CHAPTER 20

Models of Wave Height and Fraction of Breaking Waves on a Barred Beach

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Abstract
    Models for wave height and the fraction of breaking waves were developed; the models employ a wave-by-wave approach, in which the shoaling, breaking and reforming of individual waves are calculated. The performance of the models calibrated with experimental data was not satisfied; the fractions of breaking waves estimated by the models were smaller than the values measured over troughs in the field. The models therefore were calibrated and verified with the field data. Furthermore, the validity of the models calibrated with the field data was confirmed by comparison with large-scale experiment data.

Introduction
    The fraction of breaking waves, defined as the ratio of the number of breaking/broken waves to the total number of waves, strongly affects various phenomena in the surf zone, such as nearshore currents, sediment suspension, and morphology changes. This is because various surf zone phenomena are mainly caused by turbulence, mass flux and momentum flux induced by breaking/broken waves, which are much greater than those induced by non-breaking waves. For example, Kuriyama (1994) carried out numerical simulations to show that the longshore current distribution over a longshore bar and trough is dependent on the cross-shore distribution of the fraction of breaking waves. Hence, to predict longshore current velocities as well as undertow velocities, suspended sediment concentrations and topography changes, it is essential to estimate accurately the fraction of breaking waves Q_b.
    Several models have been proposed to estimate Q_b and wave height H. Battjes and Janssen (1978) simulated variations in H within the surf zone, assuming a modified Rayleigh distribution truncated at the breaking wave height H_b, where breaking and broken waves have the same value of H_b. According to the assumption, the value of Q_b is estimated with the root-mean-square wave height H_{rms} and H_b. Battjes and Stive (1985), Roelvink (1993) and Southgate and Nairn (1993) compared Q_b estimated by Battjes and Janssen's model with results of field and laboratory measurements on

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planar beaches. Although the estimated values of Q_b were smaller than those measured, the estimated cross-shore distributions of Q_b qualitatively agreed with the measurements.

Thornton and Guza (1983) improved Battjes and Janssen's model; an unmodified Rayleigh distribution, not truncated at the breaking wave height, was assumed at every location inside and outside the surf zone and wave breaking was allowed to occur at any wave height. The value of Q_b was taken to be a function of H, H_{rms} and water depth h. They measured Q_b on a planar beach in the field, and found that Q_b calculated by their model agreed well with the measured values.

Recently, Dally (1992) simulated variations in wave height by applying another algorithm, a wave-by-wave approach, in which the shoaling, breaking and reforming of individual waves are calculated. Because the mode of wave breaking of an individual wave (breaking, broken, or non-breaking) is clarified at any point through the calculation for the individual wave, Q_b can be directly estimated as the ratio of the number of breaking/broken waves to the total number of waves. By comparing calculated results with those measured over a longshore bar in the field by Ebersole (1987), Dally showed that Q_b calculated at the seaward slope of the bar correlated well with measured values, though the calculated values were smaller than the measured values.

The models reviewed above accurately predicted the cross-shore distributions of Q_b quantitatively or at least qualitatively on planar beaches and on the seaward slopes of longshore bars. Over troughs, however, the models could not predict the distributions of Q_b even qualitatively. Rivero et al. (1994) as well as Southgate and Nairn (1993) compared Q_b estimated by Battjes and Janssen's model with the values measured in experiments, and showed that Battjes and Janssen's model considerably underestimated Q_b over troughs. Dally's model also significantly underestimated Q_b over a trough (Dally, 1992; Nishi, 1994). Although Thornton and Guza's model has not been compared with measurements over troughs, their model is expected to have the same problem because it considers only wave breaking and not wave reforming.

To overcome the weakness of these models, Southgate and Wallace (1994) introduced the "persistence length" into Battjes and Janssen's model; beyond the persistence length, a broken wave reforms regardless of the wave condition. This length was assumed to be proportional to H_b, where the coefficient of the proportionality was determined by fitting the model results to Q_b measured in large-scale experiments. The use of the "persistence length" improved the accuracy of determining Q_b over troughs (Southgate and Wallace, 1994). The values of Q_b over troughs, however, were still underestimated. To predict the fraction of breaking waves more precisely, a more reliable criterion for wave reforming is required.

In this study, I hence develop two models for predicting H and Q_b that include a criterion for wave reforming proposed on the basis of field data. The models are calibrated and verified with both field and large-scale experiment data.

Formulation of models

The two models developed here employ a wave-by-wave approach in the manner of Dally (1992); the shoaling, breaking and reforming of individual waves are calculated. The values of significant wave height $H_{1/3}$ and Q_b are estimated using the simulation results of individual waves. The only difference between the two models is in the estimation of wave energy dissipation due to wave breaking. One, called Model 1, contains a periodic bore dissipation sub-model used by Thornton and Guza (1983), while the other, called Model 2, uses a dissipation sub-model developed by

Dally et al. (1985), in which stable wave height is included.

(1) Model 1

The shoaling of a wave is calculated with a shoaling coefficient proposed by Shuto (1974); the coefficient has been derived under the consideration of wave nonlinearity.

As a criterion for wave breaking, Model 1 adopts a formula proposed by Seyama and Kimura (1988), who experimentally measured wave height deformation of individual, irregular waves in the surf zone, and investigated the wave height-water depth ratio at wave breaking H_b/h_b. The formula is expressed in terms of beach slope $\tan\beta$ and the ratio of breaking water depth to the offshore wavelength h_b/L_0 as

$$\frac{H_b}{h_b} = 0.16\frac{L_0}{h_b}[1 - \exp(-0.8\pi\frac{h_b}{L_0}(1 + 15\tan^{4/3}\beta))] - 0.96\tan\beta + 0.2. \tag{1}$$

Because the experiment data of Seyama and Kimura (1988) are scattered around the values predicted by Eq.(1) with a standard deviation of $0.08H_b/h_b$, the values of H_b/h_b in Model 1 are assumed to distribute around those predicted by Eq.(1) with a normal distribution having a standard deviation of $0.08H_b/h_b$.

After wave breaking, the energy dissipation of a wave of frequency f, energy E and group velocity C_g is evaluated using

$$\frac{\partial(EC_g)}{\partial x} = \frac{1}{4}\rho g f\frac{(BH)^3}{h}, \tag{2}$$

where x is the cross-shore coordinate directed positive offshore, ρ is the density of sea water, g is the acceleration of gravity, and B is a dimensionless coefficient.

Thornton and Guza (1983) used Eq.(2) to calculate the total wave energy dissipation applying the same value of B to all waves irrespective of wave heights. By comparing H_{rms} measured in the field with the values estimated by Eq.(2), they determined that $B=1.5$. Model 1, however, predicts variations in H of individual waves, for which $B=1.5$ has not been proved to be optimum. The optimum B for individual waves has therefore been investigated with Seyama and Kimura's experimental results on wave deformation in the surf zone (1988), and has been found to be

$$B = 1.6 - 0.12\ln(H_0/L_0) + 0.28\ln(\tan\beta). \tag{3}$$

Wave reforming is judged by a formula proposed by Kuriyama and Ozaki (1996) on the basis of field data. They measured water surface elevations and the modes of wave breaking (non-breaking, breaking, or broken) of individual waves over longshore bars and troughs at the Hazaki Oceanographical Research Station (HORS), which is a 427 m long field observation pier on the Kashima-nada coast of Japan facing the Pacific Ocean, and investigated the wave height-water depth ratio at wave reforming H_r/h_r. The formula proposed is expressed as

$$H_r/h_r = -0.0624\ln(h_r/L_0) + 0.142. \tag{4}$$

Because the field data are scattered around the values predicted by Eq.(4) (Kuriyama and Ozaki, 1996), H_r/h_r in Model 1 are assumed to distribute randomly in the region between Eq.(4)-0.2 and Eq.(4)+0.2.

While wave reforming is determined on the basis of Eq.(4), a transition zone where a broken wave does not reform even though the wave height-water depth ratio is less than that estimated by Eq.(4) is introduced in the model for the following reason. After wave breaking, a bore gradually develops on the front of a broken wave. Bore development appears to be strongly influenced by the wave condition at the wave breaking point, whereas it appears to be only slightly influenced by the wave condition shoreward of the wave breaking point. Consequently, even though $H < H_r$, the bore under development is supposed to advance toward the shore without vanishing.

The length of the transition zone l is determined based on experimental data of Seyama and Kimura (1988); they reported that the variation in H within the surf zone consists of three phases: an increase in H immediately after wave breaking, a sharp decrease in H after reaching the maximum wave height and a moderate decrease in H from the middle of the surf zone to the shoreline. The length l is assumed to be equal to the distance between the point of the maximum wave height and the point where a change occurs in the rate of decrease of H/H_b versus h/h_b because bore development is supposed to end at the latter point; Seyama and Kimura (1988) reported the appearance of stable bores at the latter point. The length of l is expressed accordingly as

$$l = \frac{h_b}{\tan\beta}[1 - 0.93\exp(-9.21\tan\beta) - \frac{0.02}{0.72\exp(6.11\tan\beta)}]. \tag{5}$$

The transition zone introduced in Model 1 seems to be equivalent to the persistence length proposed by Southgate and Wallace (1994). However, these are slightly different; Model 1 assumes that a broken wave cannot reform if $H>H_r$, regardless of wave position, whereas Southgate and Wallace's model assumes that a broken wave must reform beyond the persistence length, regardless of wave condition.

Naturally, wave shoaling is calculated again after wave reforming.

(2) Model 2

Because Model 2 uses the same methods as Model 1 to calculate shoaling, wave breaking and wave reforming of an individual wave, only the method for estimating the wave energy dissipation due to wave breaking is described here.

Wave energy dissipation is estimated by the following equation, proposed by Dally et al. (1985),

$$\frac{\partial (EC_g)}{\partial x} = (EC_g - E_s C_g)\frac{K}{h},$$

(6)

$$E_s = \rho g H_s^2/8, \quad H_s = \Gamma h,$$

where K and Γ are dimensionless coefficients, and H_s represents the stable wave height, defined as the value of H at which wave breaking ends on a shelf beach composed of an upward sloping bottom and a flat bottom. Dally et al. (1985) showed that by setting $K=0.15$ and $\Gamma=0.4$, H could be estimated in good agreement with the experimental data by Horikawa and Kuo (1966).

Since H_s represents the height of a stable wave, which is non-breaking after reforming, Γ can be considered as the wave height-water depth ratio at wave reforming. In this model, hence, Γ is replaced by a value estimated by Eq.(4), while $K=0.15$ is maintained as a constant.

Calibration

The values of $H_{1/3}$ and Q_b estimated by Models 1 and 2 were compared with 11 sets of field data obtained at HORS (Kuriyama and Ozaki, 1996). The offshore boundaries in the calculations were set at the most seaward measurement points where few waves were broken. Figure 1 shows four examples of the comparisons of $H_{1/3}$ and Q_b measured in the field with values estimated with Model 1 and Model 2. Table 1 lists the wave conditions for the measurements shown in Figure 1.

Table 1 Wave and wind conditions during the measurements: significant wave heights in deep water $(H_{1/3})_0$, significant wave periods in deep water $(T_{1/3})_0$, and the wave directions visually observed at the bar crests θ_b, which are defined relative to the shoreward direction and are positive counterclockwise.

Case	Time	$(H_{1/3})_0$ (m)	$(T_{1/3})_0$ (s)	θ_b (deg.)
1	Mar. 3, 1994, 13:20-14:50	1.28	11.2	10
2	Mar. 10, 1994, 13:10-14:40	2.27	9.5	5
3	June 14, 1994, 13:20-14:40	1.50	9.2	-5
4	Nov. 22, 1994, 10:10-11:40	1.68	7.0	0

Good agreements exist between the results of Model 1 and Model 2 for Q_b and $H_{1/3}$. The values of Q_b estimated by the models are, however, smaller than the measured values over the troughs, and even over the seaward slopes of the longshore bars. Furthermore, $H_{1/3}$ estimated by the models are smaller than the measured values over the troughs. These results show that waves simulated in the models tend to break less and decay more than actual waves in the field. I attribute this tendency of the waves in the models to a scale effect; all coefficients in the models, expect those of the wave reforming criterion, were determined on the basis of experimental data.
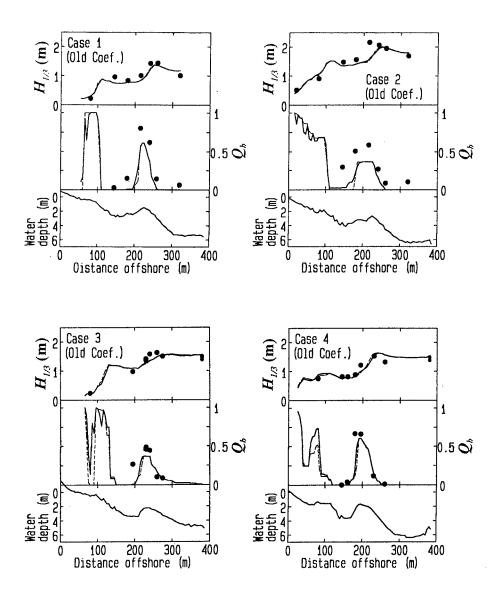
Figure 1 Comparison of $H_{1/3}$ and Q_b measured in the field (solid circles) with those estimated by Model 1 (thick solid lines) and Model 2 (thin dashed lines).

Hence, new coefficients were introduced and the models were calibrated with the field data.

Instead of Eq.(1), which is the criterion for wave breaking in the models, the following equation with a dimensionless coefficient C_{br} was introduced.

$$\frac{H_b}{h_b} = C_{br}(0.16\frac{L_0}{h_b}[1-\exp(-0.8\pi\frac{h_b}{L_0}(1+15\tan^{4/3}\beta))]-0.96\tan\beta+0.2). \quad (7)$$

Equation (3), used for the calculation of wave energy dissipation in the Model 1, was replaced by

$$B = C_B(1.6-0.12\ln(H_0/L_0)+0.28\ln(\tan\beta)), \quad (8)$$

where C_B is a new dimensionless coefficient.

A calibration for Model 1 was conducted by varying the values of C_{br} and C_B, and by determining the optimum values that minimize errors between predicted and measured values of $H_{1/3}$ ad Q_b; the value of C_{br} was varied from 0.8 to 1.1 at intervals of 0.05, and C_B was varied from 0.5 to 1.2 at intervals of 0.1. An error index $\epsilon(C_{br}, C_B)$ was defined to determine the optimum coefficients. This error index was calculated according to the following procedure.

1) The error in $H_{1/3}$ for all data sets, denoted as $\epsilon_H(C_{br}, C_B)$, and that in Q_b, denoted as $\epsilon_Q(C_{br}, C_B)$, are given by

$$\varepsilon_H(C_{br},C_B) = \sum_{i=1}^{N_s}\sqrt{\sum_{n=1}^{N_{H,i}}((H_{1/3,p})_n-(H_{1/3,m})_n)^2/N_{H,i}}\,/\,N_s,$$

$$\varepsilon_Q(C_{br},C_B) = \sum_{i=1}^{N_s}\sqrt{\sum_{n=1}^{N_{Q,i}}((Q_{b,p})_n-(Q_{b,m})_n)^2/N_{Q,i}}\,/\,N_s, \quad (9)$$

where $N_{H,i}$ and $N_{Q,i}$ are the numbers of values of $H_{1/3}$ and Q_b in the i-th data set, and N_s is the number of the data sets. The subscripts p and m denote the values predicted and measured, respectively.

2) If ϵ_H and S_H are the mean and the standard deviation of $\epsilon_H(C_{br}, C_B)$ at $0.8 \le C_{br} \le 1.1$ and $0.5 \le C_B \le 1.2$, and ϵ_Q and S_Q are those of $\epsilon_Q(C_{br}, C_B)$, then the error index $\epsilon(C_{br}, C_B)$ is defined as

$$\varepsilon(C_{br},C_B) = \frac{(\varepsilon_H(C_{br},C_B)-\overline{\varepsilon_H})}{S_H} + \frac{(\varepsilon_Q(C_{br},C_B)-\overline{\varepsilon_Q})}{S_Q}, \quad (10)$$

In a calibration for Model 2, an error index $\epsilon(C_{br}, K)$ was similarly defined; C_{br} was varied from 0.8 to 1.1 at intervals of 0.05, and K was varied from 0.025 to 1.5 at intervals of 0.025.
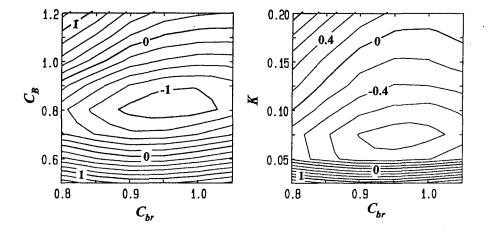
Figure 2   Contour plots of $\epsilon(C_{br}, C_B)$ and $\epsilon(C_{br}, K)$.

Figure 2 shows contour plots of $\epsilon(C_{br}, C_B)$ and $\epsilon(C_{br}, K)$. The optimum values of the coefficients were found to be $C_{br}=0.95$ and $C_B=0.8$ for Model 1, and $C_{br}=0.95$ and $K=0.075$ for Model 2.

I compared $H_{1/3}$ and Q_b measured in the field with those calculated with Models 1 and 2 containing the new coefficients, and show the results in Figure 3. The shortcomings of the models containing the old coefficients were lessen; the accuracy of Q_b estimated with the new coefficients increased. The values of Q_b estimated by the models containing the new coefficients agree with the field data over the longshore bars as well as over the troughs.

The differences between Q_b as estimated by the two models are small, but Q_b estimated by Model 2 decrease more over the troughs toward the shore and are more sensitive to the change in water depth near the shorelines than those estimated by Model 1.
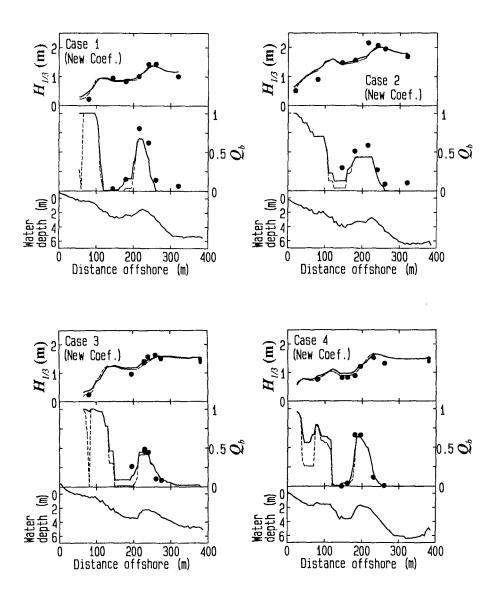
Figure 3 Comparison of $H_{1/3}$ and Q_b measured in the field (solid circles) and those estimated by the newly calibrated Model 1 (thick solid lines) and Model 2 (thin dashed lines).

Verification

The models were then compared with large-scale experiment data of the Delta Flume '93 Experiments (Rivero et al., 1994). The data shown by Rivero et al. (1994) are Q_b and H_{m0}, defined as

$$H_{m0}=4.004\eta_{rms}.$$ (11)

The present models cannot predict H_{m0} because that variable is estimated from a series of water surface elevation, which the present models cannot predict. Thus, a relationship between η_{rms} and $H_{1/3}$ is required to compare the results of the present models with those of the large-scale experiments.

Outside the surf zone, using field data, Goda (1983) has investigated the relationship between $H_{1/3}/\eta_{rms}$ and a wave nonlinearity parameter $\pi_{1/3}$, defined as

$$\pi_{1/3}=\frac{H_{1/3}}{L}(\tanh\frac{2\pi h}{L})^{-3},$$ (12)

where L is the wavelength at the water depth of h. He reported that $H_{1/3}/\eta_{rms}$ increases with increasing $\pi_{1/3}$ when $\pi_{1/3}\geq 0.1$, while $H_{1/3}/\eta_{rms}$ is constant and about 3.8 when $\pi_{1/3}<0.1$.

Although the relationship between $H_{1/3}/\eta_{rms}$ and $\pi_{1/3}$ for waves out of the surf zone was investigated, no relationship between them for waves in the surf zone has been reported. Hence, I investigated this relationship with the field data for the 11 cases mentioned above, and show the results in Figure 4.
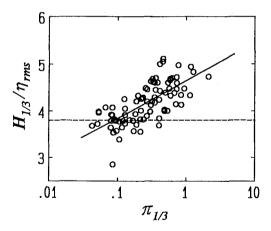

Figure 4   Relationship between $H_{1/3}/\eta_{rms}$ and $\pi_{1/3}$. The solid line shows the relationship obtained with the method of least squares, and the dashed line shows $H_{1/3}/\eta_{rms}$=3.8.

The value of $H_{1/3}/\eta_{rms}$ in the surf zone increases with increasing $\pi_{1/3}$ when $\pi_{1/3} \geq 0.1$, while $H_{1/3}/\eta_{rms}$ is remaining constant at about 3.8 when $\pi_{1/3} < 0.1$. Thus I assumed that

$$H_{1/3}/\eta_{rms} = 0.349 \ln \pi_{1/3} + 4.648, \quad \pi_{1/3} \geq 0.1,$$

$$H_{1/3}/\eta_{rms} = 3.8, \qquad\qquad\qquad \pi_{1/3} < 0.1. \tag{13}$$

The upper equation of Eq.(13) for $\pi_{1/3} \geq 0.1$ was obtained by the method of least squares.

With the relationship expressed as Eq.(13), $H_{1/3}$ predicted by Model 1 and 2 were translated to H_{m0}, and the translated H_{m0} and the predicted Q_b were compared with the values measured in cases 1A, 1B, and 1C of the Delta Flume '93 Experiment (Rivero et al., 1994). The comparisons are shown in Figure 5. In all cases, the values of H_{m0} predicted by Model 1 are almost equal to those predicted by Model 2, and both agree with the measured values quite well. Hence, the discussion is focused on the fraction of breaking waves. On the planar beach of case 1A, the predicted Q_b agree with the measured values although the predicted values are somewhat smaller. On a barred beach of case 1B, Q_b predicted over the trough do not decrease toward the shore, and agree with the measured values, while the present models overestimated Q_b out of the trough. On another barred beach, case 1C, although the predicted values of Q_b decrease slightly toward the shore over the trough, the predicted Q_b agree with the measured values. Out of the trough, the present models also overestimated Q_b. Compared with Q_b predicted by Southgate and Wallace (1994) for cases 1A, 1B and 1C, Q_b predicted by the present models are insensitive to the change in water depth over the troughs, and Q_b variations predicted by the present models are smooth.

Discussion
The models developed in this study incorporate the wave reforming criterion proposed on the basis of field data and the coefficients calibrated with the field data. Through the calibrations of the models, scale effects were recognized in H_b/h_b and in the rate of wave energy dissipation due to wave breaking. In the field, values of H_b/h_b and the energy dissipation seem to be smaller than those in small-scale experiments.

While the cause of the scale effects on H_b/h_b is unknown, the scale effects on wave energy dissipation are considered to be due to the size of the vortex and turbulence generated by wave breaking. Differences in the size of the vortex and turbulence probably results in difference in the energy dissipation process.

Although scale effects on wave energy dissipation due to wave breaking, which strongly influence wave height variation in the surf zone, were recognized in this study, a comparison of wave height variations at different scales on planar beaches (Stive, 1985) revealed that scale effects on wave height variations in the surf zone were negligible. On a planar beach, waves continuously break and few waves reform. On the other hand, at a trough on a barred beach, few waves newly break and some waves reform. The difference in wave conditions seems to result in the difference in the appearance of scale effects.

Although the present models predict well $H_{1/3}$ and Q_b in the surf zone, models employing the wave-by-wave approach, like the present models, have a serious limitation; changes in wave period due to wave decomposition cannot be predicted
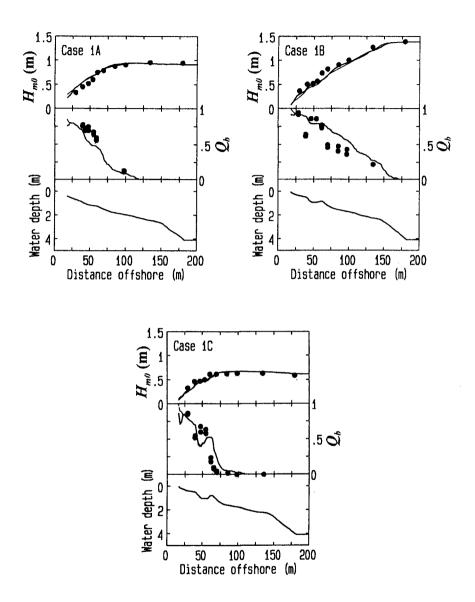
Figure 5 Comparison of H_{m0} and Q_b measured in large-scale experiments (solid circles) and those estimated by Model 1 (thick solid lines) and Model 2 (thin dashed lines).

because the models assume the number of waves to be constant. Wave decomposition frequently occurs over longshore bars, and results in a decrease in wave period. Future improvement of the models will be required to overcome this limitation.

Summary and Conclusions

Models for calculating $H_{1/3}$ and Q_b employing a wave-by-wave approach were developed. The performance of the models containing sub-models for wave breaking and energy dissipation calibrated with experimental data was not satisfactory; Q_b values estimated by these models were smaller than those measured over troughs. New coefficients were therefore introduced, and the models were calibrated with the field data. The models calibrated with the field data predict $H_{1/3}$ and Q_b well over the longshore bars as well as over the troughs. The validity of the models was also verified by comparison with large-scale experiment data.

Acknowledgements

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