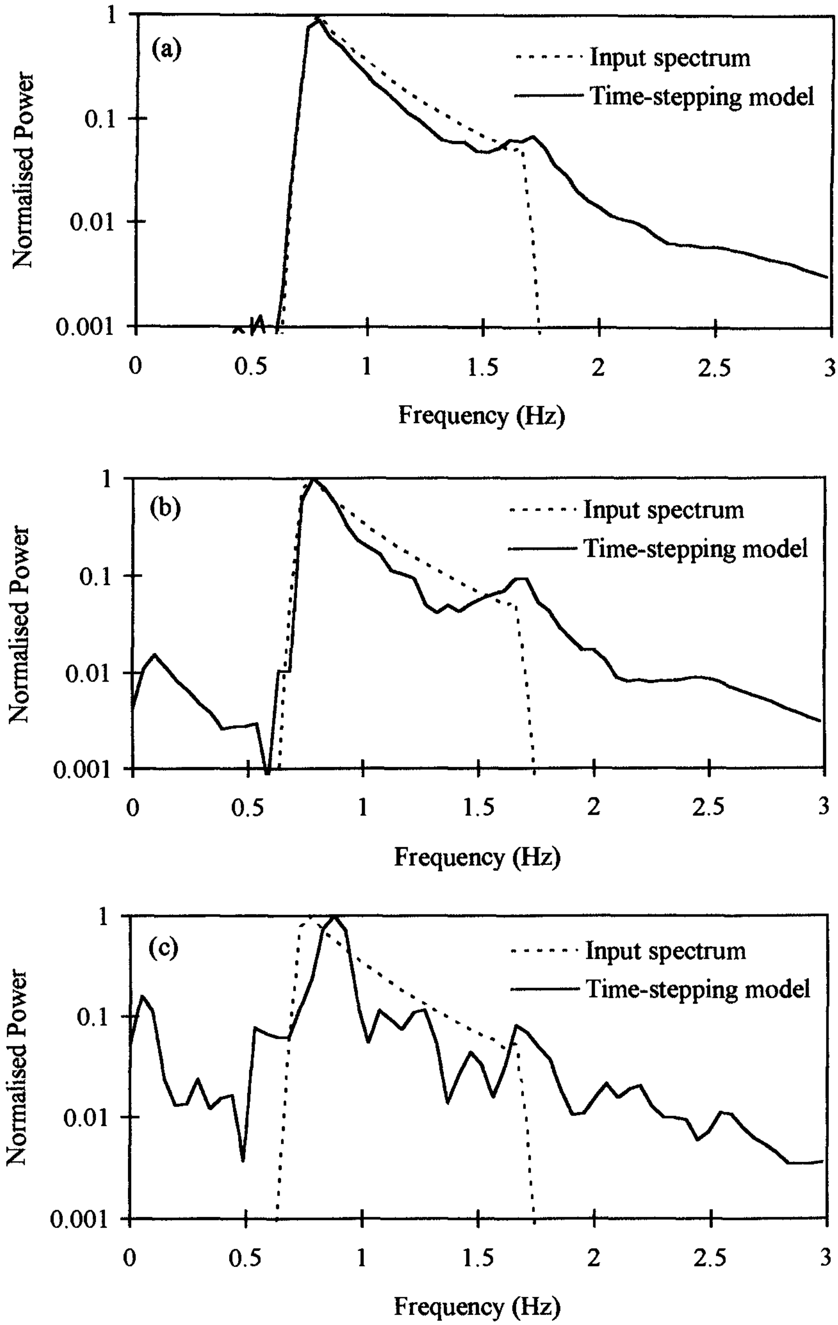
A similar pattern of results is also presented in figures 2b and 2c corresponding to water depths of d=0.3m and 0.15m respectively. In each of these cases (and indeed many other cases which have also been investigated) the local Fourier series solution provides a good description of the maximum horizontal velocities arising close to the water surface. However, with increasing depth beneath the surface, the description becomes poor. In contrast, the double Fourier series solution provides reasonable agreement over most of the water column, but consistently under-predicts the largest velocities arising close to the water surface.



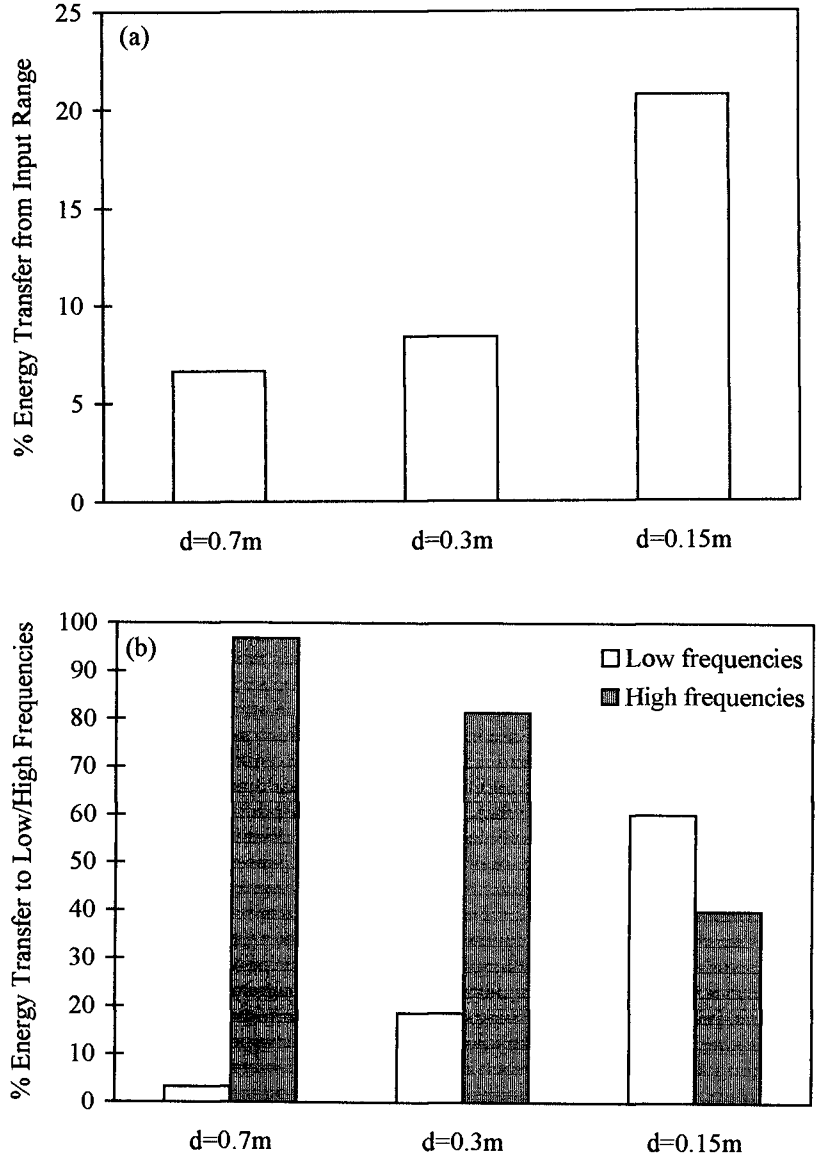
Figures 2a-2c. Horizontal velocities beneath the wave crest, u(z).
(a) Deep water: d=0.7m (b) intermediate water: d=0.3m (c) shallow water: d=0.15m.

To explain the varying success of these solutions, and in particular its dependence on the water depth, the normalised power spectra derived from a Fourier transform of the numerically predicted water surface elevations are presented on figures 3a-3c. In the deep water case (figure 3a) it is clear that the dominant nonlinear interactions generate high frequency or frequency-sum terms which in turn produce a large increase in the local energy density in the vicinity of the large wave crest. This is consistent with the experimental study presented by Baldock, Swan and Taylor (1996), and explains the large local increase in the maximum water surface elevation. However, as the water depth reduces it becomes apparent that there is a larger transfer of energy into the low frequency, or frequency-difference, terms. These results are consistent with the analysis of measured wave records presented by Mansard et al (1988), and with recent laboratory data presented by Baldock and Swan (1996). The relative importance of the total nonlinearity within the numerically calculated wave profiles is further considered in figure 4a. Within this figure each water depth has been considered separately, and the total transfer of energy out of the input range (ie. $0.6 \leq T \leq 1.4$ or $4.5 \leq \omega \leq 10.5$) has been determined from the wave spectra given in figures 3a-3c. At first sight these results are perhaps unexpected since it suggests that the shallow water case (d=0.15m) represents the most nonlinear wave form, at least defined in terms of energy transfers, despite the fact that the maximum crest elevation in this case shows no significant increase above the linearly predicted value (figure 1c). However, the reasons for this are clarified in figure 4b where the relative importance of the local and global nonlinearity is addressed. In this case the transfer of energy into the high and low frequencies are expressed as a percentage of the total energy transferred out of the input range for each water depth. These results clearly suggest that in deep water the local nonlinearity is dominant; while a reduction in the water depth leads to an increase in the importance of the frequency-difference terms, and a reduction in the importance of the frequency-sum terms. As a result, in the shallow water case (d=0.15m) the transfer of energy into the frequency-difference terms (or global interactions) is larger than the transfer of energy into the frequency-sum terms (or local interactions).

This variation in the nature of the nonlinearity explains the success (or otherwise) of the kinematics models. For example, the double Fourier series solution is based upon a best fit to a measured (or in this case predicted) time-history of the water surface elevation which includes several complete wave forms. As such, it is able to resolve both the local and the global nonlinearity, and is only restricted in the sense that the total number of harmonics is small when compared to the peak of the wave spectrum. As a result, this solution will give satisfactory results provided the transfer of energy to either the high frequencies or the low frequencies is not too large. For example, if the local nonlinearity is very strong the double Fourier series solution will typically underestimate the maximum horizontal velocity arising close to the water surface (figure 2a); while if the global nonlinearity is excessive, this solution will typically underestimate the set-down beneath the wave group, and will therefore tend to overestimate the horizontal velocities arising close to the bottom boundary (figure 2c). In contrast, the local Fourier series solution is fitted to a small section of



Figures 3a-3c. Normalised power spectra based on predicted η(t).
(a) Deep water: d=0.7m (b) intermediate water: d=0.3m (c) shallow water: d=0.15m.



Figures 4a-4b. Energy transfers arising due to nonlinear wave-wave interactions. (a) Energy transferred out of the input range. (b) Relative importance of the high and low frequencies.

the water surface profile within an individual window. As a result, it can successfully model the high frequency, or local nonlinearity, since this will determine the local curvature of the surface profile. However, given the input in any one window, it is unable to resolve the global nonlinearity, since the window length is significantly shorter than the length-scale appropriate to the set-down beneath the wave group. As a result, in those cases where the global nonlinearity is important, one would typically expect the local Fourier series solution to overestimate the velocities beneath the still water level (figures 2b and 2c).

Concluding Remarks.

The present paper has considered the description of highly nonlinear and unsteady (or transient) wave events, and has demonstrated that the nature of the nonlinear wave-wave interactions is strongly dependent upon the water depth. In particular, in deep water the exact numerical calculations indicate that the local nonlinear wave-wave interactions, corresponding to the frequency-sum terms, are dominant. However, as the water depth reduces the global nonlinearity, represented by the frequency-difference terms, become progressively more important. This result has important implications for the prediction of the underlying kinematics. For example, a double Fourier series solution (Baldock and Swan, 1994) is limited in respect of the total number of Fourier components which can practically be included; while the local Fourier series solution is unable to model the global non-linearity and thus becomes less effective in shallow water. However, despite these reservations, it should be stressed that these models will typically provide an improved description of the water particle kinematics when compared to the existing design solutions which either ignore the unsteadiness of the wave form, or the nonlinearity. In particular, the double Fourier series solution can accurately incorporate both the increased crest-trough asymmetry due to the local nonlinearity, and the set-down beneath the wave group due to the global nonlinearity, provided the extreme wave event is not too close to its breaking limit.

Acknowledgements

The authors gratefully acknowledge the assistance provided by Professors Peregrine and Sobey for allowing access to their respective numerical codes, and for the financial support provided by the UK Engineering and Physical Sciences Research Council (EPSRC) via the Marine Technology Directorate Ltd.

References.

Baldock, T.E. and Swan, C. 1994. Numerical calculations of large transient water waves. *Applied Ocean Research*, **16**: 101-112.

Baldock, T.E. and Swan, C. 1996. Extreme waves in shallow and intermediate water depths. *Coastal Engineering*, **27**: 21-46.
Baldock, T.E., Swan, C. and Taylor, P.H. 1996. A laboratory study of non-linear surface waves on water. *Philos. Trans. Royal Society, London*. Ser. A, **354**: 1-28.
Benjamin, T.B. and Feir, J.E. 1967. The disintegration of wavetrains on deep water. Part 1. Theory. *J. Fluid Mech.*, **27**: 417-430.
Craig, W., and Sulem, C. 1993. Numerical simulation of gravity waves. *J. Comp. Phys.* **108**: 73-83.
Dean, R.G. 1965. Stream function representation of non-linear ocean waves. *J. Geophys. Research*, **70**: 4561-72.
Dold, J.W. and Peregrine, D.H. 1984. Steep unsteady waves: an efficient computational scheme. *Proc. 19th Int. Conf. on Coastal Engineering*, ASCE, **1**: 955-67.
Fenton, J.D. 1985. A fifth order Stokes' theory for steady waves. *J. Waterways Port Coastal & Ocean Engineering*, ASCE, **111**: 216-234.
Fenton, J.D. 1992. Polynomial approximations and water waves. Proc. 23rd. Inter. Conf. Coastal Eng. **1**: 193-207. Venice 1992.
Fenton, J.D. and Rienecker, M.M. 1982. A Fourier method for solving nonlinear water wave problems: applications to solitary wave interactions. *J. Fluid Mech.*, **118**: 411-443.
Johannessen, T.B. & Swan, C. 1996. Numerical calculations of large unsteady water waves. Part 1: Comparisons with laboratory data. Submitted to Appl. Ocean Res.
Lambrakos, K.F. 1981. Extended velocity potential wave kinematics. *J. Waterways Port Coastal & Ocean Engineering*, ASCE, **107**: 159-174.
Longuet-Higgins, M.S. and Stewart, R.W. 1960. Changes in the form of short gravity waves on long waves and tidal currents. *J. Fluid Mechanics*, **8**: 565-583.
Longuet-Higgins, M.S. and Cokelet, E.D. 1976. The deformation of steep surface waves on water. A numerical method of computation.. *Proc. Roy. Soc.* Ser. A. **350**: 1-26.
Mansard, E.P.D., Sand, S.E. and Klinting, P. 1988. Sub and super-harmonics in natural waves. *J. Offshore Mechanics and Arctic Engineering*, **110**: 270-217.
Peregrine, D.H., 1983. Wave jumps and caustics in the propagation of finite-amplitude water waves. *J. Fluid Mechanics*, **136**: 435-452.
Rozario, J.B., Tromans, P.S., Taylor, P.H. and Efthymiou, M. 1993. Comparisons of loads predicted using NewWave and other wave models with measurements on the Tern structure. *Advances in underwater tech., ocean sci., & offshore engng.* **29**: 143-58.
Sobey, R.J. 1992. A local Fourier approximation method for irregular wave kinematics. *Applied Ocean Research*, **14**: 93-105.
Stansberg, C.T. 1993. Second-order numerical reconstruction of laboratory generated random waves. Proc. 12th. OMAE conf., Glasglow, Scotland. **2**:143-51.
Swan, C. 1990. An experimental study of waves on a strongly sheared current profile. Proc. 22nd Inter. Conf. Coastal Eng. **1**: 489-502. Delft. 1990.
Swan, C. 1992. A stream function solution for waves on a strongly sheared current. Proc. 23rd. Inter. Conf. Coastal Eng. **1**: 684-697. Venice 1992.
Tromans, P.S., Anaturk, A. and Hagemeijer, P. 1991. A new model for the kinematics of large ocean waves - application as a design wave. *Proc. 1st Int. Offshore & Polar Engng. Conf.* ISOPE.