CHAPTER 138

Numerical Simulation of Finite Amplitude Shear Waves and Sediment Transport

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Abstraet

The shear instability of a uniform longshore current and the fully developed shear waves are studied in the numerical hydrodynamic model MIKE-21. The effect of the shear waves on the sediment transport, the cross-shore momentum transfer and mean velocity profile and the dispersion of suspended or dissolved matter is studied. The strength of the shear waves is calculated for a varying flow resistance and a varying momentum exchange coefficient. The formation of shear waves can be suppressed by modest rip channels in a longshore bar.

Introduction

Shear waves or far infra gravity waves is a phenomenon which was first observed by Oltman-Shay et al. (1989) from field measurements of wave-driven currents along an almost uniform coast. By analysing the correlation between simultaneous current measurements from different locations it was found that variations in the longshore current were actually a wave motion. The observed waves were only found in connection with a longshore current, propagating in the flow direction with a celerity of about half the maximum longshore current velocity. The observed wave lengths were so small when considering the wave periods that no theory for surface gravity waves can explain the dispersion relation for these waves. Therefore they were termed "far infra gravity waves".

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The formation of these waves was explained by Bowen and Holman (1989) as the result of an instability of an initially uniform longshore current. A linear stability analysis showed that the crosshore gradient (shear) in the longshore velocity profile is the cause of the instability, similar to the instability of a free shear layer. The instability of a uniform longshore current was apparently first noted by Hino (1974), who had to disregard the unsteady terms in the hydrodynamic equations in order to describe the morphological instability of a uniform sand coast. Since the paper of Bowen and Holman (1989) more refined stability analyses have been performed e.g. Dodd and Thornton (1990) and Svendsen & Putrevu (1992). One of the most important stabilizing factors is the flow resistance due to bed friction, which may cause a longshore current to be stable, analogous to a Hele-Shaw flow.

A linear stability analysis, as the ones listed above, assumes the deviation from the steady, uniform longshore current to be infinitely small and the strength of the shear waves cannot be predicted. By considering a situation very close to a neutrally stable longshore current, a non-linear perturbation analysis has been made (Dodd and Thornton, 1992), which can describe a stationary situation with shear waves. For situations which are far from neutral stability chaotic behaviour may occur, and it is not certain that analytical methods can describe such flows satisfactorily.

The formation of shear waves is strongly dependent on the coastal topography e.g. the presence of longshore bars cf. Svendsen & Putrevu (1992). The stability analyses consider a uniform coastal profile. It is not known if shear waves are formed on a non-uniform coast as for example a longshore bar with rip channels.

The present study

The purpose of this study is to investigate some properties of shear waves by use of a numerical hydrodynamic model, that simulates the unsteady wave driven currents along a coast. The hydrodynamic model used is MIKE-21 developed by the Danish Hydraulic Institute. The model solves the complete, unsteady depth integrated flow equations. In this way it is possible to make a simulation of the fully developed shear waves on a longshore current driven by waves breaking on a given topography.

An example of a topography is given in Fig. 1 that shows a cross section of a long straight uniform coast. The profile is barred, composed of a plane profile with a slope of $\tan(\beta) = 1:33.3$, superposed by a bar with a shape described by a Gaussian function. The still water depth is given by

$$D(x) = D_{p}(x) - (D_{p}(x) - D_{c}) \exp\left(-\chi \frac{(x - x_{c})^{2}}{x_{c}^{2}}\right)$$
(1)

where $D_p(x)$ is the plane profile:

$$D_{p}(x) = D_{0} - x \tan(\beta)$$
⁽²⁾

x is the cross-shore coordinate. The parameters D_c and x_c determine the still water depth over the bar crest and its position. χ determines the width of the bar. A similar bar profile was applied in the stability analysis by Svendsen & Putrevu (1992). For the profile in Fig. 1 D_c is 1.5 m, x_c is 200 m, and χ is equal to 30.

The wave description

The wave conditions are modelled by the MIKE-21 NSW module, which describes refraction and breaking of irregular waves with directional spreading. The wave heights are assumed to be Rayleigh-distributed, and the wave breaking is described by the model of Battjes and Janssen (1978). The wave modules are run for a rectangular modelling domain of 400 m × 3600 m with the long side parallel with the coast line, which is situated at the east boundary. Constant boundary conditions are prescribed at the west boundary: Significant wave height $H_s = 3.0$ m, mean wave period $T_m = 9.0$ s, wave direction $\alpha_0 = 45^\circ$ relative to the coast normal.

The north and south boundary conditions assume uniform wave conditions along the coast. The simulation is run on a grid $\Delta x = 1.0$ m and $\Delta y = 5.0$ m. Results from the wave module are shown in Fig. 1. The wave breaking is seen to reduce the wave height at the bar and at the shore line. The shore-normal radiation stress S_{yy} and the shear radiation stress S_{xy} vary according to the variation in wave height and direction.

The hydrodynamic simulation

The wave module gives the basis for calculating the radiation stress field, and by differentiation of the radiation stresses, the forcing from the waves is determined. The wave-driven flow is simulated by the hydrodynamic module MIKE-21, which solves the depth-integrated equations for conservation of mass and momentum.

The flow resistance is described by the Manning formula, and the bed shear stress $\tau_{\rm b}$ is calculated as

$$\frac{\tau_b}{\rho} = \frac{gV^2}{M^2 h^{1/3}}$$
(3)

where M is the Manning number, which is determined by the bed roughness, ρ is the density, g is the acceleration of gravity, V is mean flow velocity and h is the water depth.



Figure 1. Bottom: coastal profile. Top: Simulated wave height variation across the profile.

The boundary conditions for the hydrodynamic simulations are specified at the offshore boundary, where the water level is kept constant, and at the upstream and down-stream boundary, where the flux is specified with a distribution across the profile corresponding to a uniform longshore current. The grid size for this hydrodynamic simulation is $\Delta x = 2m$, $\Delta y = 4m$. The time step is $\Delta t = 1.25s$ and the simulation period is 3.0 h. In order to avoid a surge caused by the wave set up, the soft start facility is used, increasing the driving forces gradually from zero to the steady state conditions during a period of 2000s. The topography and the boundary conditions have thus been designed - as closely as possible - to give a steady, uniform longshore current after the warm up period. It appears, however, that the instability mechanism creates an unsteady meandering motion of the longshore flow, which is similar to the shear waves observed by Oltman-Shay et al. (1989).

Figure 2a shows a vector plot of the velocity field at the downstream point of the modelling area at time t = 3200 and 5200 s. Fig. 2b shows the instantaneous water surface elevation along the bar crest for fully developed shear waves. The instability mechanism can clearly be seen, the shear waves become visible at some distance from the upstream boundary. They grow in amplitude to reach an approximately uniform level.



Figure 2. A: Vector plots of the velocity field at the down stream part of the bar. B: Water surface elevation along the bar crest, fully developed shear waves.

Figure 3 shows time series of the longshore and the cross-shore velocity. The time series are taken in the point on the bar crest, 125 m from the downstream boundary. The period of the shear waves is approximately 200s and the wave length is 190 m, giving a phase velocity of 0.95m/s, which is about 55% of the maximum longshore velocity speed. It is seen that the oscillations have all the characteristics of the shear waves observed in the field measurements and described by the perturbation analyses. It has been attempted to make a faster growth of the instabilities by making an abrupt change in the topography near the upstream boundary. It was found that the irregular topography did not enhance the formation of the shear waves significantly.



Figure 3. Time series of longshore (dotted line) and cross-shore (full drawn line) velocities at the bar crest.

The sediment transport

Based on the results from the wave module and the hydrodynamic module, MIKE-21 can calculate the instantaneous sediment transport rate in every grid point of the hydrodynamic model. The sediment transport is calculated on basis of the water depth, the wave-averaged flow velocity, the wave height, period and direction and the wave breaking. The sediment transport model is based on the model by Fredsøe et al. (1985) including the effect of breaking waves according to the model by Deigaard et al. (1986). Fig. 4 shows the distribution of the calculated longshore sediment transport across the coastal profile. The time-averaged transport at a distance of 125m from the downstream boundary is shown together with a calculation corresponding to the steady uniform conditions that would be found if shear waves were not found. It can be seen that the formation of shear waves causes a reduction in the calculated longshore sediment transport of about 17% compared to the result that would be obtained by assuming steady uniform longshore current.

Due to the non-linearity of the hydrodynamics and the sediment transport the time average of the cross-shore sediment transport deviates from zero. Fig. 5 shows the time averaged cross-shore transport across the profile. The distribution is closely related to the distribution of the driving forces, i.e. the longshore current velocity profile. It tends to modify the bar profile, removing the material from the front of the bar and depositing it offshore and at the crest of the bar. The maximum cross-shore transport rate is of the order 1 m³/m hr, which is not negligible compared to other contributions.

Conditions close to neutral stability

The stabilizing mechanisms which may prevent the formation of shear waves in the model are the bed shear stress and the horizontal momentum exchange, modelled as a turbulent eddy viscosity term. In all runs (except when specificly mentioned) the momentum exchange coefficient E has been specified as zero.



Figure 4. The distribution of the time mean longshore sediment transport, q_s across the bar profile. Full drawn line: actual value; dotted line: assuming steady uniform conditions.

The flow resistance

The dependence of the shear waves on the flow resistance has been investigated by varying the Manning number in the example considered above. For a very large flow resistance (small Manning numbers) the longshore current is steady and uniform, but at a certain value ($M = 21.75 \text{ m}^{1/3}$ /s in the present case) shear waves are formed with an increasing intensity for increasing Manning numbers. The intensity of the shear waves is characterized by the standard deviation of the velocity fluctuations at the bar crest. The variation of the intensity with the Manning number is illustrated in Fig. 6. It may be noted that the flow resistance in Fig. 6 corresponds to very large bed roughnesses. A M of 21.75 m^{1/3}/s corresponds to a hydraulic bed roughness of 2.5 m, a M of 25 m^{1/3}/s corresponds to 1.1 m, 30 m^{1/3}/s to 0.37 m and 35 m^{1/3}/s to 0.17 m. The actual flow resistance will depend on the physical bed roughness and on the turbulent interaction between the wave boundary layer and the current, which will cause an increase in the flow resistance compared to a pure current situation. This mechanism has not been considered in details in the present study.



Figure 5. The distribution of the time-mean cross-shore sediment transport.

In this investigation shear waves have not been formed on a constant slope beach. A longshore current on a bar is expected to be more unstable, because it has two shear zones and can move freely onshore as well as offshore. For a given coastal profile the stability of a longshore current will depend on the formulation of the flow resistance and the distribution of the driving force $(\partial s_{xy}/\partial x)$ across the profile. For small deviations, u and v, from the uniform longshore current V_0 , the shear stress can be linearized to give:

$$\vec{\tau}_{b} = \frac{\rho g}{M^{2} h^{1/3}} \vec{V} | \vec{V} | = \frac{\rho g}{M^{2} h^{1/3}} \left\{ \begin{matrix} u \\ V_{0}^{+} v \end{matrix} \right\} \sqrt{u^{2} + (V_{0}^{+} v)^{2}} \approx$$

$$\frac{\rho g}{M^{2} h^{1/3}} \left\{ \begin{matrix} V_{0} u \\ V_{0}^{2} + 2 V_{0} v \end{matrix} \right\} = \left\{ \begin{matrix} 0 \\ \tau_{b0} \end{matrix} \right\} + \tau_{b0} \left\{ \begin{matrix} u/V_{0} \\ 2v/V_{0} \end{matrix} \right\}$$
(4)

where τ_{b0} is the bed shear stress corresponding to V_0 . τ_{b0} is thus identical with the driving force. For given wave conditions τ_{b0} is constant, and the flow resistance for the perturbations is proportional to M^{-1} or the Chezy coefficient. This flow resistance has many similarities to the 'strong current case' considered by Dodd

(1994). The assumption of irregular waves and the use of the Battjes and Jansen (1978) model for wave breaking give a rather smooth distribution of the driving force and the mean longshore current across a plane beach, which may give a rather weak instability mechanism.



Figure 6. The intensity of the shear waves as function of the Manning number. Full drawn line: longshore velocity, dotted line: cross-shore velocity.

The stabilizing effect of the momentum exchange coefficient

The dependence of the shear waves on the momentum exchange coefficient has been illustrated by runs similar to the runs with variable flow resistance. The Manning number is kept at $M = 25 \text{ m}^{1/3}$ /s, which gave shear waves for $E = 0 \text{ m}^2$ /s. Fig. 7 shows the intensity of the shear waves as a function of E. E is constant over the entire modelling area. The intensity of the shear waves is seen to decrease with increasing E, until a very low level is reached at $E = 0.1 \text{ m}^2$ /s. An increase in E may stabilize the flow in two ways: by giving a more smooth velocity distribution of the longshore current or directly by dampning the fluctuations. In the present case with a very smooth distribution of the driving force across the profile it is expected that the latter mechanism is the most important.

The exchange of momentum caused by the shear waves

The shear waves cause a redistribution of the momentum across the coastal profile, analogous to the Reynolds stresses in a turbulent flow. The momentum balance is considered at a location where the shear waves are fully developed. If there is no momentum exchange, the driving and the retarding forces are in balance, and the time-averaged bed shear stress will be identical to the radiation stress gradient $\partial S_{xy}/\partial x$.



Figure 7. The intensity of the shear waves as function of the momentum exchange coefficient E.

The cross shore momentum transport due to the shear waves can be calculated

$$\overline{\rho Duv}$$
 (5)

where an overbar signifies a time average.

Fig. 8 gives examples of "phase plots" showing the traces of the velocity vectors at three locations: at the bar crest, 40 m offshore and 40 m inshore of the crest. It is clearly seen that the correlation between the longshore and cross-shore velocity fluctuations give a non-zero cross-shore flux of momentum (defined by eq. 5) at the crest and - more pronounced - at the offshore location. At the inshore location the momentum transfer is apparently zero. Traces made at locations further offshore than the three shown here are similar to the inshore trace without any momentum transfer.

Fig. 9a shows the distribution across the coastal profile of the three elements in the momentum balance:

$$-\frac{\partial S_{xy}}{\partial x} = \bar{\tau}_b + \frac{\partial}{\partial x} \bar{\rho} D u v$$
(6)

as



Figure 8. "Phase plots" showing the traces of the current velocity vectors at three locations. v is the longshore velocity and u is the crossshore velocity.

It can be seen that the distribution of the mean bed shear stress is more smooth than the driving force, and the redistribution of the momentum corresponds to the transfer by the shear waves. Fig. 9b shows the mean velocity profile at this cross section together with the velocity distribution corresponding to a steady uniform longshore current. The reduction in the peak of the mean bed shear stress due to the shear waves is closely related to the reduction in the peak of the longshore velocity and the reduction in the sediment transport illustrated in Fig. 4.

In longshore current models the momentum exchange is often modelled by an eddy viscosity term. The maximum of the momentum transfer defined by Eq. 5 is close to the point of maximum gradient in the mean longshore current. The magnitude of the momentum transfer by the shear waves can be illustrated by the eddy viscosity that would be necessary to give a similar transfer of momentum. In the example considered this equivalent eddy viscosity E_{eq} is found to be

$$E_{eq} = \max(\overline{Duv})/\max\left(D\frac{d\overline{V}}{dx}\right) \approx 0.7 \ m^2/s$$
 (7)

This is a very large value, considering for example the drastic effect of a much smaller eddy viscosity on the flow regime, cf. Fig. 7.



Figure 9. (A) full drawn line: $\partial(S_{xy}/\rho)/\partial x$ broken line: $\overline{\tau}_b/\rho$, dotted line: $\partial(\overline{Duv})/\partial x$. (B) Time mean longshore velocity. Full drawn line: actual velocity, dotted line: assuming steady, uniform conditions.

Longshore non-uniformity of the coastal topography

The bar has been made non-uniform in the longshore direction by giving it a sinusoidal perturbation, representing very weak rip channels. The shear waves have been analysed for varying amplitudes of the perturbation. The strength of the shear waves is characterized by the standard deviation of the velocity fluctuations at the bar crest. Fig. 10 shows the strength of the shear waves as a function of the amplitude of the perturbation of the bar in the profile of Fig. 1. The longshore wave length of the perturbation is 400m. It is seen that the non-uniformity of the bar effectively suppresses the shear waves and that they have disappeared completely for an amplitude of 0.4 m.



Figure 10. The strength of the shear waves as function of the amplitude of the perturbation of the bar.

Dispersion by shear waves

In addition to the exchange of momentum, the shear waves are effective for dispersing suspended or dissolved matter in the surf zone. This is illustrated by a simulation with the advection-dispersion module of MIKE-21. The hydrodynamic simulation corresponds to the example considered previously. A constant source is placed at the bar crest near the upstream boundary. The dispersion coefficient has been specified to be zero. Figure 11 shows the instantaneous concentration field of the released matter. It can be seen that the plume is deformed by the shear waves and broken up in unconnected units - 'cat eyes', which are also observed in flow visualizations of shear layers. In the case where the source is located at a point offshore of the point of maximum mean longshore current velocity, the "cat eyes" will be formed around another row of vortices.

Conclusions

Shear waves can be simulated in a numerical model for wave-driven currents. The simulations can be used to study the formation of shear waves and the properties of the fully developed shear waves.

The shear waves are found to be effective in the cross shore exchange of momentum and matter. They reduce the longshore sediment transport and causes a cross shore transport. The present simulation can be improved in a number of ways, e.g. by including wave-current interaction in the description of the flow resistance and by including the effects of time-and-space lag in the description of the sediment transport.



Figure 11. Dispersion of suspended material dischanged at the bar crest.

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