

CHAPTER 171

A RELATIVE INTERCOMPARISON BETWEEN VARIABLE WAVE SHOALING, BREAKING AND TRANSITION ZONE FORMULATIONS

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Abstract

The description of nearshore structural and sedimentary dynamics is highly dependant on a quantification of the nearshore wave regime, which is subject to bathymetry induced transformation effects. Assuming straight and parallel bottom contours, computed wave heights and water levels using variable wave shoaling, incipient breaking, wave decay and transition zone formulations are compared with measurements.

1 Introduction

Waves approaching the shoreline undergo transformations due to shoaling and refraction effects. The most significant such transformation, however, occurs coincident with the breaking process, where a dramatic change in wave form occurs. Associated with wave height decay is an internal transition from predominately oscillatory, irrotational motion to a highly rotational state where breaker generated turbulent kinetic energy (TKE) is strongly dissipated.

As reviewed by Ham *et al* (1993), complex numerical methods have been relatively successfully applied to the highly non-linear shoaling region. However, in order to have more efficient tools for engineering application a number of analytical formulations have been proposed. In the present study a comparison is made between linear theory and two of the simplest such formulations, namely first order cnoidal and parametrized solutions proposed by Isobe (1985) and Swart (1978) respectively. For defining the onset of wave breaking, the earlier criteria of Weggel (1972) is compared with formulations of Moore (1982) and Larson and Kraus (1989).

Energy dissipation in a breaking wave is often formulated (Battjes and Janssen, 1978) as being equivalent to that across an hydraulic jump. An alternative approach proposed by Dally *et al* (1984) relates the rate of energy dissipation to the excess of energy between the actual and a stable wave energy flux. Despite generally favourable wave height comparisons with measured values, a consistent lag between the maximum gradient of wave energy and that of setup and return flows has been

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observed (Roelvink and Stive, 1989).

Such findings relate primarily to the manifestation of a transition zone immediately after breaking across which waves undergo transformation into turbulent bores. Svendsen (1984) defined the extent of this zone to be equivalent to the area of near constant mean water level. It was proposed by Basco and Yamashita (1986) that the width of this zone would vary in relation to the surf similarity parameter.

Svendsen (1984) incorporated the effect of the transition zone in the energy conservation formulation by including a storage term representing an initial conversion of organized wave motion into forward momentum flux in the roller. A similar term proposed by Roelvink and Stive (1989) considers a local imbalance of production and dissipation of TKE. Drawing on the concept (Svendsen, 1984) of a surface roller "riding" on a wave front, Nairn *et al* (1990) incorporate a relation proposed by Deigaard and Fredsoe (1989) whereby dissipation is modelled as the work performed by a shear stress at the interface between the roller and the organized wave motion. In a fundamentally different approach Okayasu *et al* (1990) incorporate a term representing the initial transfer of kinetic energy to organized large vortices before subsequent conversion to dissipative TKE for an evaluation of the various transition zone relations.

Lacking explicit measurements of energy dissipation rates in the surfzone, the present study deduces the cross-shore distribution of this parameter through an inverse modelling approach that draws on measurement of surfzone TKE.

2 Experimental Data

The experimental cases discussed are primarily confined to measurements of regular waves over planar bottom slopes. Test conditions for these cases are summarized in Table 1. The set of measurements constitutes a total of 14 test cases with breaking conditions ranging from weakly spilling to strongly plunging.

For each of the data parameters considered, a root mean square error e denoting the deviation of predictions from measurements is computed. This statistical parameter, along with a computed bias b are defined as follows:

$$e = \frac{1}{N} \left[\sum_{i=1}^N (X_{pred,i} - X_{meas,i})^2 \right]^{\frac{1}{2}} \quad (1)$$

$$b = \frac{1}{N} \sum_{i=1}^N (X_{pred,i} / X_{meas,i}) \quad (2)$$

Table 1 Beach slopes and deepwater wave characteristics of various laboratory experiments used in model simulations

Test	slope	H _o (m)	T (s)	H _o /L _o	breaker type
Hansen & Svendsen B	1:34	0.103	1.00	0.0710	spilling
Hansen & Svendsen E	1:34	0.098	1.25	0.0440	spilling
Hansen & Svendsen H	1:34	0.099	1.67	0.0240	spilling
Hansen & Svendsen I	1:34	0.091	1.67	0.0220	plunging
Hansen & Svendsen K	1:34	0.080	1.67	0.0190	plunging
Hansen & Svendsen L	1:34	0.070	1.67	0.0170	plunging
Hansen & Svendsen N	1:34	0.066	2.00	0.0110	plunging
Hansen & Svendsen P	1:34	0.071	2.50	0.0080	plunging
Hattori & Aono 1	1:20	0.030	1.00	0.0210	plunging
Hattori & Aono 3	1:20	0.029	1.60	0.0080	plunging
Nadaoka & Kondoh 1	1:20	0.216	1.32	0.0792	spilling
Nadaoka & Kondoh 5	1:20	0.219	2.34	0.0257	plunging
Stive 1	1:40	0.159	1.79	0.0320	spilling
Stive 2	1:40	0.142	2.99	0.0100	plunging

3 Wave Shoaling

Transformed wave heights and water levels (η) are defined by the 1-D depth-averaged energy and momentum conservation equations:

$$\frac{\partial}{\partial x} (EC_g) = -D_b \tag{3}$$

$$\frac{\partial S_{xx}}{\partial x} = -\rho g d \frac{\partial \bar{\eta}}{\partial x} \tag{4}$$

where E is the wave energy, C_g the group velocity, D the energy dissipation and S_{xx} the radiation stress. In addition to linear wave theory, an adaptation (Isobe, 1985) of the first order cnoidal theory is considered:

$$EC_g = \rho g H^2 (gd)^{\frac{1}{2}} f_2$$

$$\text{with } f_2 = \text{func}(U_s) \sim \frac{32}{15(3U_s)^{\frac{1}{2}}} \left(1 - \frac{15}{(3U_s)^{\frac{1}{2}}} + \frac{20}{U_s} \right) \text{ for } U_s \gg 1 \tag{5}$$

$$\text{and } U_s = \frac{gHT^2}{d^2}$$

as well as the vocoidal theory of Swart (1978):

$$EC_g = (e_p + e_k) \rho g H^2 n C, \quad \text{with } e_p \text{ and } e_k = \text{func} \left(\frac{H}{d}, \frac{L}{d} \right) \quad (6)$$

Computed wave height and mean water level variations were compared for several laboratory experiments. An example of such a comparison against the measurements of Stive (1980) is shown in Figure 1. Figure 2 presents a comparative summary of statistical errors for both wave height and water levels in the shoaling region. Results of the error and bias calculations for wave heights and mean water levels for the experimental data simulated are summarized in Figure 3.

No pre-breaking measurements for either the Hattari and Aono(1985) or Nadaoka and Kondoh (1982) cases were available and comparisons could thus not be made. For the rest of the test cases the cnoidal technique provided the best overall estimates of both wave height and mean water levels in most cases. The vocoidal theory, on the other hand, seemed to fare the worst of the three theories tested. However even though the deviation is highest across the shoaling region, the vocoidal theory provided the closest estimation of both wave height and mean water level at the breaking point for most of the test cases. The bias calculations indicated that the linear theory both estimates wave height and overpredicts setdown. The cnoidal theory performs slightly better in estimating both wave height and mean water levels, whilst the vocoidal theory tends to overestimate both these parameters slightly.

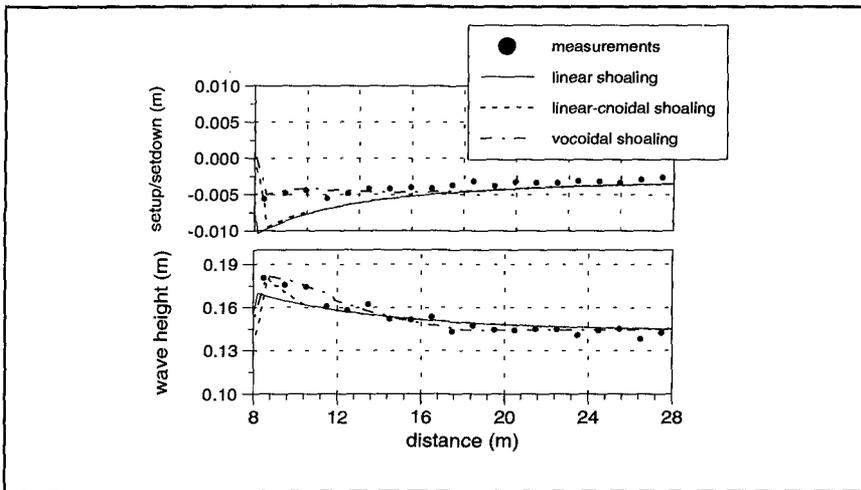


Figure 1 Comparison between different shoaling techniques for Stive Test 1

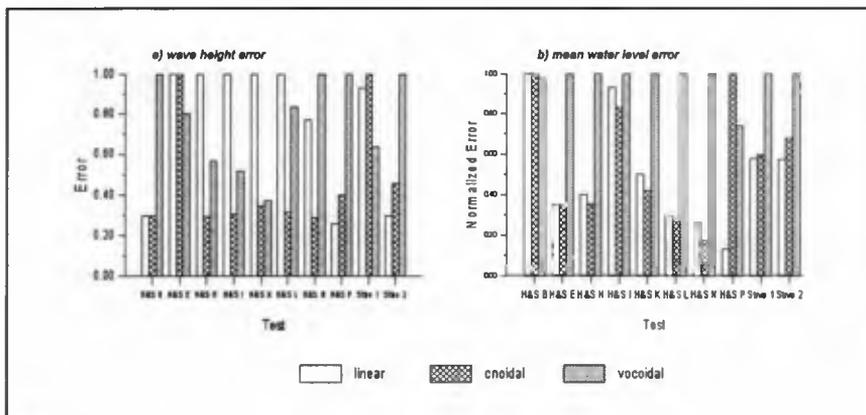


Figure 2 Normalized root mean square errors for wave heights and mean water levels in the shoaling region

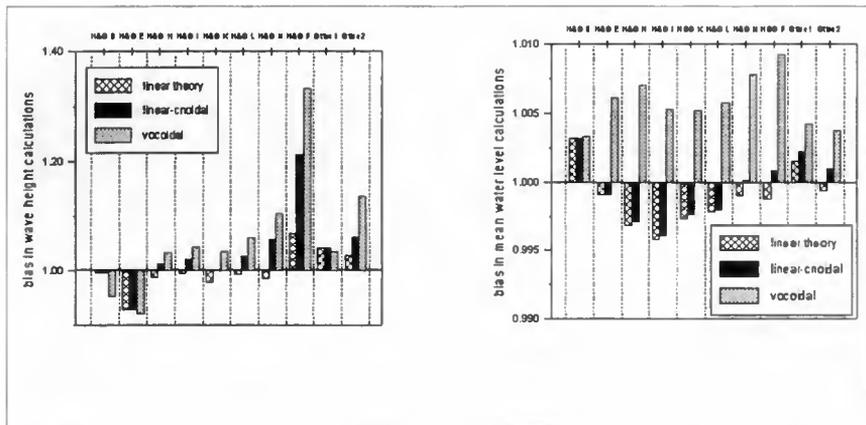


Figure 3 Bias statistics for wave heights and mean water levels in the shoaling region

4 Wave Breaking

Incipient Breaking

Three relatively well-established breaking criteria were assessed, namely the criterion of Weggel (1972),

$$\frac{H_b}{d_b} = b_m - a_m \cdot \frac{H_b}{gT^2} \tag{7}$$

where H_b = wave height at breaking, d_b = water depth at breaking

$$a_m = 43.8 (1.0 - e^{-19m}), \quad b_m = \frac{1.56}{1.0 + e^{-19.5m}}$$

m = bottom slope

Moore (1982),

$$\frac{H_b}{d_b} = b_m - a_m \cdot 0.083 \left(\frac{H_0}{L_0} \right)^{0.8} \quad (8)$$

and Larson and Kraus (1989):

$$\frac{H_b}{d_b} = 1.14 \zeta^{0.21} \quad (10)$$

with ζ = surf similarity parameter = $m \left(\frac{H_0}{L_0} \right)^{-\frac{1}{2}}$

An example of a comparison for a case of Hansen and Svendsen (1979) is presented in Figure 4. The results for all the test cases indicate that there is very little difference between the 3 breaking criteria. The Larson-Kraus (1989) criterion tends to predict the breaking point in slightly deeper water than either the Moore (1982) or Weggel (1972) criteria. For all intents and purposes the oldest and probably most established of the criteria, namely that of Weggel, seems perfectly adequate to use. The Weggel criterion has the advantage of using only local information in determining the breaking position as the deep water wave steepness does not have to be known. This implies that a wave transformation model using the Weggel criterion can be run using input conditions at any point seaward of the breaker zone.

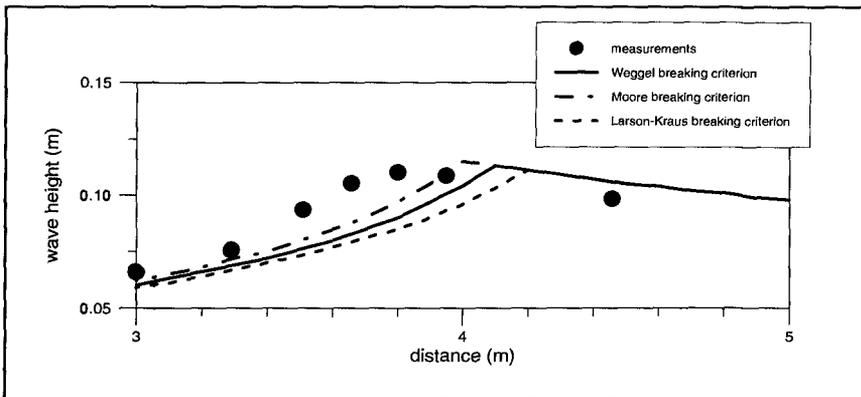


Figure 4 Comparison between breaking criteria for Hansen and Svendsen Case N

Wave Decay

With the steady state conservation of wave energy flux described by equation (3) it is necessary to quantify the rate of energy dissipation D_b due to breaking, which is the dominant energy dissipation mechanism in the surfzone. Considering an equivalence to a hydraulic jump the average breaking wave dissipation per unit area has been expressed for random waves by Battjes and Janssen (1978).

In the event of regular waves the above formulation reduces to the bore dissipation expression of Stive (1984):

$$D_b = A_b \frac{\rho g c}{4} \frac{H^3}{d} \quad (10)$$

where A_b should ideally be greater than unity so as to compensate for the tendency of the hydraulic jump formulation to underestimate wave breaker dissipation.

Dally *et al* (1984) propose a simple algorithm that models waves as undergoing energy dissipation at a rate equivalent to the excess of energy above some stable limit. This approach, which has been referred to as the excess-energy (EE) concept, has been formulated as follows:

$$D_b = \frac{\kappa}{d} [ECg - F_s] \quad (11)$$

where κ = decay coefficient $\sim 0.15 - 0.2$
 F_s = stable wave energy flux $= \frac{1}{8} \rho g H_s^2 Cg$
 H_s = Γd , (with $\Gamma \sim 0.35 - 0.40$)

Both approaches were applied over the whole surfzone using standard coefficients ($A_b = 1$, $\kappa = 0.15$, $\Gamma = 0.4$). The possible reformation of waves was incorporated in the model using a stable wave definition, as proposed by Horikawa and Kuo (1966). An example of results obtained from the two formulations is presented in Figure 5. Relative wave height and water level error and bias statistics are charted in Figures 6 and 7 respectively.

The results indicate that using standard parameters the bore model generally describes the wave height change through the surfzone slightly better than the EE model, with the bias calculations showing the latter tends to underestimate breaker dissipation. An exception to this is in the case of plunging breakers (H&S Case P, H&A Case 3, Stive Case 2) where the bore expression tends to underestimate dissipation rates. However, either of the models can be made to fit the plane slope measurements well using slight parameter adjustments. In the event of a highly irregular bathymetry, particularly under random waves, a limited sensitivity analysis shows neither method to be entirely adequate. As has been previously observed the problem in this regard relates primarily to the effect of wave reformation over trough features. This is

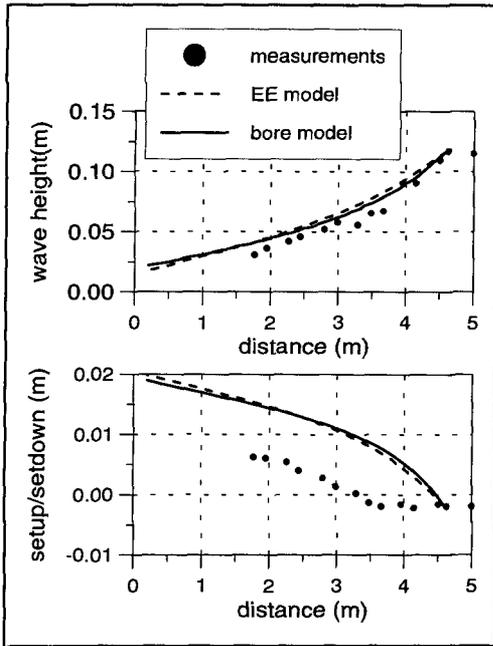


Figure 5 Comparison between wave heights and mean water levels with the EE and bore breaker models (H&S, Case K)

reflected in an incorrect estimate of the percentage of broken waves, as found by Mocke *et al* (1994) in an inverse modelling exercise using undertow and suspended sediment measurements.

The comparison of calculated mean water levels with measurements indicated quite clearly that the use of the momentum equation as defined in equation (4) is not able to describe the setup in the surfzone adequately at all. This serves to underline the importance of including the transition zone and the "lag" effect on the setup in the formulation.

5 The Transition Zone

Transition zone length

Defining the transition zone length (l_t) as the distance from the breakpoint to a point of

abrupt change in mean water level slope Nairn *et al* (1990) (hereafter NRS) use a data set of laboratory experiments to define the following empirical relation:

$$\begin{aligned}
 l_t &= 0.556 m_b L_b \zeta_b^{-1.465} & \text{for } \zeta_b \geq 0.05 \\
 &= 0 & \text{for } \zeta_b < 0.05
 \end{aligned}
 \tag{12}$$

Zhang and Sunamura (1990) (hereafter ZS) use visual laboratory observations for assessing the process described by Nadaoka *et al* (1989) of a progressive transition from strongly two-dimensional vortices at breaking to oblique vortices. Adapting a temporal relation proposed by the authors, an equivalent expression in terms of distance results:

$$l_t = \left[0.02 \frac{H_b}{g T^2 m} \left(\frac{H_b L_b}{\nu T} \right)^{\frac{1}{3}} + 0.12 \right] T c
 \tag{13}$$

Also using visual laboratory observations Okayasu *et al* (1990) (hereafter OWI) define the transition point as the position of roller establishment:

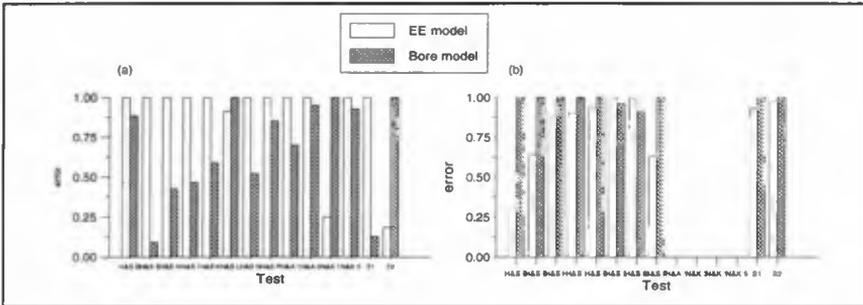


Figure 6 Normalized root mean square errors for (a) wave height and (b) mean water levels using the EE and bore breaker decay models

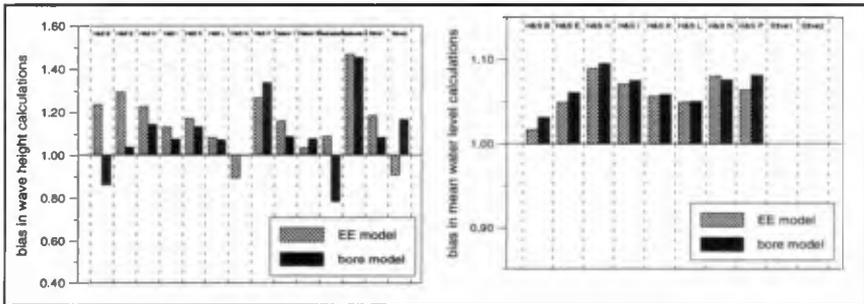


Figure 7 Bias in a) wave heights and b) water levels for breaker decay models

$$l_t = \left(\frac{1}{m} + 4 \right) d_b \tag{14}$$

Quantifying the transition zone length as the area of depressed setdown the above relations are compared to our experimental data set in Figure 8. Being the only formulation based on a similar definition of the transition zone the NRS (1990) expression not surprisingly provides the closest correspondence to measurements. The ZS definition, however, also provides a relatively close comparison. The more qualitatively determined OWI scale, however, tends to consistently predict a transition zone distance that extends beyond the point of change in setup slope. Lacking a more explicit definition of the transition zone, it is impossible at this juncture to make a relative judgement on the proposed relations. So as to assure a certain level of consistency between the comparisons that follow, the empirical NRS relation is considered to define the transition zone length within each formulation.

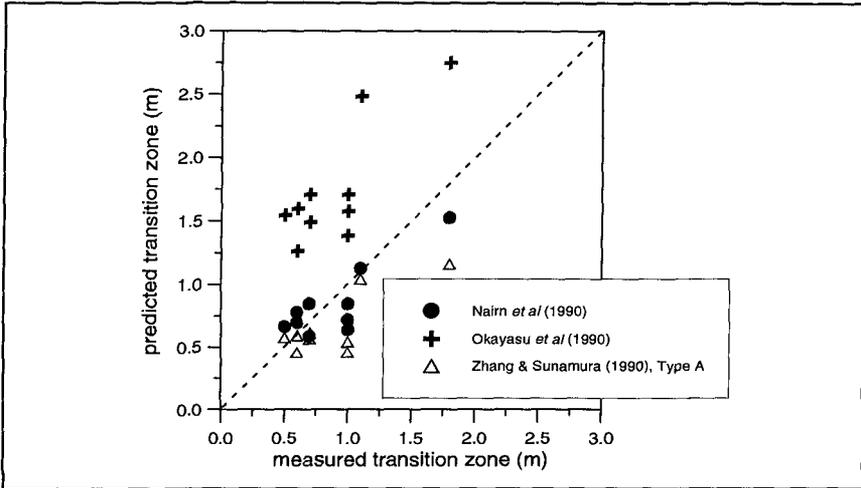


Figure 8 Comparison between measured transition zone lengths and predicted lengths using different empirical transition zone length formulations

Analytical formulations

In recent years attention has been focussed on obtaining an analytical formulation to describe the contribution of the transition zone after the onset of breaking. These formulations all include the inclusion of a lag/storage term in the energy conservation equation. Our previous equations (3) and (4) are therefore adapted to the form below:

$$\frac{\partial}{\partial x} (E_w C_g) + \Gamma_x = D_b \tag{15}$$

$$\frac{\partial}{\partial x} S_{xx} + M_r + \rho g d \frac{\partial \bar{\eta}}{\partial x} = 0, \tag{16}$$

where Γ_x = lag/storage term
 M_r = additional momentum flux term

Where the additional energy and momentum equation terms have been formulated as follows by the various investigators:

Svendsen (1984):

$$\Gamma_x = \frac{\partial}{\partial x} E_r c \text{ and } M_r = \frac{\partial}{\partial x} (2E_r) \tag{17}$$

where E_r = roller energy = $\rho \frac{Ac}{2T}$, and A = roller area $\sim 0.9H^2$

Roelvink and Stive (1989):

$$\Gamma_x = \frac{\partial}{\partial x} \beta_r k d c, \quad D_b = -\rho \beta_d k^{\frac{3}{2}}, \quad \text{and} \quad M_r = \frac{\partial}{\partial x} \beta_s \rho k d \quad (18)$$

with $\beta_r = \beta_d = 1$ and $\beta_s = 0.22$ and $k = \text{turbulent kinetic energy}$

NRS:

$$\Gamma_x = \frac{\partial}{\partial x} E_r c, \quad D_b = -2\beta g \frac{E_r}{c} \quad \text{and} \quad M_r = \frac{\partial}{\partial x} (0.22 E_r) \quad \text{with} \quad \beta = 0.1 \quad (19)$$

OWI:

$$\Gamma_x = \frac{\partial}{\partial x} E_v c_g, \quad D_b = \frac{\partial}{\partial x} E_t c_g, \quad \text{and} \quad M_r = \frac{\partial}{\partial x} \left(\frac{5}{3} E_v \right) \quad (20)$$

where $E_v = \text{energy of organized vortices}$, $E_t = \text{total wave energy} = E_v + E_w$

OWI proposed that the energy transferred to the vortices is dissipated over a "dissipation distance" proportional to the water depth. Comparisons between the analytical formulations of Svendsen, NRS and OWI were carried out on Stive (1980) case 1. Figure 9 depicts the calculated energy dissipation (D_b) and gradient in energy flux ($\partial EC_g / \partial x$) across the surfzone using the three formulations. It may be remarked that both terms are non-zero prior to the breakpoint for the OWI case. This is because energy transfer in the OWI model starts at a position slightly seaward of the breaking point primarily as a means of avoiding an abrupt energy step, and associated numerical instabilities, across this point.

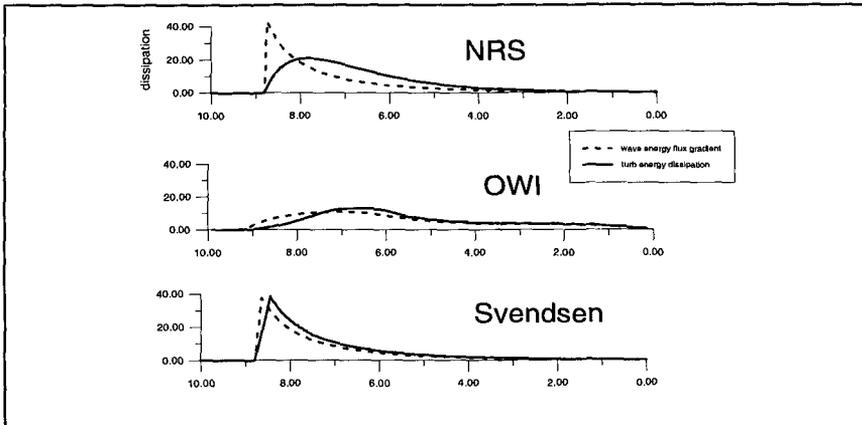


Figure 9 Calculated gradient in energy flux and energy dissipation for Stive Case 1 using the Svendsen, Nairn *et al* and Okayasu *et al* analytical transition zone formulations

In Figure 10 is shown the computed setup distributions for the same test case. As may be seen from a comparison with the setup computed without transition zone effects, the Svendsen relation shows negligible improvements. Although the other relations are found to be somewhat more satisfactory, comparisons with measurements are found to be generally inadequate. It is difficult to pursue the

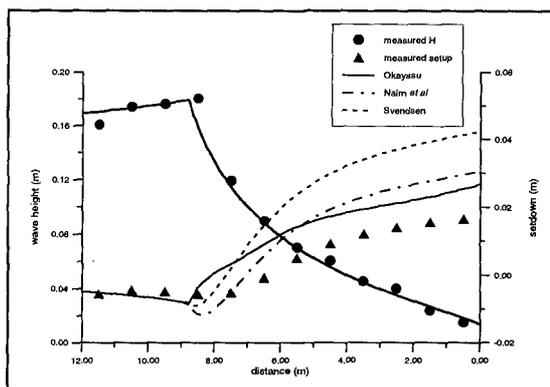


Figure 10 Comparison between setup calculated for Stive Case 1 using different analytical transition zone formulations

analysis in terms of setup measurements as inadequacies may exist in both the adapted energy and momentum relations. Lacking explicit energy dissipation measurements reference is made to available TKE measurements.

Depth-dependant TKE

In employing a two equation (k, ϵ) turbulence model as described in Mocke (1991), D_b rates as determined by the various methods are imposed as a near surface production source of TKE. Considering a dominant diffusion/dissipation balance, computed turbulent intensities for the formulations of OWI and NRS are compared with computations lacking a transition effect. Comparisons with measurements from Stive 1 are shown in Figure 11.

With the breakpoint at approximately $X = 9$ m, limited transition effects appear to be necessary for the three inner measurement stations. At $X = 7.5$ m, however, turbulence intensities tend to be overpredicted with the OWI relation showing the most favourable lag effects. Although no measurements were made at $X = 8.5$ m, the significant discrepancy between predictions at this point immediately following breaking is apparent, with the OWI relation once again showing the most lag effects. Similar findings were observed comparisons against for case 2 of Stive (1980) as well as for the eddy viscosity estimates of OWI.

Depth averaged TKE

Further comparative analyses have been made using depth average/equation ($k-l$) and two equation (k, ϵ) turbulence closure modelling. The details pertaining to this approach are comprehensively described in Mocke *et al* (1993). Referring once again to Stive case 1 measurements, depicted in Figure 12 are comparisons between

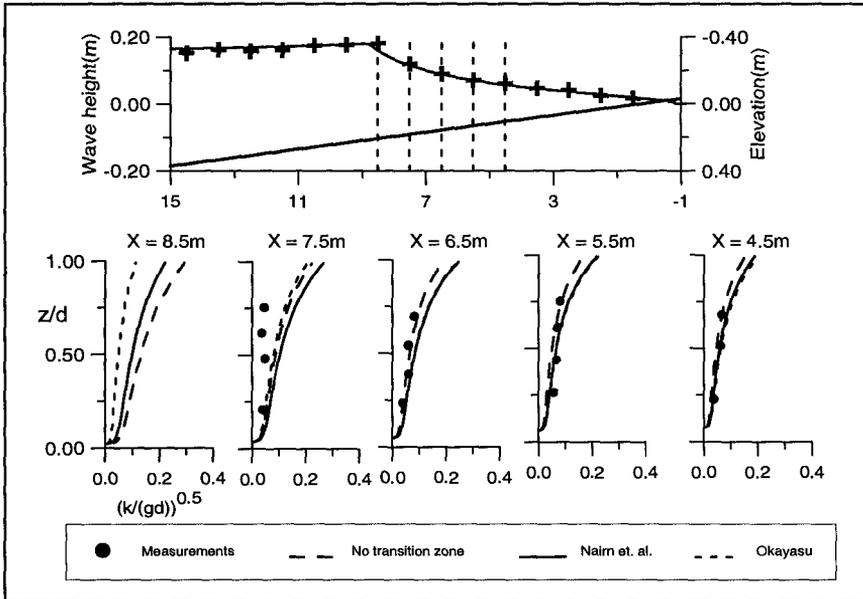


Figure 11 Comparison between TKE profiles through the surfzone using the NRS and OWI transition zone formulations

simulations and estimates of depth averaged TKE.

Note the discernible lag after breaking in the increase in TKE, an effect not reflected in simulations made without consideration of transition zone effects. Although the OWI relation provides for more lag influence than the NRS expression, neither approach satisfactorily represents the measurements.

Examining the NRS expression we note that a fundamental assumption is that the effective roller slope used for determining the dissipation rate remains constant throughout the full extent of the surfzone. This assumption inherently assumes no roller development area nor any changes in the form of this feature throughout the surf zone. In an effort to more closely reflect the physical reality, transformation effects are incorporated by artificially varying the roller slope factor β in the cross-shore direction. A limited sensitivity analysis shows a variation of β from 0.01 to 0.07 within the NRS relation provides an improved estimate of the measurements under consideration.

Conclusions

The comparison between the different shoaling techniques indicates that the first order cnoidal technique provides the closest approximation of wave heights and mean water levels in most of the test cases. However the vocoidal theory provides the

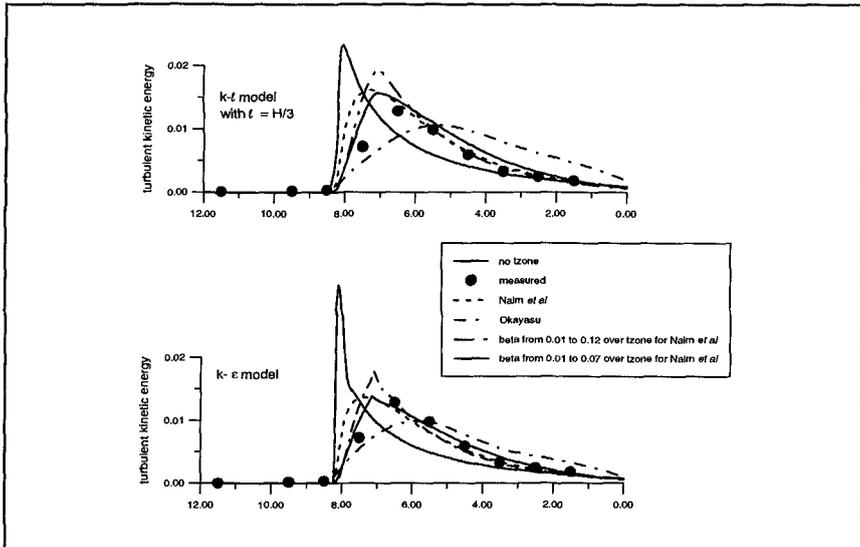


Figure 12 Calculated depth averaged TKE using the Naim *et al* and Okayasu transition zone formulations compared to measurements

closest estimate of wave height and setdown at the breaking point itself. The comparison between the different incipient breaking criteria indicated that there is little to choose between them, and the Weggel criterion, which has the advantage of requiring only local information, is adequate for most applications.

The EE and bore models were found to provide reliable wave decay predictions for regular waves over simple geometries. Simulations not discussed here however reflect previous findings of inadequate predictions for more complex cases. Many of the integral surfzone processes associated with breaking are furthermore not satisfactorily predicted without incorporation of transition zone effects following initiation of breaking. The formulation of Okayasu *et al* (1990), which attempts to quantify an initial transfer of energy to organized large vortices, appears to be physically and empirically the most appropriate approach. An adaptation of the Naim *et al* (1990) formulation to reflect a changing bore configuration, however provides the best correspondence with measured turbulence intensities.

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