CHAPTER 174

SUSPENDED SEDIMENT PARTICLE MOTION IN COASTAL FLOWS

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Abstract

The motion of suspended sediment particles is discussed with the aim of determining which aspects are important and which ones can be neglected when the goal is to model coastal sediment transport. It is shown that the key lies in the structure of the flow, i e, vortices versus wave orbital motion, rather than in details of the drag law. With a proper description of the flow structure the most important phenomena can be modelled under simple assumption that the sediment velocity is everywhere the sum of the local, instantaneous fluid velocity and the still water settling velocity of the sediment: $u_s = u + w_o$. Guide lincs are also given for dealing with sediment in numerical models. This paper considers only non-cohesive particles in thin suspensions where the interaction of sediment particles can be safely neglected.

Introduction

To model the transport of (suspended) sediment one must first understand the way in which suspended particles move with the flow. For many years, this was however considered to be too difficult. - The turbulence was filed away as "complicated" and little progress was made with respect to the motion of suspended particles in organised flows because it was assumed that the general non-linearity of the of drag forces would have to be accounted for in the first approximation.

Recent research has however shown that simple kinematic considerations suffice to describe the most important aspects of suspended particle motion. For example, the important difference between vortices and wave motion, which is illustrated in Figure 1, can be understood through the simple kinematic model

$$u_s = u + w_o. \tag{1}$$

which expresses the assumption that the sediment particle velocity vector u_s can at all

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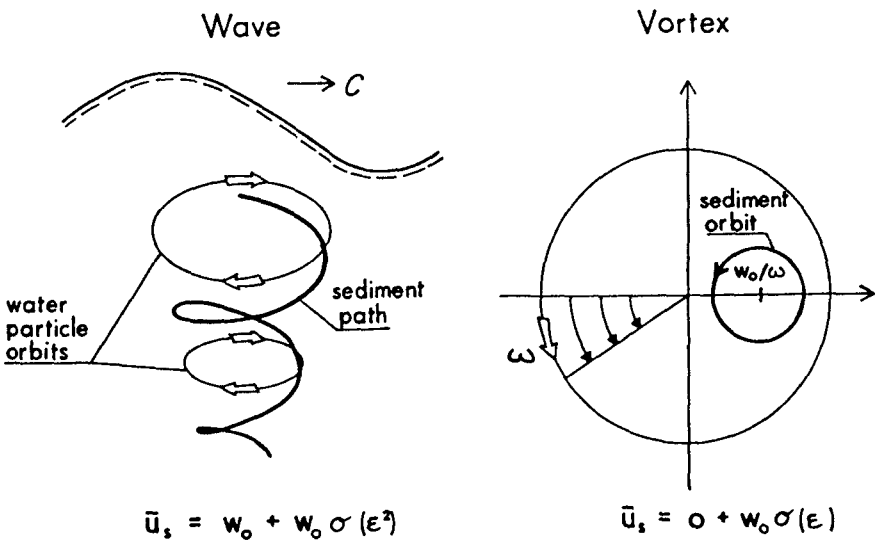

Figure 1: Although vortices and wave motion both have closed eliptical fluid orbits, their influence on suspended sediment are quite different. Vortices will practically eliminate the settling velocity while the effect of wave motion is small of order ε^2 (Equation 2).

times be obtained by adding the still water settling velocity w_o to the local fluid velocity vector $\boldsymbol{u}$.

Under the assumption (1) it can be shown that sediment particles can be trapped along closed paths in most types of vortices as illustrated in figure 3. On the other hand, a wave motion with elliptical fluid orbits has no effect: $\overline{u_s} = w_o$, see e g Nielsen (1992). No consideration of the linear/non-linear nature of the fluid forces is needed to explain these differences. What is important however, is the nature of the organised flow structures like vortices. The essential difference between the vortices and the wave motion in this respect is that the vortices are inhomogeneous while the wave motion is (approximately) homogeneous.

In the following we shall consider, and as far as possibly quantify, the various mechanisms which affect the motion of sediment particles.

Non-linear Drag

It has long been realised that if the fluid drag force on the particle is non-linear then

this will affect the mean vertical velocity of a particle that settles through a vertically oscillating flow. The effect will be a delay compared to settling through still water.

Several numerical studies, e g Ho (1964) and Murray (1970) have been conducted on this effect and an analytical solution was given by Nielsen (1979, 1992). The effect has also been shown experimentally by Ho (1964) who monitored the settling of single particles inside a water filled container which was shaken as a whole in the vertical direction but not stirred. Other types of experiments, which include stirring of the fluid, are not suitable for showing the effect of non-linear drag because the other effects such as vortex trapping and/or fast tracking are likely to dominate in such flows.

The perturbation solution of Nielsen (1979, 1992) shows that the reduction in settling velocity due to quadratic drag on a particle in vertically oscillating fluid with amplitude R and angular velocity Ω is approximately given by

$$\overline{w} = w_o \left[1 - \frac{1}{16} \left(\frac{R\Omega^2}{g} \right)^2 \right] \tag{2}$$

showing that the effect the effect is small of order ε^2 where

$$\varepsilon = \frac{1}{g} \left| \frac{du}{dt} \right| \tag{3}$$

The solution (2) is compared to Ho's (1964) data in Figure 3. Note that in near-bed coastal flows $\varepsilon \lesssim 1/6$ (Bagnold 1946) and that Equation (2) with $\varepsilon = 1/6$ gives a delay of only $0.002 w_o$, which is clearly negligible.

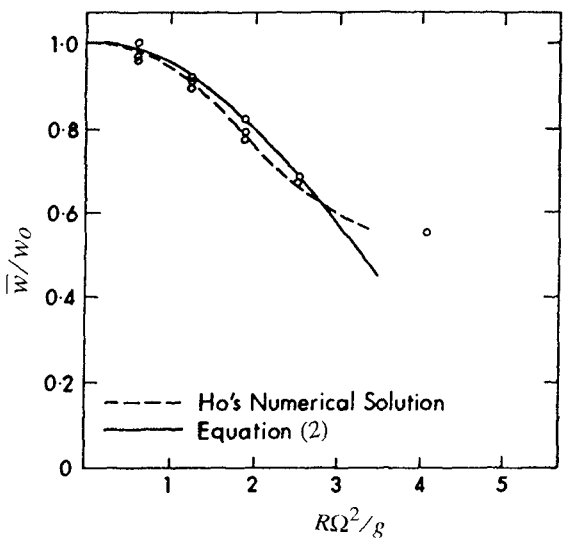

Figure 3: The settling delay due to no-linear drag is well described by equation (2) and it is seen that the effect is insignificant when the fluid accelerations are small compared to g. This is always the case near the bottom in a wave motion.

Vortex trapping

An effect which can cause much greater delay to sediment settling is the trapping in vortices. If a particle is trapped in a vortex for ever, its settling velocity is obviously eliminated. The potential for vortex like flows to trap particles and bubbles was probably first noted by Stommel (1949) who considered the motion of particles inside a box shaped convection cell. However, this motion is much more complicated mathematically than that of a circular solid body vortex. Tooby et al (1977) and Nielsen (1979) considered the latter and showed that if a sediment particle moves in accordance with the assumption (1) in such a vortex it can be trapped for ever on any circular orbit around the point where the fluid velocity balances its still water settling velocity, and the orbit angular velocity is that of the vortex, cf Figure 3.

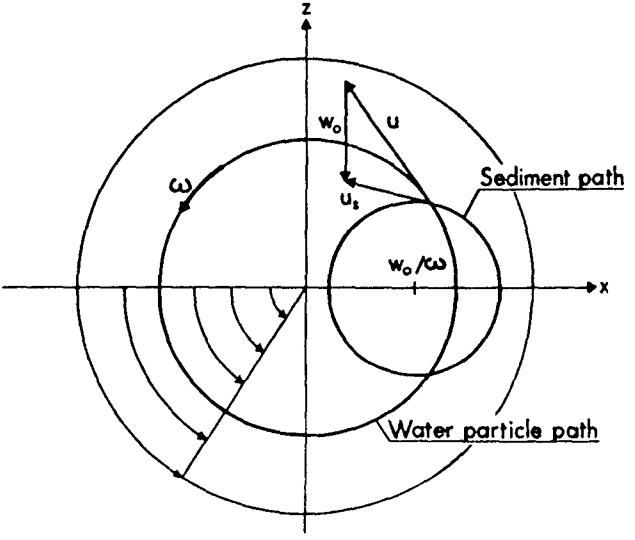

Figure 3: A sediment particle with settling velocity w_o can, under the assumption (1) be at rest at the point $(w_o/\omega, 0)$ or move along any circle around this point with the angular velocity of the vortex motion. See also the photograph by Tooby et al (1977).

In a simple forced vortex the sediment paths are circles around the point P_o where the fluid velocity is equal and opposite to w_o. In Rankine vortices and irrotational vortices, the closed sediment orbits still exist in the vicinity of P_o but they are not circular. See e g Nielsen (1992).

The fact that also irrotational vortices can trap sediment shows that the essence

of the effect is not that the flow is rotational. Rather, it lies in the inhomogenuity of the flow. In fact, the trapping effect is a special case of the loitering effect which is discussed below.

Centrifugal effects

The assumption (1) and hence the solution shown in Figure 3 is valid if the fluid accelerations are small in the sense that

$$\varepsilon = \frac{1}{g}\left|\frac{d\boldsymbol{u}}{dt}\right| << 1 \tag{4}$$

At order ε^1 another interesting feature of particle motion in flow with eddies becomes apparent. Due to centrifugal effects a heavy particle in a vortex will spiral outwards from the closed path shown in Figure 3 and as a result, sand particles will tend to become concentrated in the border areas between eddies. This concentration effect was noticed by Maxey & Corrsin (1986) through a computor simulation, but the essence of it, i e, the centrifugal drift of heavy particles towards the edge of the vortices was quantified already by Nielsen (1979) who showed that the time scale T_{sp} of the spiralling process is given by

$$T_{sp} = \left[\frac{1}{r}\frac{dr}{dt}\right]^{-1} = \frac{g}{w_o\,\Omega^2} \tag{5}$$

where r is the distance from the vortex centre and Ω is the angular velocity of the vortex, see also Nielsen (1992) p 186. The centrifugal effect also affects bubbles, but since they are lighter than water, they spiral inwards. This makes the trapping mechanism particularly efficient for bubbles.

Fast tracking between vortices

The fact that dense particles will spiral out towards the vortex boundaries can lead to fast tracking of small, dense particles in strong turbulence. The reason, as shown in Figure 4, is that the particle speed along the preferred tracks is the maximum vortex velocity which may be much larger than the particles still water settling velocity.

The result of this fast traching is an increase in effective settling velocity $\overline{w}$ which is theoretically unlimited: the relative, effective settling velocity is proportional to the relative turbulence intensity for strong turbulence. For grid turbulence the factor of proportionality is approximately 0.3

$$\frac{\overline{w}}{w_o} \approx 0.3\,\frac{\sigma}{w_o} \tag{6}$$

where σ is a typical turbulent velocity, see Figure 5. A perfectly regular and steady array of vortices would lead to a factor of about *0.4*,

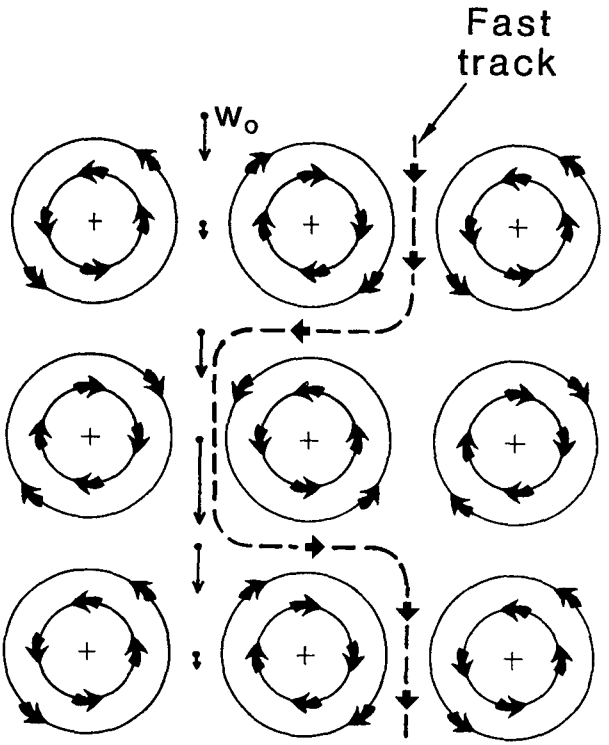

Figure 4: Due to centrifugal effects, sediment particles will get concentrated along the boundaries between vortices and experience a settling velocity increase if the vortices are persistant enough and strong enough.

A sceptic is of course justified in asking whether the behaviour of particles in a highly artificial flow field as the one in Figure 4 has any relevance to sediment motion in natural turbulence. The answer from experimental evidence is that it has. The available data from grid turbulence experiments is shown in Figure 5 but also experiments in steady open channel flow by Jobson & Sayre (1970) show the fast tracking effect. The Jobson & Sayre experiments show up to a factor 2 increase in

settling velocity for fine sand [w_o = 1.1cm/s] and a more moderate increase of only a few percent for coarse sand [w_o = 6.3cm/s] - a picture which is in qualitative agreement with that of the grid turbulence data.

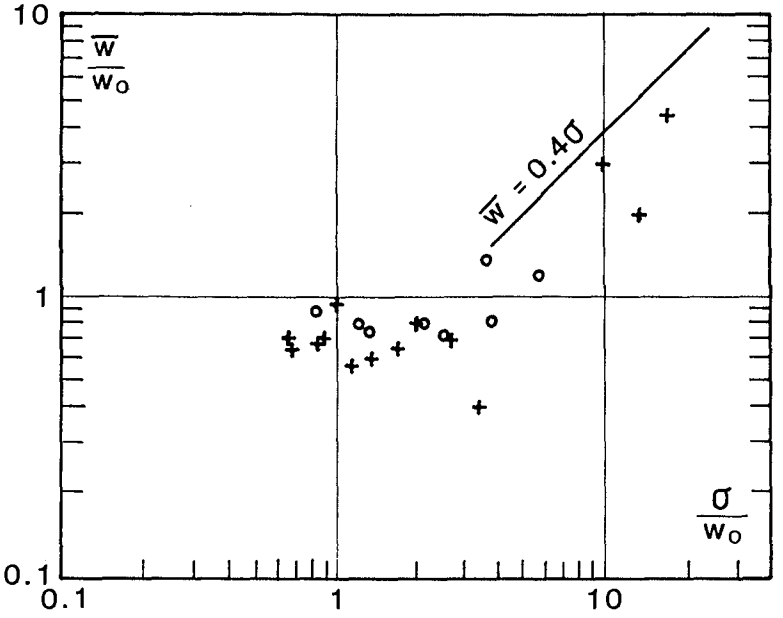

Figure 5: Measured change in settling velocity due to grid turbulence. For weak turbulence the effect is generally a settling velocity decrease due to the loitering effect and trapping, while stong turbulence results in a (theoretically unlimited) increase. + Murray (1970), o Nielsen (1993).

Trapping or fast tracking?

In the two previous sections we have seen that vortices can have two opposite, strong effects on the settling of particles. The particles can be trapped inside a vortex or they can be fast tracked along the boundaries of vortices. The question then naturally arises: *Which effect is going to be dominant?*

The answer is that if the sand is fed into the vortex while it is being formed such as it happens with the lee vortices on a rippled sand bed, see e g Bijker et al (1976). Then it is going to be trapped for a while and will travel with the vortex for some considerable distance without settling out.

On the other hand, there is no chance of a heavy particle coming from the outside of a vortex and getting onto one of the "trapping" closed trajectories inside. Sediment particles which settle towards a vortex will get swept past the potential

trapping area of the vortex as illustrated in Figure 4. See also Nielsen (1992) Figure 4.6.3.

The general loitering effect

Considering the particle which settles along the vertical symmetry line between the vortices in Figure 4 it can be seen that that even though the spatial average of its velocity ($\boldsymbol{u} + \boldsymbol{w_o}$) is $\boldsymbol{w_o}$, the time averaged velocity will be less than $\boldsymbol{w_o}$. Hence particles will settle more slowly through an inhomogeneous flow field than through still (or uniformly flowing) water. This is because the particle spends more time (loiters) with the fluid which moves opposite to its settling velocity.

The loitering effect was quantified by Nielsen (1992, 1993) for a special example and it was shown how this effect might slow the settling of particles in random walk simulated, "stuctureless" turbulence. That is, while the loitering effect is due to the non-uniformity of the flow field, it is not necessary to deal explicitly with the flow structure.

The loitering represents the essential reason why vortices delay settling while a (almost) uniform wave motion, does not. - Figure 1.

Sediment in wave motions

With coastal engineering applications in mind, it is natural to ask whether a pure wave motion will delay the settling of sediment or the rise of bubbles significantly. It is very difficult to give a complete answer to this. However, for the simple velocity field of a linerar shallow water wave

$$\boldsymbol{u} = \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \frac{Hc}{2h}\cos(\omega t - kx) \\ \frac{H\omega z}{2h}\sin(\omega t - kx) \end{pmatrix} \tag{7}$$

the possible effect must be as small as effect of non-linear drag i e without significance. In Equation (7) H is the wave height, h the water depth, c the wave ceklerity, ω the angular velocity and k the wavc number.

The cquation of vertical motion of a small (with linear drag) particle can be conveniently written as

$$\frac{dw_r}{dt} + \frac{\alpha g}{w_o} w_r + \frac{\partial w}{\partial t} = -\alpha g - \alpha \frac{dw}{dt} \tag{8}$$

where w_r is the vertical velocity relative to the fluid and $\alpha = \dfrac{s-1}{s+C_m}$, s being the relative density of the sediment and C_m being the added mass coefficient (approximately 0.5 for nearly spherical particles).

If the expression above is inserted for w, this gives

$$\frac{dw_r}{dt} + \frac{\alpha g}{w_o}\left[1 + \frac{H\omega w_o}{2hg}\sin \omega t\right]w_r \approx -\alpha g - \alpha\frac{\omega^2 Hz}{2h}\cos \omega t \qquad (9)$$

Here it seems safe to neglect the second term in the bracket since its typical magnitude for sand in waves is 10^{-3}. This small term represents the effect of the vertical non-uniformity of the wave motion.

Then, by writing the relative velocity w_r in terms of the still water settling velocity w_o: $w_r = w_o + w_1$ and using

$$z = h - w_o t\,[1 + O(\varepsilon)] \qquad (10)$$

we get the following equation for w_1

$$\frac{dw_1}{dt} + \frac{\alpha g}{w_o}w_1 \approx -\alpha\frac{\omega^2 Hz}{2h}\,[h - w_o t\,]\cos \omega t \qquad (11)$$

The solutions to this equations are of the form $A\cos(\omega t + \varphi) + Bt\cos(\omega t + \psi)$ showing that at the first level of approximation there is no net change of the settling rate due to the wave motion.

Any net change of the settling velocity is thus either due to the non-uniformity mentioned above or due to non-linear drag. In both cases its magnitude will be of the order $10^{-3}w_o$ for sand in wave motions.

Sediment in numerical models

For the purpose of dealing with sediment particles in numerical models it is useful to note that the sediment velocity can be written as

$$\boldsymbol{u_s} = \boldsymbol{w_o} + \boldsymbol{u}(t-\delta_t) + w_o\,\boldsymbol{O}(\varepsilon^2) \qquad (12)$$

where $\boldsymbol{O}(\varepsilon^2)$ denotes a vector function of magnitude ε^2. The time lag is given by $\delta_t = w_o/g$ for small particles that settle under the laminar drag law and $\delta_t = w_o/2g$ for larger particles that settle with quadratic drag, see Nielsen (1979, 1992).

The use of the approximation

$$\boldsymbol{u_s} \approx \boldsymbol{w_o} + \boldsymbol{u}(t-\delta_t) \qquad (13)$$

is thus usualy justified in natural sediment transport processes and it is very convenient because it does not require explicit consideration of the particle dynamics at all. All

that is needed is a stored value $\boldsymbol{u}(t-\delta_t)$ of the local fluid velocity at a previous time.

Frequency response

The expression (13) corresponds to a linear frequency response function $F(\omega)$ defined by

$$u_s - w_o = F(\omega)\, U e^{i\omega t} \tag{14}$$

for the dynamic part of the velocity of a particle that moves in a homogeneous, simple harmonic flow $u(t) = Ue^{i\omega t}$. In terms of the time lag quantified above this frequency response function is given by:

$$F(\omega) = \frac{1}{1 + i\omega\delta_t} \tag{15}$$

and a corresponding gain function is

$$G(\omega) = \frac{1}{\sqrt{1 + (\omega\delta_t)^2}} \tag{16}$$

Conclusions

The considerations above cover the behaviour of suspended sediment particles in small concentrations where particle-particle interactions can be neglected and the analytical approximations are obtained using perturbation expansions in the parameter $\varepsilon = \frac{1}{g}\left|\frac{d\boldsymbol{u}}{dt}\right|$.

It has been shown that the most important aspects of the motion of such sediment can be understood on the basis of the simple assumption $\boldsymbol{u_s} = \boldsymbol{u} + \boldsymbol{w_o}$ which is valid for $\varepsilon \to 0$.

It has also been shown that the settling delay due to drag non-linearity is negligible (about $10^{-3}w_o$) for sand in wave motions and that the effect of non-uniformity of the wave motion is likely to be equally small.

Vortices can have two strong but opposite effects on the settling of sediment: *Trapping inside the vortices* and *fast tracking between vortices*. The former occurs only when the sand is fed into the vortex during the vortex formation as it happens with lee vortices behind sharp-crested bedforms. Fast tracking occurs when the sediment is settling through a field of vortices. Both of these effects are limited to situations where the typical vortex velocities u_v are large compared to w_o. Trapping is possible as long as $u_v > w_o$. Fast tracking is not dominant unless $u_v/w_o \gtrsim 4$. For

smaller vortex strengths, the particles tend to cut through the vortices (see the illustrations of Maxey & Corrsin (1986)) and doing so they are delayed through the loitering effect and/or traping.

Numerical modelling of particles in flows where $\varepsilon^2 << 1$ can be done very easily using equation (13) which does not require explicit consideration of the particle dynamics.

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