CHAPTER 166

A THREE DIMENSIONAL MODEL FOR WAVE INDUCED CURRENTS

BIN LI and ROGER J MADDRELL

SIR WILLIAM HALCROW & PARTNERS LTD, BURDEROP PARK, SWINDON, WILTSHIRE, SN4 0QD, UK

1 Introduction

In this paper, a three-dimensional model which is based on the Navier-Stokes equations is developed. The finite volume method and a time-splitting technique are used to solve the equations. Longshore and crossshore currents induced by waves are reproduced by the model and the results are compared with existing experimental data.

2 Governing Equations

If, apart from the wave-induced orbital motion, the current involves no strong vertical accelerations, the time-mean pressure can be approximated by:

$$p = p_h - \rho\langle w_w^2\rangle \tag{1}$$

in which p is pressure; p_h is hydrostatic pressure

$$p_h = \rho g(\eta - z) \tag{2}$$

ρ is mass density of the fluid; w_w vertical component of the wave orbital velocity; <..> averaged over one wave cycle.

If, in addition, the Boussinesq-hypothesis is adopted to model the Reynolds stress terms, the wave-and turbulence-averaged horizontal momentum equations can be written as:
Momentum equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}(\nu_h\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\nu_h\frac{\partial u}{\partial y})+\frac{\partial}{\partial z}(\nu\frac{\partial u}{\partial z})+W_x \tag{3}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\partial}{\partial x}(\nu_h\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(\nu_h\frac{\partial v}{\partial y})+\frac{\partial}{\partial z}(\nu\frac{\partial v}{\partial z})+W_y \tag{4}$$

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0 \tag{5}$$

where u, v and w are the velocities in the x, y and z directions, respectively; P is pressure; Wx and Wy are the radiation stress terms in the x and y directions, respectively. These radiation stress components will be discussed in the next section.

3 Radiation Stresses

The radiation stress formulae are:

$$W_x = -\frac{\partial}{\partial x}(\langle u_w^2\rangle-\langle w_w^2\rangle)-\frac{\partial}{\partial y}(\langle u_w v_w\rangle) \tag{6}$$

$$W_y = -\frac{\partial}{\partial x}(\langle u_w v_w\rangle)-\frac{\partial}{\partial y}(\langle v_w^2\rangle-\langle w_w^2\rangle) \tag{7}$$

and it should be noted that

$$\frac{\partial}{\partial z}(\langle u_w w_w\rangle) = 0 \qquad and \qquad \frac{\partial}{\partial z}(\langle v_w w_w\rangle) = 0 \tag{8}$$

due to the phase shift between (u_W, w_W) and (v_W, w_W), where <..> means time average.
Formulae for the depth averaged radiation stresses are also given here because they will be used later on when the time splitting technique is applied.

$$S_{xx} = \int_{z_b}^{\langle z_s\rangle} \rho(\langle u_w^2\rangle-\langle w_w^2\rangle)\,dz+\frac{1}{2}\rho g\langle(z_s-\langle z_s\rangle)^2\rangle = [N(\cos^2\theta+1)-1]E \tag{9}$$

$$S_{yy} = \int_{z_b}^{\langle z_s\rangle} \rho(\langle v_w^2\rangle-\langle w_w^2\rangle)\,dz+\frac{1}{2}\rho g\langle(z_s-\langle z_s\rangle)^2\rangle = [N(\sin^2\theta+1)-1]E \tag{10}$$

$$S_{xy} = S_{yx} = \int_{z_b}^{\langle z_s\rangle} \langle\rho u_w v_w\rangle = [N\sin\theta\cos\theta]E \tag{11}$$

where N is the ratio of the wave group velocity to the wave phase speed.

The water surface roller induced by wave breaking has not been taken into account in the above expressions. If the surface roller due to wave breaking is taken into account, the corresponding stresses can be explicitly expressed as (Svendsen, 1984):

$$\tau = -\frac{1}{h}\frac{\partial}{\partial x}(7\frac{h}{L}E) \tag{12}$$

where L is the wave length at wave breaking and it is assumed that the roller induced stress is in the cross-shore direction.

4 Numerical Modelling

The three dimensional model for wave induced currents is governed by the equations given in section 2. Due to the complicated nature of the model, efficient numerical schemes are definitely required to reduce run times as much as possible. In this section, we will use the time splitting technique and the finite volume method to solve the equations.

4.1 Application Of The Time Splitting Method

From the governing equation for wave induced currents it is understood that the surface variation is a function of x and y, but not z. The radiation stresses below wave trough level are hardly varying with z.
First, we introduce depth averaged velocities,

$$\overline{u} = \frac{1}{h}\int_{z_b}^{z_s} u\,dz \qquad and \qquad \overline{v} = \frac{1}{h}\int_{z_b}^{z_s} v\,dz \tag{13}$$

Then the original governing equations can be split into two main parts. The first part corresponds to water surface set-up and set-down and the mean movement of the wave-induced current:

$$\frac{\partial\eta}{\partial t} = -\int\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dz \tag{14}$$

$$\frac{\partial u}{\partial t} = -g\frac{\partial\eta}{\partial x}-\frac{1}{\rho h}\frac{\partial S_{xx}}{\partial x}-\frac{1}{\rho h}\frac{\partial S_{xy}}{\partial y}-\tau \tag{15}$$

$$\frac{\partial v}{\partial t} = -g\frac{\partial\eta}{\partial y}-\frac{1}{\rho h}\frac{\partial S_{xy}}{\partial x}-\frac{1}{\rho h}\frac{\partial S_{yy}}{\partial y} \tag{16}$$

Defining

$$(u',v') = (u,v)-(\overline{u},\overline{v}) \tag{17}$$

yields

$$\frac{\partial\eta}{\partial t}+\frac{\partial\overline{u}h}{\partial x}+\frac{\partial\overline{v}h}{\partial y} = 0 \tag{18}$$

$$\frac{\partial\overline{u}}{\partial t} = -g\frac{\partial\eta}{\partial x}-\frac{1}{\rho h}\frac{\partial S_{xx}}{\partial x}-\frac{1}{\rho h}\frac{\partial S_{xy}}{\partial y}-\tau \tag{19}$$

$$\frac{\partial\overline{v}}{\partial t} = -g\frac{\partial\eta}{\partial y}-\frac{1}{\rho h}\frac{\partial S_{xy}}{\partial x}-\frac{1}{\rho h}\frac{\partial S_{yy}}{\partial y} \tag{20}$$

The second part involves solving for the velocity field

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} = \frac{\partial}{\partial x}\left(\nu_h\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_h\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)+W_x+RS_x \tag{21}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z} = \frac{\partial}{\partial x}\left(\nu_h\frac{\partial v}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_h\frac{\partial v}{\partial y}\right)+\frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right)+W_y+RS_y \tag{22}$$

$$w = -\int_{-h}^{z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dz \tag{23}$$

where RSx and RSy are depth averaged radiation stresses

$$RS_x = -\frac{1}{\rho h}\frac{\partial S_{xx}}{\partial x} - \frac{1}{\rho h}\frac{\partial S_{xy}}{\partial y} \tag{24}$$

$$RS_y = -\frac{1}{\rho h}\frac{\partial S_{xy}}{\partial x} - \frac{1}{\rho h}\frac{\partial S_{yy}}{\partial y} \tag{25}$$

The first set of the equations is used to obtain the mean velocity field and the water surface elevation, while the second one to obtain the depth-varying velocities u, v and w. Both sets of equations can be expressed based on time splitting as follows (from ndt to (n+1)dt):
At the first step

$$\overline{u}^n = \frac{1}{h}\int_{-h}^{\eta^n} u^n dz \tag{26}$$

$$\overline{v}^n = \frac{1}{h}\int_{-h}^{\eta^n} v^n dz \tag{27}$$

and

$$u' = u^n - \overline{u}^n \tag{28}$$

$$v' = v^n - \overline{v}^n \tag{29}$$

then

$$\frac{\eta^{n+1}-\eta^n}{\Delta t} + \frac{\partial \overline{u}^{n+\frac{1}{2}} h}{\partial x} + \frac{\partial \overline{v}^{n+\frac{1}{2}} h}{\partial y} = 0 \tag{30}$$

$$\frac{\overline{u}^{n+\frac{1}{2}} - \overline{u}^n}{\Delta t} = -g\frac{\partial \eta^{n+1}}{\partial x} - \frac{1}{\rho h}\frac{\partial S_{xx}}{\partial x} - \frac{1}{\rho h}\frac{\partial S_{xy}}{\partial y} - \tau \tag{31}$$

$$\frac{\overline{v}^{n+\frac{1}{2}} - \overline{v}^n}{\Delta t} = -g\frac{\partial \eta^{n+1}}{\partial y} - \frac{1}{\rho h}\frac{\partial S_{yx}}{\partial x} - \frac{1}{\rho h}\frac{\partial S_{yy}}{\partial y} \tag{32}$$

$$u^{n+\frac{1}{2}} = \overline{u}^{n+\frac{1}{2}} + u' \tag{33}$$

$$v^{n+\frac{1}{2}} = \overline{v}^{n+\frac{1}{2}} + v' \tag{34}$$

At the second step, we have

$$\frac{u^{n+1}-u^{n+\frac{1}{2}}}{\Delta t} + U_i^{n+\frac{1}{2}}\frac{\partial u^{n+1}}{\partial X_i} = \frac{\partial}{\partial x}(\nu_h\frac{\partial u^{n+1}}{\partial x}) + \frac{\partial}{\partial y}(\nu_h\frac{\partial u^{n+1}}{\partial y}) + \frac{\partial}{\partial z}(\nu\frac{\partial u^{n+1}}{\partial z}) + W_x + RS_x \tag{35}$$

$$\frac{v^{n+1}-v^{n+\frac{1}{2}}}{\Delta t} + U_i^{n+\frac{1}{2}}\frac{\partial v^{n+1}}{\partial X_i} = \frac{\partial}{\partial x}(\nu_h\frac{\partial v^{n+1}}{\partial x}) + \frac{\partial}{\partial y}(\nu_h\frac{\partial v^{n+1}}{\partial y}) + \frac{\partial}{\partial z}(\nu\frac{\partial v^{n+1}}{\partial z}) + W_y + RS_y \tag{36}$$

$$w^{n+1} = -\int_{-h}^{z}(\frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y})\,dz \tag{37}$$

The two sets of equations given above will be solved by different methods. The depth averaged equations will be treated by the alternating direction implicit method (ADI) and governing equations for the depth-varying velocities will be solved by the finite volume technique.

4.2 ADI Solver

It is well known that the Alternative Direction Implicit method, or ADI, has been widely used in two dimensional flow problems. It is used here to solve the depth averaged equation set. Hence, we have:
First step:

$$\frac{\eta^{n+\frac{1}{2}}-\eta^{n}}{\Delta t} + \frac{\partial \overline{u}^{n+\frac{1}{2}}h}{\partial x} = -\frac{\partial \overline{v}^{n}h}{\partial y} \tag{38}$$

$$\frac{\overline{u}^{n+\frac{1}{2}}-\overline{u}^{n}}{\Delta t} + g\frac{\partial \eta^{n+\frac{1}{2}}}{\partial x} = -\frac{1}{\rho h}\frac{\partial S_{xx}}{\partial x} - \frac{1}{\rho h}\frac{\partial S_{xy}}{\partial y} - \tau \tag{39}$$

Second step:

$$\frac{\eta^{n+1}-\eta^{n+\frac{1}{2}}}{\Delta t} + \frac{\partial \overline{v}^{n+\frac{1}{2}}h}{\partial y} = -\frac{\partial \overline{u}^{n+\frac{1}{2}}h}{\partial x} \tag{40}$$

$$\frac{\overline{v}^{n+\frac{1}{2}}-\overline{v}^{n}}{\Delta t} + g\frac{\partial \eta^{n+1}}{\partial y} = -\frac{1}{\rho h}\frac{\partial S_{yx}}{\partial x} - \frac{1}{\rho h}\frac{\partial S_{yy}}{\partial y} \tag{41}$$

It is clear that the solution process presented here is exactly the same as that for a two dimensional flow model like, for example, tidal model. The equations above satisfy the continuity equation from which the water surface variable is obtained.

4.3 Coordinate Transformation

The purpose of the transformation is to restructure the vertical coordinate viz:

$$\sigma = \frac{z-z_b}{\eta-z_b} \tag{42}$$

Using this relationship, the water column at any location between water surface and the sea bottom is transformed into a layer of thickness 1. This transformation introduces additional terms into the equations of motion. However, most of the additional terms introduced by the stretching are contained in the horizontal diffusion terms. Since horizontal diffusion is generally small compared to the vertical diffusion and horizontal advection, only the leading terms need to be retained in general. The derivatives can be obtained as follows:

$$\frac{\partial\sigma}{\partial z} = \frac{1}{h} \tag{43}$$

$$\frac{\partial}{\partial z} = \frac{1}{h}\frac{\partial}{\partial\sigma} \tag{44}$$

$$\frac{\partial^2}{\partial z^2} = \frac{1}{h^2}\frac{\partial^2}{\partial\sigma^2} \tag{45}$$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t}+\frac{\partial\sigma}{\partial t}\frac{\partial}{\partial\sigma} \tag{46}$$

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x}+\frac{\partial\sigma}{\partial x}\frac{\partial}{\partial\sigma} \tag{47}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial y}+\frac{\partial\sigma}{\partial y}\frac{\partial}{\partial\sigma} \tag{48}$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial x^2} + \textit{higher order terms} \approx \frac{\partial^2}{\partial x^2} \tag{49}$$

(50)

$$\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial y^2} + \textit{higher order terms} \sim \frac{\partial^2}{\partial y^2}$$

$$\frac{\partial \sigma}{\partial x} = -\frac{1}{h}\left(\sigma\frac{\partial \eta}{\partial x} + (1-\sigma)\frac{\partial z_b}{\partial x}\right) \tag{51}$$

$$\frac{\partial \sigma}{\partial y} = -\frac{1}{h}\left(\sigma\frac{\partial \eta}{\partial y} + (1-\sigma)\frac{\partial z_b}{\partial y}\right) \tag{52}$$

In general from the transformation formulae given above one can derive the governing equations in the new coordinate system. However, it is convenient to define a vertical velocity as

$$w^* = \frac{\partial \sigma}{\partial t} + u\frac{\partial \sigma}{\partial x} + v\frac{\partial \sigma}{\partial y} + w\frac{\partial \sigma}{\partial z} \tag{53}$$

We can finally obtain the momentum equations in x and y directions:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w^*\frac{\partial u}{\partial \sigma} = \frac{\partial}{\partial x}\left(\nu_h\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_h\frac{\partial u}{\partial y}\right) + \frac{1}{h^2}\frac{\partial}{\partial \sigma}\left(\nu\frac{\partial u}{\partial \sigma}\right) + W_x^* + RS_x^* \tag{54}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w^*\frac{\partial v}{\partial \sigma} = \frac{\partial}{\partial x}\left(\nu_h\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_h\frac{\partial v}{\partial y}\right) + \frac{1}{h^2}\frac{\partial}{\partial \sigma}\left(\nu\frac{\partial v}{\partial \sigma}\right) + W_y^* + RS_y^* \tag{55}$$

where the radiation stress terms with a star mark use values of the new coordinate. In the time-splitting scheme, the equations are

$$\frac{u^{n+1} - u^{n+\frac{1}{2}}}{\Delta t} + U_i^{*\,n+\frac{1}{2}}\frac{\partial u^{n+1}}{\partial X_i} = \frac{\partial}{\partial x}\left(\nu_h\frac{\partial u^{n+1}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_h\frac{\partial u^{n+1}}{\partial y}\right) + \frac{1}{h^2}\frac{\partial}{\partial \sigma}\left(\nu\frac{\partial u^{n+1}}{\partial \sigma}\right) + W_x^* + RS_x^* \tag{56}$$

$$\frac{v^{n+1} - v^{n+\frac{1}{2}}}{\Delta t} + U_i^{*\,n+\frac{1}{2}}\frac{\partial v^{n+1}}{\partial X_i} = \frac{\partial}{\partial x}\left(\nu_h\frac{\partial v^{n+1}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_h\frac{\partial v^{n+1}}{\partial y}\right) + \frac{1}{h^2}\frac{\partial}{\partial \sigma}\left(\nu\frac{\partial v^{n+1}}{\partial \sigma}\right) + W_y^* + RS_y^* \tag{57}$$

where U^* is the vector velocity in the new coordinate system and

$$U_i^*\frac{\partial}{\partial X_i} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w^*\frac{\partial}{\partial \sigma}$$

It should be noted that the vertical velocity in the old coordinate system is calculated by

$$w^{n+1} = -\int_{-h}^{z}\left(\frac{\partial u^{n+1}}{\partial x} + \frac{\partial v^{n+1}}{\partial y}\right)dz \tag{58}$$

where no transformation is rsquired.

4.4 Application Of The Finite Volume Method

Aftsr transforming the momentum equations into a new coordinate system where the calculation domain is in regular form, the finite volume method is a suitable numerical solver due to its excellent conservation characteristics. The details of this method have been very well dsscribed by Patankar in his book (Patankar 1977). The discretization equation for velocity u is:

$$a_P u_P = a_E u_E + a_W u_W + a_N u_N + a_S u_S + a_T u_T + a_B u_B + b \tag{59}$$

where indexes E,W,N,S,T and B mean the east,west,north,south,top and bottom neighbours respectively, P means central point, and

$$a_E = D_e A(|P_e|) + [\![-F_e, 0]\!] \tag{60}$$

$$a_W = D_w A(|P_w|) + [\![F_w, 0]\!] \tag{61}$$

$$a_N = D_n A(|P_n|) + [\![-F_n, 0]\!] \tag{62}$$

$$a_S = D_s A(|P_s|) + [\![F_s, 0]\!] \tag{63}$$

$$a_T = D_t A(|P_t|) + [\![-F_t, 0]\!] \tag{64}$$

$$a_B = D_b A(|P_b|) + [\![F_b, 0]\!] \tag{65}$$

$$a_P^o = \frac{\Delta x \Delta y \Delta \sigma}{\Delta t} \tag{66}$$

$$b = S_C \Delta x \Delta y \Delta \sigma + a_P^o u_P^o \tag{67}$$

$$a_P = a_E + a_W + a_N + a_S + a_T + a_B + a_P^o \tag{68}$$

whsre S_C is the source term including contributions from radiation stresses. The flow rates and conductances are defined as

$$F_e = u_e \Delta y \Delta \sigma \qquad D_e = \frac{\nu_{he} \Delta y \Delta \sigma}{\Delta x} \tag{69}$$

$$F_w = u_w \Delta y \Delta \sigma \qquad D_w = \frac{\nu_{hw} \Delta y \Delta \sigma}{\Delta x} \tag{70}$$

(71)

(72)

$$F_n = v_n \Delta\sigma \Delta x \qquad D_n = \frac{\nu_{hn}\Delta\sigma\Delta x}{\Delta y}$$

$$F_s = v_s \Delta\sigma \Delta x \qquad D_s = \frac{\nu_{hs}\Delta\sigma\Delta x}{\Delta y}$$

$$F_t = w_t^* \Delta x \Delta y \qquad D_t = \frac{\nu_t \Delta x \Delta y}{h^2 \Delta\sigma} \tag{73}$$

$$F_b = w_b^* \Delta x \Delta y \qquad D_b = \frac{\nu_b \Delta x \Delta y}{h^2 \Delta\sigma} \tag{74}$$

The Peclet number P is to be taken as the ratio of F and D; thus, $P_e=F_e/D_e$, and so on. The power-law formulation is used here, that is

$$A(|P|) = [\![0, (1-0.1|P|)^5]\!] \tag{75}$$

which means taking the largest of the values in the brackets.
More details can be found in Patankar's book.

5 Boundary Conditions

Wave breaking on the water surface produces extra momentum. Svendsen (1984) introduced the concept of the surface roller which actually takes into account the effect of wave breaking. The formulae derived by him have been used by many researchers. The essential point of Svendsen's idea is that the surface roller contains a certain amount of shoreward directed momentum, which causes a return flow below the wave trough in the offshore direction. According to his work, the flux induced by the water surface roller can be related to the wave energy. If linear wave theory is used, the wave induced mass flux including the water surface roller contribution can be written as

$$M = \left(1+\frac{7h}{L}\right)\frac{E}{\rho c} \tag{76}$$

where h is the water depth, L is the wave length, c is the wave celerity, E is the wave energy density. Actually this formula has already been given in section 3 when discussing radiation stresses. However, in the calculation domain, we need to define a boundary at the water surface, and assumptions are required to tackle this requirement. Here it is assumed that:

1. The thickness of the surface layer is approximately equal to the wave height.
2. The time-averaged velocity in the layer is uniform and it is

$$u_s = \left(1+\frac{7h}{L}\right)\frac{E}{\rho c}/(\langle\eta_s\rangle - z_{tr}) = \left(1+\frac{7h}{L}\right)\frac{E}{\rho c}/H \tag{77}$$

3. The mean water surface is located halfway between the wave crest and the wave trough.

The momentum equations solved by the finite volume method are actually only applied to the area below the wave trough. The velocity at the wave trough is then known from assumption (2) made above, which can be used as the boundary condition at the water surface.

The boundary condition at the sea bottom should include the effects of the wave oscillatory velocities. Liu and Dalrymple (1977) derived a formula for the bottom shear stress under the condition of a weak current and large incident wave angle. Their formula is used here.

6 Model Tests And Discussions

Aspects of model stability are considered first before verification tests are carried out. The model is a combination of two sub-models: the first is the ADI model and the second is the finite volume based model for the advection terms. It is well known that the ADI model is stable and accurate if the Courant number is not very large (usually less than 4). On the other hand, the finite volume method used in this paper is also a stable numerical solver. The power-law formula proposed by Patankar (1972) can ensure unconditional stability. It can therefore be concluded that the wave-induced current model is stable.

The model proposed in this paper is tested against available experimental data. The model includes the time variable, and a steady state solution can be obtained if the model is run for a sufficiently long period. Of course for a problem like the simulations of tidal flow, the model can quickly reach a three-dimensional velocity field. Before the model performance was compared against experimental data, a series of numerical tests were undertaken for some simple cases in order to ascertain the capabilities of the model. The cases include horizontal channel flow and a plane beach with a constant slope. As a rough criterion for accessing the model performance, continuity of the flow must be satisfied. After these numerical experiments had been done successfully, the experimental data presented by Stive and Wind (1982,1986) were used to test whether the model could predict cross-shore circulation and water surface set-up.

The experiments were conducted in a wave flume 55m long, 1m wide and 1m deep. A plane, concrete beach with a 1:40 slope was installed. Two experiments have been reported by Stive and Wind in 1982 and in 1986. In the first experiment, two test conditions were used. The first of these was also used for the experiment carried out in 1986. The experimental results from the first test of the first experiment and of the second experiment are used for testing the model. For more details for these experiments the interested reader can consult the original papers.

Comparisons of the results of the model presented in this paper against Stive and Wind's experimental data for water level set-up and cross-shore circulation are shown in figures 1 to 4. In the case of the water surface set-up, it is clear that the agreement between the experimental data and the computational results is reasonable. The velocity distribution over the water depth obtained from the model is plotted against the experimental data. The trend of both experimental and computational results is the same, however, there are obvious differences. If the results from other models are considered, for example, Stive and Wind's model (1986) and Svendsen's model (1986), the results from the present model are acceptable. Some reasons for such differences between the experimental data and the model results are as follows.

1). Wave breaking is a nonlinear process. Although Svendsen (1984) presented a formula dealing with the surface roller and the results from this formula is reasonable, a much more detailed description is really required.

2). The eddy viscosity formula used in the present model can be improved if the turbulence in the surf zone is better described. It is well known that the turbulent processes in the surf zone are far from understood. Bottom friction is another subject which many researchers are still studying. The bottom friction formula used in the present model is only one of the many available formulae.

3). Wave-current interaction is ignored in the present model and it is ignored in all existing wave-induced current models. How to incorporate wave-breaking into wave-current interaction is a problem which has yet to be tackled.

4). The calculated velocities at water surface are smaller than the values of the experimental data which is probably caused by the inaccurate simulation of the air-water mixture at the surface. Improvement is under way.

From the above statements it can therefore be concluded that the wave-induced current problem is so complicated that numerical results are only rough estimates of the true solutions.

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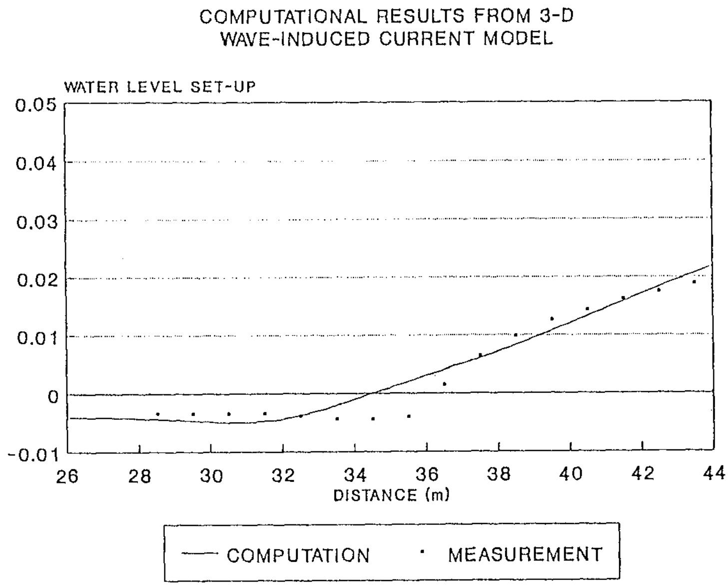
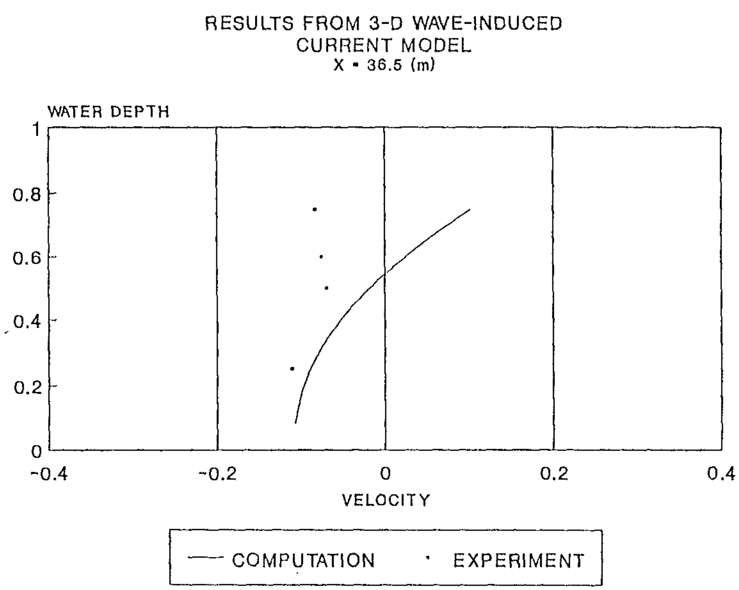
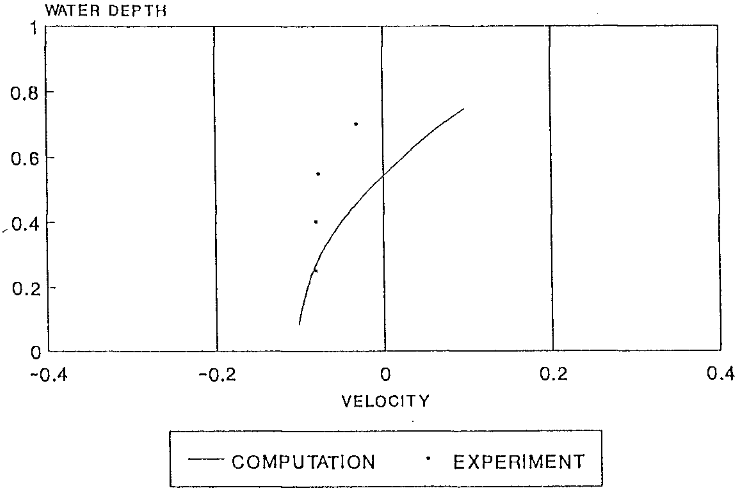
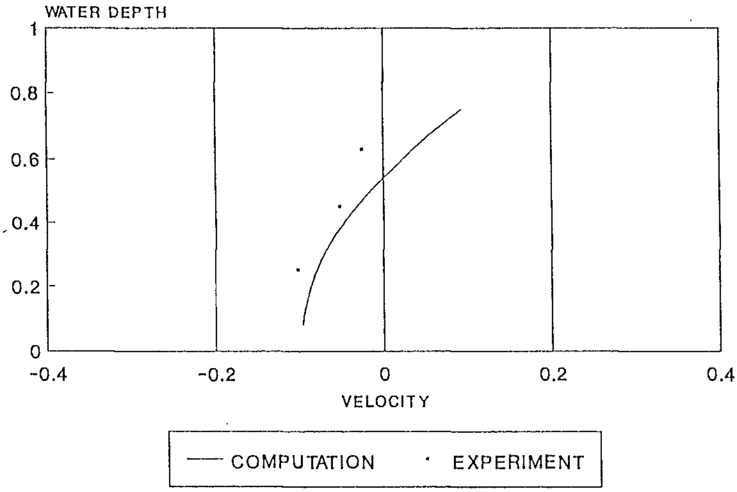

COMPUTATIONAL RESULTS FROM 3-D
WAVE-INDUCED CURRENT MODEL
WATER LEVEL SET-UP
0.05
0.04
0.03
0.02
0.01
0
-0.01
26
28
30
32
34
36
38
40
42
44
DISTANCE (m)
COMPUTATION
MEASUREMENT

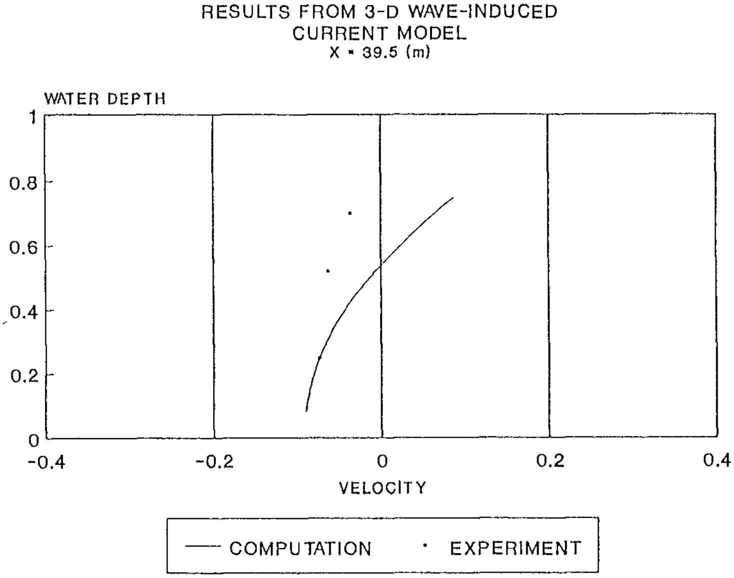

RESULTS FROM 3-D WAVE-INDUCED
CURRENT MODEL
X = 36.5 (m)
WATER DEPTH
1
0.8
0.6
0.4
0.2
0
-0.4
-0.2
0
0.2
0.4
VELOCITY
COMPUTATION
EXPERIMENT

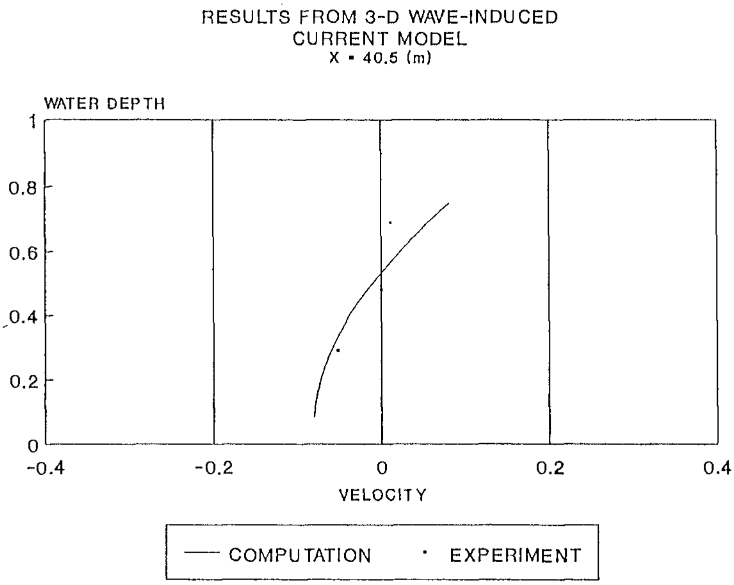
RESULTS FROM 3-D WAVE-INDUCED
CURRENT MODEL
X = 37.5 (m)



RESULTS FROM 3-D WAVE-INDUCED
CURRENT MODEL
X = 38.5 (m)




RESULTS FROM 3-D WAVE-INDUCED
CURRENT MODEL
X = 39.5 (m)
WATER DEPTH
1
0.8
0.6
0.4
0.2
0
-0.4
-0.2
0
0.2
0.4
VELOCITY
COMPUTATION
EXPERIMENT


RESULTS FROM 3-D WAVE-INDUCED
CURRENT MODEL
X = 40.5 (m)
WATER DEPTH
1
0.8
0.6
0.4
0.2
0
-0.4
-0.2
0
0.2
0.4
VELOCITY
COMPUTATION
EXPERIMENT