## CHAPTER 246

# Behaviors of Fluid Mud under Oscillatory Flow

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*Abstract*

Characteristics of settling velocity of fine particles and movements and the mechanism of maintenance of the fluid mud layer under oscillatory flow are investigated to control the transport flux of suspended solids in estuaries and the coastal zone. Settling velocity of fine particles under oscillatory flow is reduced as a function of only the local concentration. Motion of fluid mud depends on shear stress acting on the interface between the overlying water layer and the fluid mud layer and the horizontal pressure gradient. Especially, the horizontal pressure gradient causes a phase lag between the overlying water layer and the fluid mud movements. Velocity profiles calculated by a linearized model agree well with the measured velocity profiles in spite of assuming the kinematic viscosity in the fluid mud layer to be homogeneous. The apparent viscosity in the fluid mud layer is approximately 200 times as large as that of pure water.

## 1.Introduction

The water layer on mud bed under waves and currents is generally divided into two parts; one is the overlying water layer which is comparatively dilute in concentration and the other is the fluid mud layer which is high in concentration. Moving fluid mud under waves and currents maintains a loose state without consolidation on the bottom. Concentrations of the fluid mud layer are 1 order to 2 orders of magnitude higher than those of the overlying water layer. The flux of suspended solids transported in the fluid mud layer is larger than that in the overlying water layer, though the transport rate of suspension in the fluid mud layer is smaller than that by advection in the overlying layer.

A series of field measurements on shoaling of Kumamoto Port, located in the west side of Kyushu Island, were carried out to confirm effects of submerged mounds against siltation (Tsuruya *et al.*, 1990 ). Three trenches were set inside the navigation pass of Kumamoto Port under construction. One was surrounded by a

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type of submerged mound, but the others were not. Although there was little difference of deposition amounts between them under normal conditions, heavy depositions were observed in the no submerged mound trenches under some severe conditions. From these temporal variations of the depositional amounts inside these three trenches during about a year, the submerged mounds were found effective against siltation. These facts also indicate that fluid mud moves along the sea bed under waves. Results of the field measurements made by Odd and Owen(1972) and Smith and Kirby(1989) also suggest the existence of a high concentration layer near the bottom. Therefore, studies on behaviors of fluid mud under currents and waves are of importance for siltation. Furthermore, so far, as is known, little work has been done on the mechanism of maintenance of fluid mud layers under waves and currents.

The purposes of this study are to explain the formation process of the fluid mud layers, the vertical transport process between suspended solids and fluid mud, the mechanism of maintenance of the fluid mud layers, settling velocity of particles, viscosity in fluid mud, the role of pressure gradient within fluid mud layers, and modeling of the fluid mud movement under oscillatory flow. These results will give useful suggestion on what maintains the fluid mud layer under oscillatory flow and the control of shoaling.

## 2.Theoretical Analysis

A fluid mud layer, according to Ross and Mehta(1989) and our experimental results, consists of both mobile and stationary fluid mud layers. Fluid mud moves as not solid but viscous fluid because the fluid mud layer is high in concentration. Around the upper interface between the overlying water and the fluid mud, a zone of high concentration gradient exists, which is referred to as "lutocline." Based on these facts, in order to accurately formulate the equations of motion of the layer, a multi-layered fluid model was used(Yamanishi and Kusuda, 1991). The model, however, could not show any phase lag of movement between both layers. In this study, in order to explain the causes of the phase lag, a simple linearized and horizontally uniform model was applied. As a model, a simplified vertical structure is adopted as illustrated in **Fig.1**, where the x axis is taken at the interface. Since the motions of the fluid mud are subjected to both shear stress acted on the interface and the horizontal pressure gradient, $\partial p/\partial x$, in the fluid mud, the motions of the fluid mud are modeled by superposing two motions induced by these two forces. The

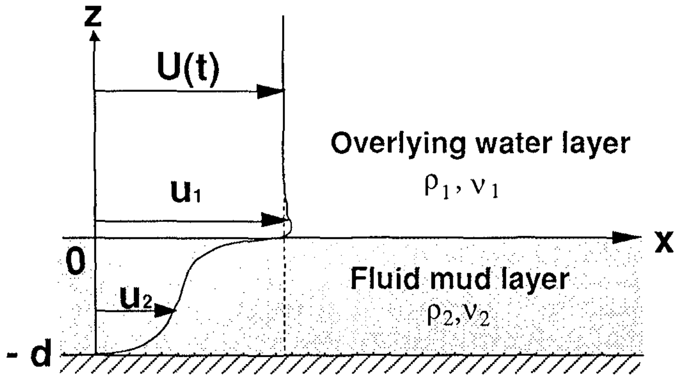


**Fig.1** Definition sketch for fluid mud model

following assumptions are made owing to model the motions of fluid mud under oscillatory flow:

(a)Each layer moves only to the horizontal direction;
(b)Velocity $U$ in the overlying water is a function of time $t$ only;
(c)The advective terms in the equations of motion are negligible small;
(d)The pressure amplitude is uniform in the vertical direction; and
(e)The kinematic viscosity in each layer is homogeneous.

Under the above assumptions, the linearized equations of motions of the fluid mud and overlying water become as follows:

**Overlying water layer( $z \geq 0$ ):**

$$\begin{cases} \dfrac{\partial U}{\partial t} = -\dfrac{1}{\rho_1}\dfrac{\partial p}{\partial x} \cdots (1) \\ \dfrac{\partial u_1}{\partial t} = -\dfrac{1}{\rho_1}\dfrac{\partial p}{\partial x} + \nu_1\dfrac{\partial^2 u_1}{\partial z^2} \cdots (2) \end{cases}$$

**Fluid mud layer( $-d \leq z < 0$ ):**

$$\frac{\partial u_2}{\partial t} = -\frac{1}{\rho_2}\frac{\partial p}{\partial x} + \nu_2\frac{\partial^2 u_2}{\partial z^2} \cdots (3)$$

where, $u_j$, $p_j$, $\nu_j$ and $\rho_j$ are the horizontal velocity, the pressure, the kinematic viscosity and the layer-averaged density, respectively. The subscript $j$ indicates each layer, so that $j$=1 for the overlying water layer and $j$=2 for the fluid mud layer.

In order to analytically solve Eqs.(1)~(3), some solutions described by using the complex representation are supposed as the following equations:

$$U = -i\,\widehat{U}e^{i\omega t} \cdots (4)$$

$$\tilde{u} = -i\widehat{\tilde{u}}\,e^{i\omega t} \cdots (5)$$

$$u_2 = -i\,\widehat{u_2}e^{i\omega t} \cdots (6)$$

$$\widehat{U} = a\omega \cdots (7)$$

$$\partial p/\partial x = -\widehat{p_*}e^{i\omega t} \cdots (8)$$

$$\widehat{p^*} = \rho_1 a\omega^2 \cdots (9)$$

where, $\tilde{u} = u_1 - U$, $a$ is the amplitude, $\omega$ is the frequency ($\omega=2\pi/T$, $T$; period), $t$ is the time, $\hat{}$ indicates the amplitude, and $i$ is the imaginary unit ($i^2= -1$).

The boundary conditions necessary to solve them are as follows:

$$u_1 = U(t)\ ,\ for\ z \to \infty \cdots (10)$$

$$u_1 = u_2,\ at\ \ z = 0 \cdots (11)$$

$$-\rho_1 \nu_1 \frac{\partial u_1}{\partial z} = -\rho_2 \nu_2 \frac{\partial u_2}{\partial z}\ ,\ \ at\ \ z = 0 \cdots (12)$$

$$u_2 = 0\ ,\ at\ \ z = -d \cdots (13)$$

The velocity $U(t)$ in the overlying water is regarded as a function of time $t$ only. These boundary conditions satisfy the continuity of the velocities and stresses across z=0 and the non-slip condition at z=-d.

The analytical solutions of $u_1$ and $u_2$ which satisfy the above boundary conditions are as follows:

$$\begin{cases} u_1 = -i\left\{\left(1 - e^{-\lambda_1 z}\right)\hat{U} + e^{-\lambda_1 z}\hat{u_i}\right\} e^{i\omega t} \cdots (14) \\ u_2 = -i\left[\left\{1 + \dfrac{\sinh \lambda_2 z}{\sinh \lambda_2 d} - \dfrac{\sinh \lambda_2 (z+d)}{\sinh \lambda_2 d}\right\} \cdot \dfrac{\hat{p}}{\rho_2 \omega} + \dfrac{\sinh \lambda_2 (z+d)}{\sinh \lambda_2 d}\hat{u_i}\right] e^{i\omega t} \cdots (15) \end{cases}$$

where,

$$\hat{u_i} = \frac{\rho_1 \nu_1 \lambda_1 \hat{U} + \rho_2 \nu_2 \lambda_2 \dfrac{1 - \cosh \lambda_2 d}{\sinh \lambda_2 d} \cdot \dfrac{\hat{p}}{\rho_2 \omega}}{\rho_1 \nu_1 \lambda_1 - \rho_2 \nu_2 \lambda_2 \dfrac{\cosh \lambda_2 d}{\sinh \lambda_2 d}} \cdots (16)$$

$$\lambda_1 = (1 + i)\sqrt{\frac{\omega}{2\nu_1}} \cdots (17)$$

$$\lambda_2 = (1 + i)\sqrt{\frac{\omega}{2\nu_2}} \cdots (18)$$

From results of the simulations, validity of the model is examined with respect to behaviors of fluid mud and the phase lag.

## 3.Experiments

**Figure 2** shows a U-shape oscillating water tunnel used in this study. This water tunnel does not allow the horizontal transport of fluid mud even for a long time experiment. Mud used was obtained at the Ariake Bay where the maximum tidal range is 5m. This fluid mud was directly sampled from the surface of stationary mud at an ebb tide. The mud consists of clay(55.6%), silt(41.7%) and sand(2.7%) as an average. Its mean diameter and density were 4.0μm and 2540kg/m$^3$, respectively.

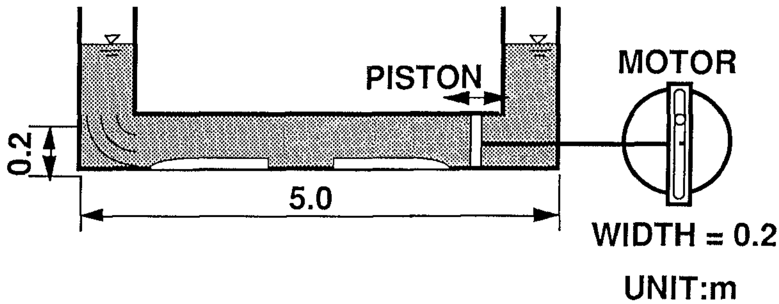


**Fig.2** U-shape oscillating water tunnel

After pouring a mud-seawater mixture with a certain concentration into the tunnel, experiments were immediately started. During the experiments, suspension in the tunnel was sampled through pipes at several points toward the vertical direction at a prescribed time interval, and concentrations of suspended solids were measured with membrane filter. The motion of fluid mud was monitored by a video camera with a closed-up lens and motor-driven camera with extension rings. Pressure in the fluid mud layer was also measured to understand pressure effects.

## 4.Results and Discussion

### (1)Settling velocity of particles

**Figure 3** indicates some temporal changes of the interface in height. In addition, its change under a quiescent state is plotted on it. These plots indicate that the fluid mud exits as suspension on the bottom. The settling velocity of the interface decreases gradually because of the decrease in the apparent velocity of particles.This settling of particles is determined by the local concentration $C$ only. In this case, the equation governing the vertical mass concentration is:

$$\frac{\partial C}{\partial t} - \frac{\partial F_a}{\partial z} = 0 \cdots (19)$$

$$F_a = w_s C - F_e = w_{as} C \cdots (20)$$

Here, $C$ is the local concentration, $F_a$ is the apparent settling flux, $w_s$ is the settling velocity, $F_e$ is the vertical solids flux and $w_{as}$ is the apparent settling velocity. The z-axis is taken to be upward from the bottom. The apparent settling velocity can be calculated by using the following equation on the basis of the concentration profiles under a quiescent state and oscillatory flow:

$$\overline{w_{as}}\overline{C}\,|_{z=h} = -\frac{\partial}{\partial t}\int_0^h C\,dz \cdots (21)$$

in which, $\overline{w_{as}}$ and $\overline{C}$ are the average apparent settling velocity and the average

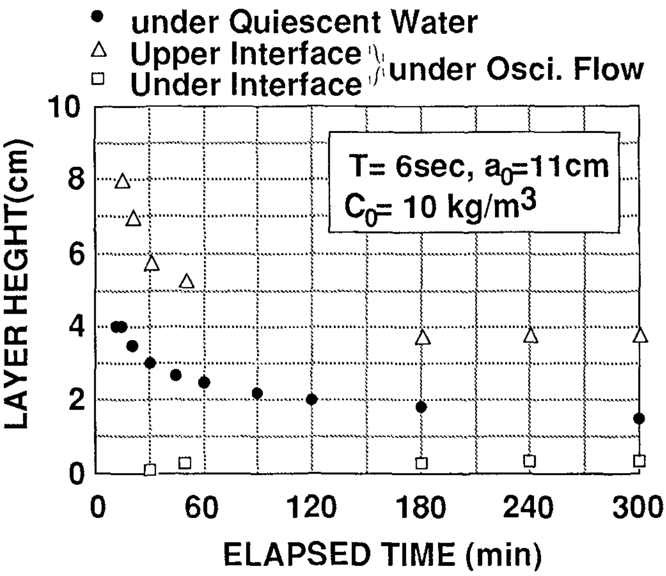


**Fig.3** Temporal changes of the interface in height

concentration at a certain height, $h$.

If Fe in Eq.(20) is equal to zero, the settling velocity $w_s$ under a quiescent state is obtained. **Figure 4** depicts the settling velocity $w_s$ and the settling flux $w_s$ under a quiescent state. If the settling velocity $w_s$ is assumed to be a function of only the

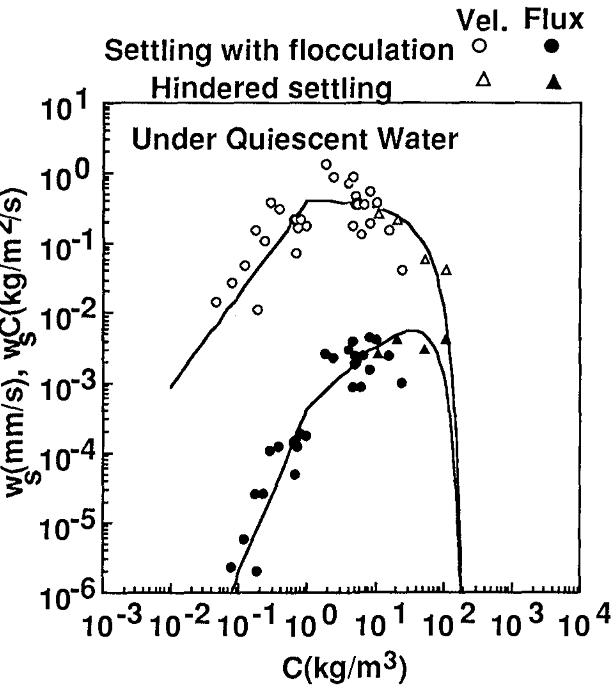


**Fig.4** Settling velocity under a quiescent water

local concentration as the formers (ex., Smith and Kirby 1989), the following best fit relation between the settling velocity and the concentration under a quiescent state is derived from Fig.4.

$$\left.\begin{aligned} w_s &= 0.4\, C^{4/3}\ (mm/s) && C \le 1\ (kg/m^3) \\ &= 0.4 && 1 < C \le 2 \\ &= 0.42\,(1 - 0.005\, C)^5 && C > 2 \end{aligned}\right\} \cdots (22)$$

Similarly, **Fig.5** shows the apparent settling velocity $w_{as}$ and the flux $w_{as}C$ calculated by Eq.(21). In order to compare the apparent settling velocity $w_{as}$ under oscillatory flow with the settling velocity $w_s$ under a quiescent state. Eq.(22) is also drown in the same figure. The apparent settling velocity $w_{as}$ is evidently smaller than $w_s$ and the observed values scatter. This results from that the settling velocity is subject to shear rate and floc size in addition to local concentration of suspended solids. Furthermore, it is possible to describe the changes in height of fluid mud layers by using the method of Kynch(1952). Toorman and Berlamont(1992) adopted this method in order to unify a hidered settling model and a soil mechanics model by estimating apparent settling velocity under a quiescent state. However, these results can not directly explain the mechanism which maintains a fluid mud layer under

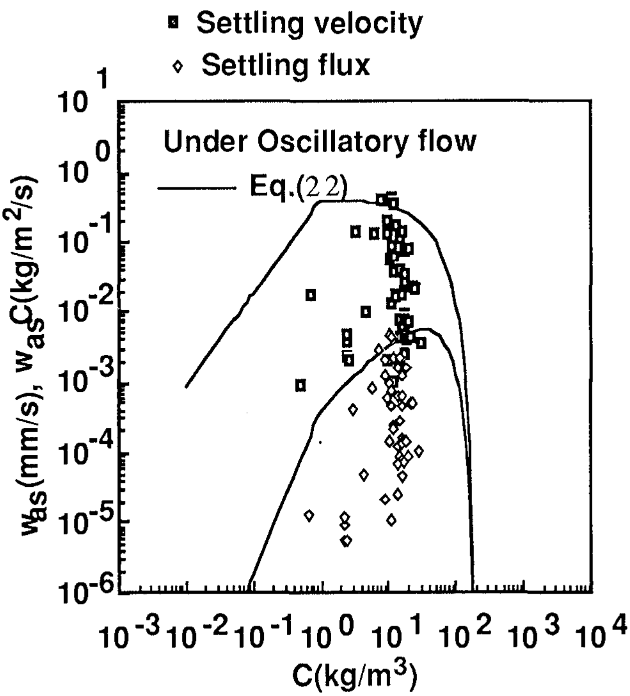


Fig.5 Settling velocity under oscillatory flow

oscillatory flow. Consequently, the maintenance mechanism of fluid mud layers is considered due to the balance between settling flux by gravity and upward flux produced by mixing. If each term could be estimated on the basis of the results, the maintenance mechanism of the fluid mud layer would become clear, and the temporal changes in height of fluid mud layer by using the mass conservation equation could be solved at the same time. This problem seems to be worthwhile for further research work.

**(2)Kinematic viscosity in fluid mud**

Some results simulated by the linearized model are shown in **Figs.6** and **7**. **Figure 7** is velocity profiles measured and calculated at different times. The calculated conditions are taken as the period T=3sec, the horizontal amplitude=5cm, the apparent kinematic viscosity $\nu_2$=2x10$^{-4}$m$^2$/s and the fluid mud layer thickness d=2.1cm in order to compare with the experimental data. It is also important to consider the wall effect. Here, the results have been corrected by assuming an oscillating Poiseuille flow.

Assuming the kinematic viscosity in the fluid mud layer to be uniform as a Newtonian fluid, these results agree well with the measured values. The viscosity in the fluid mud layer used for the calculation notices about 200 times as large as that of pure water. There is an Einstein equation applicable to a dilute suspension but not for dense suspension. Furthermore, the present theoretical analyses for the viscosity of

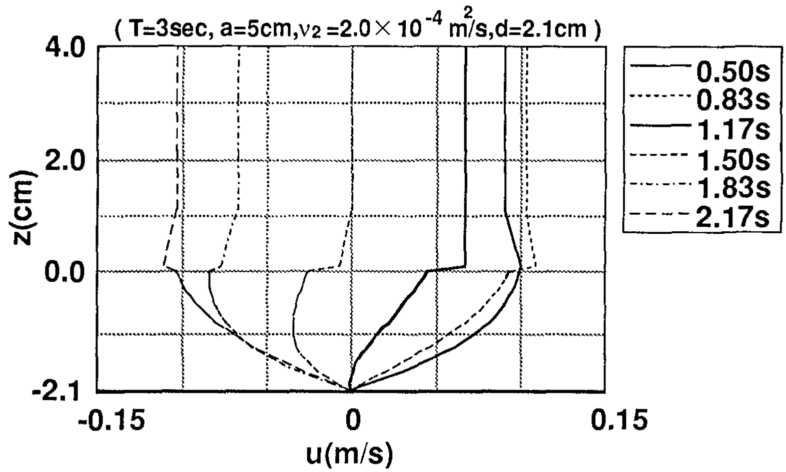


Fig.6 Analytical solutions

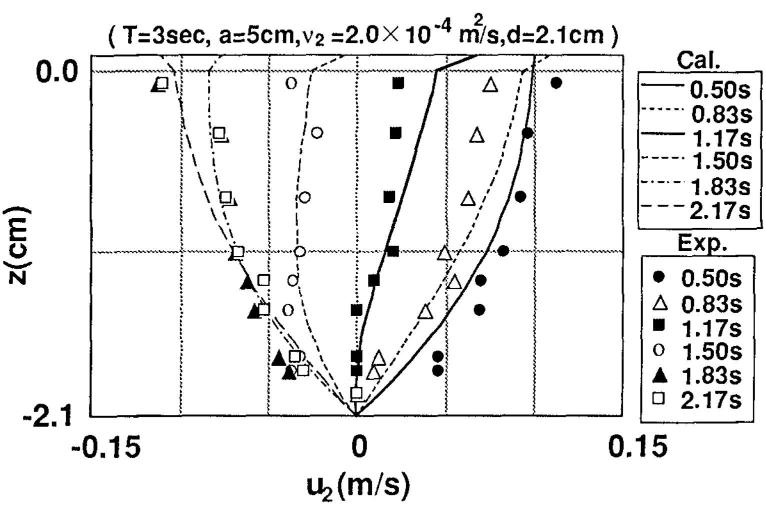


Fig.7 Calculated velocity profiles compared with measurements in the fluid mud

suspension are insufficient for engineering purposes, yet. In general, the viscosity (or kinematic viscosity ) is a function of void fraction and shear rate. Here, based on the constitutive equation (Eq.(23)) obtained experimentally by Kusuda *et al.* (1994), the apparent viscosity in the fluid mud was evaluated.

$$\mu_a = \mu_w(6.1D^{-0.66} + 1)\times\left\{3.8\times10^3\left(\frac{\rho_2 - \rho_w}{\rho_2 - \rho_w}\right)^{1.7} + 1\right\} \quad (\mathrm{Pa\,.\,s})\cdots \quad (23)$$

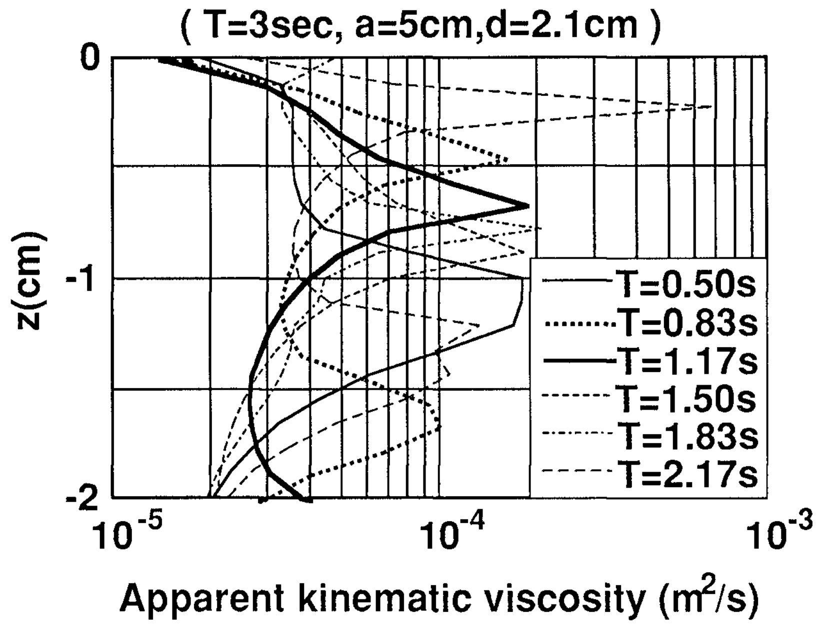


**Fig.8** Apparent kinematic viscosity profiles in the fluid mud

in which, $\mu_a$ is the apparent viscosity, $\mu_w$ is the viscosity of water, $D$ is the shear rate, $\rho_S$ is the particle density and $\rho_W$ is the water density. After the flow field was calculated by Eqs.(1) and (2) and the following equation (Eq.(24)), the apparent viscosity was estimated.

$$\rho_2 \frac{\partial u_2}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial z}\left(\mu_a \frac{\partial u_2}{\partial z}\right) \cdots \quad (24)$$

Calculation was repeated until the apparent viscosity calculated by Eq.(23) converges.

**Figure 8** expresses a result of the apparent kinematic viscosity obtained by Eq.(23). The apparent viscosity is also about 10 to 100 times as large as that of pure water. Because a power exponent function of shear rate is involved in Eq.(23), the apparent kinematic viscosity at a small shear rate in **Fig.8** becomes extremely large. In order to obtain the apparent viscosity with high precision, the fluid mud should be regarded as an elastic or a visco-elastic fluid at small shear rates or considered the change in apparent density under oscillating shear stress.

### (3)Phase lag between overlying water and fluid mud

**Figures 6** and **7** also indicate the phase lag between the overlying water layer and the fluid mud layer velocities. This implies that the motion of fluid mud is different from that of viscous fluid under shear flow. So that, the motions of fluid mud

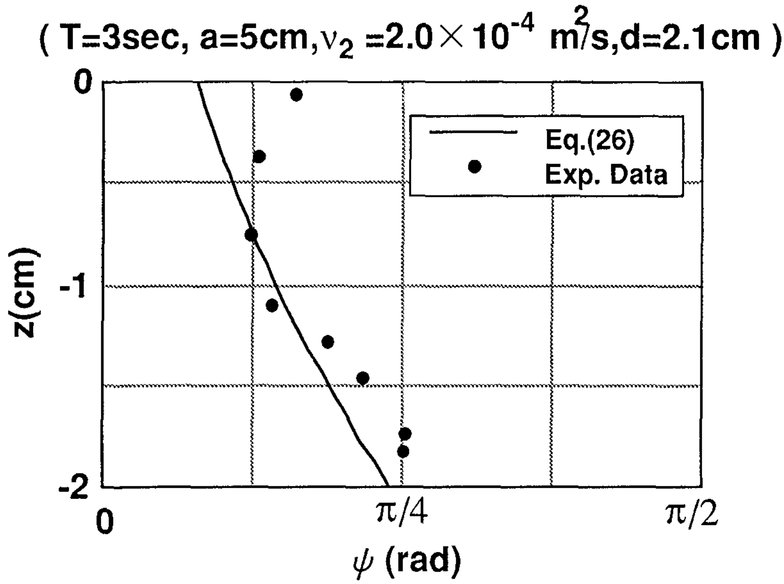


**Fig.9** Observed and calculated phase lag profiles

layer are governed by both horizontal pressure gradient and shear stress acted on the interface between the overlying water and the fluid mud layers. Especially, the horizontal pressure gradient causes a phase lag between them. Therefore, when velocity in the fluid mud decreases under a positive pressure gradient, it becomes slower to lose the kinematic energy by high viscosity in the fluid mud. As a result, the fluid mud may move opposite against the overlying water. This phase lag may be derived from Eq.(6). Therefore, as $\hat{u}_2$ is a complex velocity amplitude, it is transformed by the following equation;

$$\begin{aligned} u_2 &= -\, i\widehat{u_2}e^{i\omega t} \\ &= -\, i\left(\widehat{u_R} + \widehat{u_I}\right)e^{i\omega t} \\ &= -\, i\sqrt{\widehat{u_R}^2 + \widehat{u_I}^2}\; e^{i(\omega t+\psi)} \cdots (25) \end{aligned}$$

$$\psi = tan^{-1}\left(\widehat{u_I}\big/\widehat{u_R}\right) \cdots (26)$$

where, $\hat{u}_R$ and $\hat{u}_I$ indicate the real part and the imaginary part of $\hat{u}_2$ , respectively.

**Figure 9** compares the experimental results with the calculated ones. There is slight difference between the measured and the calculated values, but the analytical solutions approximately explain the experimental results. This simple model proposed here describes well the phase lag at the fluid layer movements. In these experiments, the phase lag ranged from $\pi/12$ to $\pi/4$.

## 5.Conclusions

The summary of the results is as follows:

(1)Settling velocity was shown as a function of only the concentration of suspended solids;
(2)The maintenance mechanism of fluid mud layers is due to the balance between the settling flux by the gravity and the upward flux produced by mixing in the fluid mud layer;
(3)The viscosity in the fluid mud layer is approximately 100 times as large as that of pure water.
(4)The motion of the upper layer in the fluid mud depends on shear stress acted on the interface between the upper water layer and fluid mud layer. The motion of the lower layer in the fluid mud layer is mainly governed by horizontal pressure gradient. This causes a phase lag between the upper layer and the fluid mud movements.
(5)The fluid mud layer moves in advance, compared to the overlying water layer and in this experiments the maximum phase lag was $\pi/4$.

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