CHAPTER 129

Line-Modeling of Shoreface Nourishment.

by
Willem T.Bakker[1], Nico F.Kersting[2] and Hanz D.Niemeyer[3]

Abstract

Using line modeling, in the scope of the NOURTEC project a prediction is made concerning the behavior of the shoreface nourishment of Terschelling and an evaluation concerning the behavior of the combined supply on beach and shoreface at Norderney. Computations suggest smaller long-shore diffusivity on the upper beach than on the zone between NAP-1 m to -3 m for Terschelling and the inverse for Norderney (groynes!). Cross-shore diffusivity at Norderney appears to be much larger than at Terschelling. If further investigations substantiate those results, these lead to the conclusion, that shoreface nourishment at Norderney is still more preferable than at Terschelling.

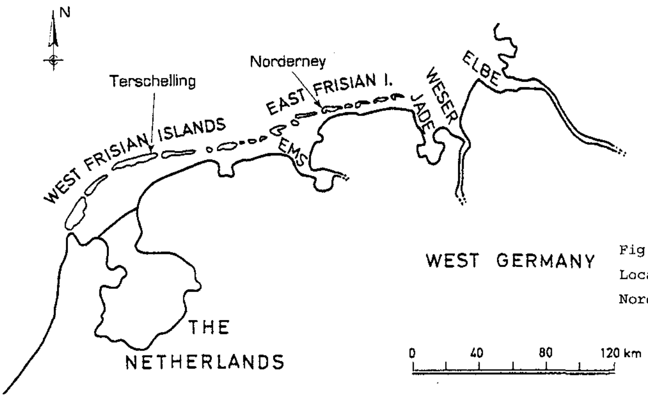


Fig.1.
Location Terschelling and Norderney

1 Introduction

In the scope of the NOURTEC project (cf.Mulder et al.,1994) national coastal authorities in three different countries around the North Sea (Germany, the Netherlands

1 Netherlands Centre for Coastal Research (on secondment from Rijkswaterstaat, The Hague); Delft Univ. of Techn., Stevinweg 1, 2628 CN Delft, The Netherlands

2 Delft Univ.of Techn., Stevinweg 1, 2628 CN Delft, The Netherlands

3 Coastal Research Station Norderney, An der Mühle 5, D2982 Norderney, Germany

and Denmark) have carried out full-scale experiments with alternative coastal nourishment techniques: shoreface nourishment and combinations of shoreface and beach nourishment. These experiments are implemented under a range of environmental conditions. Extensive modelling and monitoring programmes form an essential part of the projects. The present paper deals with the prediction of the coastal behaviour with the aid of line models and comparison with prototype data for the Dutch and German supply: at Terschelling and Norderney (fig.1) respectively. In the future, also an analysis of the Danish supply will be given. A more detailed description of the investigations is given by Bakker & Kersting (1994) and by Kersting, Bakker & Niemeyer (1995) respectively.

2 Terschelling

2.1 Situation

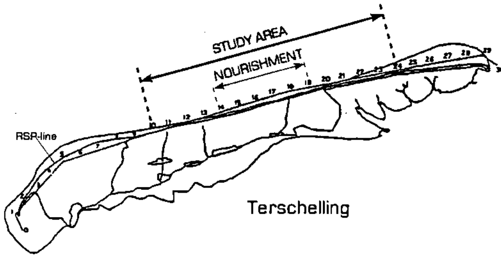


Fig.2 Location study area Terschelling

An underwater supply of ca. 2 Mm^3 has been carried out in spring and summer of 1993 on the Dutch Wadden isle of Terschelling. Fig. 2 shows the site. Three breaker bars are gradually moving seaward along the whole stretch; the motion is periodic. Mutual distance between the breaker bars is 500 m; seaward propagation velocity is 50 m/year and period 10 years. Depth-averaged (between NAP + 3 m and NAP - 6 m; "NAP" denotes a level, which is about mean sea level) the area under consideration shows a coastal retreat of about 2.75 m/year.

2.2 Three-line model

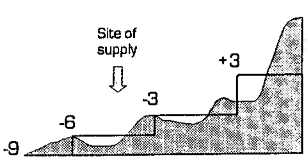


Fig.3. Schematization 3-line theory

Concerning the principles of line modeling is referred to the literature (Bakker, 1968a etc.). In the three-line model used the coastal zone is schematized in three zones (fig.3), which will be called: "beach", "inshore" and "deeper part".

In each zone it is assumed that all contour lines remain parallel and that the profile in this zone only moves horizontally. It is sufficient to compute only an average line in each zone, because it characterizes this zone. There is assumed to be a horizontal separation plane

between the zones. The sediment transport between the lines is assumed to be proportional to the difference between the equilibrium distance and the actual distance between the characterizing lines.
In the present application an initial equilibrium position (before any supply takes place) is assumed, where the three lines are straight and parallel (fig.4). Distances $y_{1,2,3}$, are the deviations of the lines of the beach, inshore and deeper part, compared with this initial position. The y-value is computed by dividing the surplus volume in a zone (with respect to the initial position) by its height. In case $y_1 = y_2 = y_3$, the profile again is an equilibrium slope. If longshore transport could be neglected (see Appendix) the coast would tend to this equilibrium, if for instance originally sand on the inshore would have been supplied (fig.4).

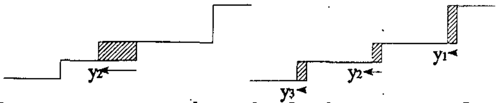


Fig.4. Definition y_1, y_2, y_3

The process, sketched above results in cross-shore diffusion of sand. Adding longshore diffusion to the cross-shore diffusion, according to the approach of Pelnard-Considere (1954) and Bakker (1968a), the following dynamic equations can be derived:

$$\frac{dy_1}{dt} = \frac{s_1}{h_1} * \frac{\partial^2 y_1}{\partial x_1^2} - \frac{s_{y1}}{h_1} * (y_1 - y_2) \tag{1a}$$

$$\frac{dy_2}{dt} = \frac{s_2}{h_2} * \frac{\partial^2 y_2}{\partial x_1^2} + \frac{s_{y1}}{h_2} * (y_1 - y_2) - \frac{s_{y2}}{h_2} * (y_2 - y_3) \tag{1b}$$

$$\frac{dy_3}{dt} = \frac{s_3}{h_3} * \frac{\partial^2 y_3}{\partial x_1^2} - \frac{s_{y2}}{h_3} * (y_2 - y_3) \tag{1c}$$

On the lefthand side is the accretion of line of beach, inshore and deeper part respectively. On the righthand side the gradient of the longshore transport is determined by the constants s_1,s_2 and s_3 respectively; the cross-shore transport ("beach to inshore" and "inshore to deeper part") by s_{y1} and s_{y2} respectively. $h_{1,2,3}$ denotes the depth allotted to the three zones. Fig 5 shows the diffusion of the (originally rectangular) supply shape in longshore direction in the course of time.

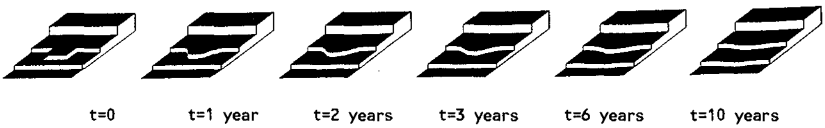


Fig. 5 Diffusion of supply

2.3 Coastal constants

Cross-shore constants: by the supply (between two breaker bars; fig.3) the breaker bars get an "antropogenic distortion". It is assumed, that by a diffusion process the sand, involved in this distortion will diffuse to the neighbouring breaker bars. Furthermore, it is assumed, that the velocity of diffusion is related to the time laps between the replacement of one bar by the next-landward bar. Per cycle, a certain percentage (determined by "engineering judgement") is retained; the rest of the sand is transported to the two adjacent breaker bars. The engineering judgement is based upon the solution of Smit (1987) for the cross-shore diffusion, reproduced in the Appendix. From his equations it can be derived, that in a period T_{01}(defined in the Appendix) the process of spreading the supply evenly over the profile is completed for almost 80 %. It does not seem illogical to assume, that the diffusion process to a similar rate is accomplished during the propagation period of the bars. The periods T_{01} and T_{02} are therefore chosen both equal to 10 years. With values for $h_{1,2,3}$ (6 m, 3 m,3 m; see fig.3), values of s_{y1} and s_{y2} of respectively .2 and .15 m/year are found. A higher cross-shore diffusitivity at the upper part is found than at the lower part, which seems reasonable.
It is assumed, that processes, dominated by the diffusion of the supply can be linearly superponed to the natural longshore and cross-shore coastal processes. The latter processes are implemented by assuming an autonomous erosion of 2.75 m/yr for the zone between NAP -6 m and NAP + 3 m (derived from Van Vessem, 1992)
In the mathematical model it is assumed, that cross-shore transport of supply below a level of NAP -9m is negligible (during the time of consideration) compared to the transport above this level. Thus, in the model no period-averaged cross-shore transport is assumed by the periodical motion of the breaker bars as such.

To determine the **longshore constants** for the line-model use is made of the CERC-formula and the assumptions of Svasek (1968): the local sediment transport between two contour lines is proportional to the longshore component of the decrease of the energy flux between those two lines. For detailed description is referred to Svasek & Bijker (1969), Bakker et al. (1971), Ten Hoopen & Bakker (1974) and to Bakker et al.(1988).

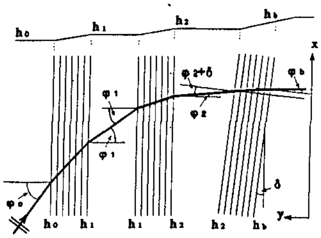


Fig.6 Schematization used for transport computations

Sloping shores (fig.6) have been assumed between terraces, separating beach, inshore and deeper zone; local height of the spilling breaker is taken proportional to local depth. Total wave climate has been taken into account, taking discrete wave classes, each defined by a certain wave height H, period T and direction ϕ and a probability of occurrence:

$$S_\phi = S_{H.T,\phi} * P_r(H,T,\phi) \qquad (2)$$

Fig. 7 shows the various classes of wave direction, translated into angles of incidence.
Vertical tide has been taken into account by schematizing it by a number of levels, combined with its probability of occurrence; furthermore it has been taken into account, that high waves from certain directions are linked with water level rise (storm surge height).

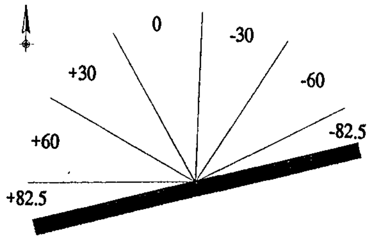


Fig.7. Directions of wave approach.

The black arrows in fig.8 give the transport rates when all contour lines are parallel. Under those circumstances the method results in a total depth-integrated longshore transport for a certain H,T,ϕ as given by the

CERC-formula: $$S_{H,T,\phi} = .040 H^2_{b,sig} n_b C_b \sin\phi_b * \cos\phi_b \qquad (3)$$

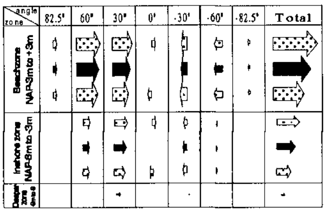


Fig.8 Transport rates along the three zones as function of angle of approach of the waves at deep water and the effect of $\delta = \pm 2°$.

	82.5°	60°	30°	0°	-30°	-60°	-82.5°	ΣS
+2°	41.42	500.4	431.4	40.53	-114.6	-84.83	-10.14	804.2
0°	40.55	478.3	387.3	0	-128.7	-87.91	-10.25	679.3
-2°	39.46	453.8	341.2	-40.53	-142.1	-90.54	-10.30	551.0
+2°	0.004	68.36	161.9	15.22	-38.16	-8.830	0	198.5
0°	0.004	67.57	148.5	0	-41.98	-8.818	0	165.3
-2°	0.004	66.30	134.4	-15.22	-45.56	-8.732	0	131.2
0°	0	0	5.518	0	-1.428	0	0	4.09

Table 1. Transport rates (10^3 m^3/yr)
Volumes of the arrows in fig.8 are proportional to numbers in table 1.

The columns in fig.8 denote the various angles of wave incidence; the 3 sets of 3 rows the 3 zones. The size of the arrows is proportional to the transport (weighted with its probability of occurrence) integrated over all classes of H and T. Probability of occurrence has been derived from SON, a measurement station in the vicinity of Schiermonnikoog. Wave data have been made available by Rijkswaterstaat (Roskam 1988).
Calculated numerical values of transport rates (in thousands m^3/year) are depicted in table 1 (the lines "0 degree "; for beach, inshore and deeper zone respectively).

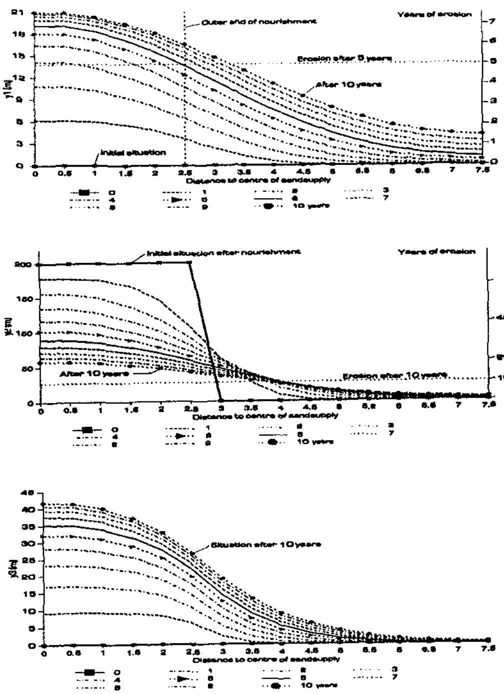
For both upper zones the longshore transport has been computed three times for each wave direction: apart from assuming parallel depth contours also the angle of the beach (inshore) has been turned over an angle δ with respect to the other contour lines. For instance: in fig 6 the angle of the beach is turned. The gray arrows in fig 8a show the effect, where δ has been chosen $\pm 2°$. Table 1 shows the numerical results.

The constants s_1 and s_2, which can be shown theoretically (Pelnard Considere, 1954) to be the derivative of the longshore transport to the coastal direction (atan $\partial y/\partial x$), are calculated from:

$$s_i = \frac{\sum_{+82.5°}^{-82.5°} S_i(\phi_i + \delta) - \sum_{+82.5°}^{-82.5°} S_i(\phi_i - \delta)}{2\delta} \tag{4}$$

2.4 Results

From the last column of table 1 s_1 is found by substitution of the total transport of $804.2*10^3$, resp. $551*10^3$ m^3/year for the terms in the numerator of (4) (δ equals $\pi/90$). A similar computation gives s_2; then for s_1, resp. s_2 values of $3.63*10^6$, resp. $0.964*10^6$ m^3/year/rad are found. A total transport of $(697.3+165.3+4.09)*10^3$= $.85*10^6$ m^3/year results.
It was found, that diffusitivity in longshore direction along the deeper part could be neglected. Effect of curvature of the inshore on the coastal constant for the beach (the inshore turns more or less in the same way at the same time as the beach) has not been taken into account. Neglecting this effect can give a rather important over-estimate of the coastal constant of the beach (Bakker & Delver, 1986; Bakker et al., 1988).
Another extreme (no longshore diffusion) is given by the formulae of Smit (1987) (Appendix)



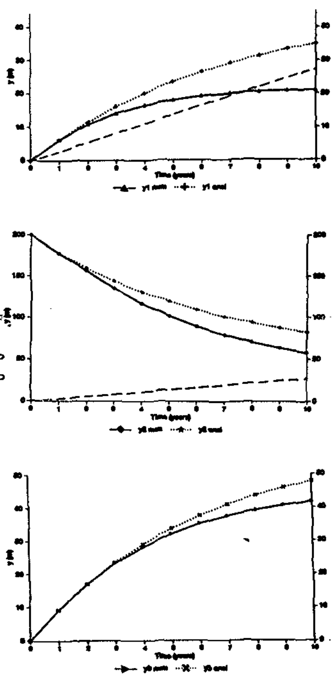


Fig 9a,b,c. Lines of beach, inshore and deeper part $y_{1,2,3}$

Fig.10a,b,c Site of beach, inshore, deeper part in the course of time (in center of supply).

Fig 9 a,b,c shows (extreme high diffusitivity) a plan view of the expected development of beach, inshore and deeper part (mind the different vertical scales). The autonomous erosion of the coast is depicted on the righthand side of the figure.

Fig.10a,b,c show the expected position (in the course of time) of the line of the beach, inshore and deeper part, in the center of the supply area.
Both extremes (no c.q. extreme longshore diffusitivity) are shown, where autonomous erosion is not taken into account.
The positive direction denotes a more seaward position. The drawn lines show the development, if one ignores the autonomous coastal erosion. The beach would increase, the inshore (where the supply was applied at t=0) would retreat from its exposed position. Finally, supply sand would spread evenly over the profile.

The interrupted lines in fig. 10 a,b,c show the rate of autonomous coastal erosion at the same scale, more exactly: the amount of cumulative supply, necessary to balance this kind of erosion.

During the time the drawn line lies above (c.q. below) the interrupted line, the beach, c.q. inshore has a more seaward (c.q. landward) position than just before the application of the supply.
It shows, that according to the computations the supply will stop the erosion of the beach during the order of a decennium, without causing considerable accretion.

3 Norderney

3.1 Conditions at the site of supply.

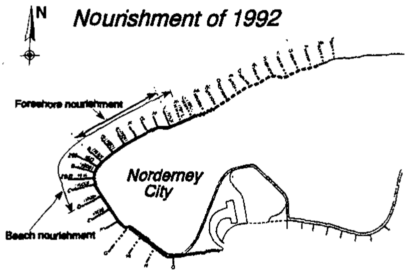


Fig.11 Combined beach and shoreface nourishment. Coastal protection.

Near the west point of the German Wadden island of Norderney, 450.000 m^3 of sand has been supplied in 1992 to the beach and the shoreface (fig. 11).
The seaward limit of the nourishment coincides with the 5 m depth contour. The coastline is protected by groynes, revetments and seawalls. For the largest part the area is inside the "Riffbogen" (bars formed by the outer delta).
Wave attack is important on the North West side.

At the South West side (Norderneyer Seegat) current dominates (cf. Thilo & Kurzak (1952), Luck (1977) and Niemeier (1986 a,b,c, 1990 and 1992)).

Since the first artificial nourishment (1.25 Mm^3 in 1951/52) (major) sand supply in this area and its vicinity was repeated 6 times (incl.1992). Fig. 12 shows the area of coastal nourishment in 1992: an amount of .5 Mm^3, of which 50 % supplied above NN-1m (as NAP the level NN is about mean water level). Ca. 20% was supplied between NN-3m and NN-5m.

3.2 Wave climate and wind set-up

The outer delta largely shelters the considered area from the deep water wave field. However, even more than at Terschelling, high waves and high surge levels are often linked together. Kersting et al.(1995) compute the sand transport using the SON deep water wave climate as well for Terschelling as for Norderney. However, storm surge levels at Norderney are higher than at Terschelling. Thus, if Norderney would have been located near Terschelling, its (present) outer delta would give better protection; it would reduce wave height at the Westpoint shore more than in the present situation. Kersting et al. estimate that the effect of location on the surf height results (for the same wind, i.e. offshore wave height) in surge top water level rises, being 22% higher at Norderney than at Terschelling. They use as well theoretical considerations (Schalkwijk (1947) and Weenink (1958)) as well as analysis of the SON wind and wave data.

3.3 Sand transport computations

3.3.1 Schematization

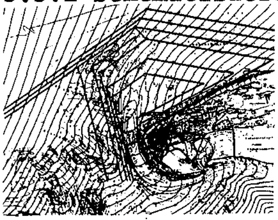


Fig.12a Refraction pattern

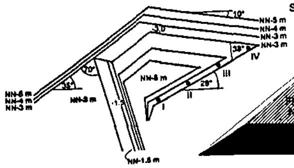


Fig.12b Schematization and points I to IV

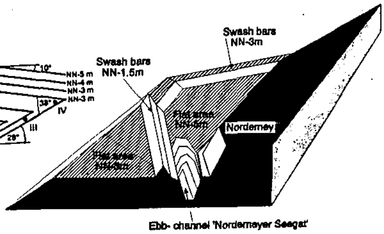


Fig 12c Schematization (3D)

Investigations concern the coastal area with a length of ca 5 km between the tidal inlet and the site where the outer delta merges with the island (between point I to IV; fig 12b).
The wave climate on the inshore and the beaches in this area is strongly influenced by the bars of this outer delta. These bars reduce the energy of the incoming waves and also influence the propagation direction of the waves by refraction.
Fig 12b,c shows the schematization chosen; contour lines are schematized to sets of parallel straight lines. Furthermore, fig. 12a displays the real contour lines and a refraction diagram, from Niemeyer (1986), which indicates analogy in similarity of hydraulic response between schematization and prototype. Further research with respect to this aspect is proceeding. Convergence and divergence of wave rays are, in the schematization, partially taken into account: difference in direction of beach and inshore is implemented. However, the effect of the curvature of beach and inshore on the refraction coefficient is not taken into account. By the chosen schematization, the three parts of the outer delta together act as a convex lens, focussing the wave rays. From the refraction diagram of fig.12a the same picture arises. Because of this, a convex beach, making everywhere (about) the same angle with the wave crests will be stable, where for an offshore topography with totally

straight contour lines, parallel to the beach, a straight beach would have been stable. This explains the convex character of the Westend of the island.
Up to now, only the transport along the shore (beach and inshore) of the island has been calculated. The transport along the seaward shoreface of the outer delta will be calculated in the future. Again, the Svasek method of computation has been used (cf. ch.2 for literature). Effects of current are not taken into account.
Preliminary computations concerning diffusitivity constants $s_{1,2}$ have been carried out (for point IV; fig.12b) by varying the direction of beach or inshore as done in ch.2. Apart from this, diffusitivity constants for Norderney will be calculated from measured data (sect.3.4)

3.3.2 Results

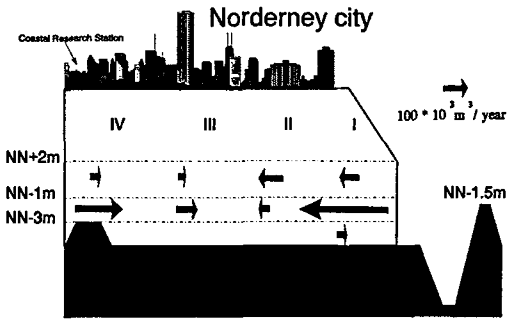


Fig.13 Transport capacity (without groynes) if the coast would be straight

Fig. 13 shows the calculated transport capacities in the points I to IV , which would occur, if the coastline would be straight. It shows, that accretion in the coastal area between points I and IV would result (according to the computations). Longshore diffusitivity constants s_1 and s_2 in point IV are found of $.6*10^6$ and $2.8*10^6$ m^3/year/rad for beach and inshore respectively. From the values calculated one would find, that a convex coast with a difference in coastal direction of ca.9° to 12° between point I and point IV (fig.12b) would give a stable coast. The present difference in coastal direction is ca 15° which is more; thus erosion (as occurs) can be expected. For the computations is referred to Kersting et al.(1995). As accuracy is low, the result has only qualitative importance.
Because of the existing groynes the real transport, along the inshore, will be much less than the transport capacity. Bakker & Joustra (1970), comparing coastal erosion along the dutch coast before and after the construction of groynes, indicate, that groynes might reduce coastal transport with a factor of 4. This would reduce the value of s_2 (in case of erosion) in the same way.

3.4 Line modeling

3.4.1 Coastal constants; relation between curvature and erosion

Bakker (1968b) describes another way to find the coastal constants: from coastal measurements. Related are the data of volumetric change and curvature; the latter taken as difference of coastal direction $\Delta\phi$ between left side and right side of the stretch. In order to obtain uniformity of data, a stretch of (only) 890 m (between groynes "A" and "H1") near the Western end of Norderney has been analyzed. Fig. 14 displays this relationship; from this figure Kersting et al.(1995) derive values of s_1=1.1*10^6 and s_2=.9*10^6 m^3/year/rad. As for the first observation (encircled in fig.14b) leading to s_2 the groynes will have been less effective (the supply extended further seaward than the groynes), this observation has been discarded, leading to s_2=.2*10^6 m^3/year/rad. Based upon fig. 14d, s_3 has been taken zero. The constants $s_{y1,2}$ (both assumed as 4 m/year) have been found by estimating T_{01} from the measurements and by iterative approximation, using the numerical three-line model (ch.2.2). Initial- and boundary conditions are imposed by prototype data.

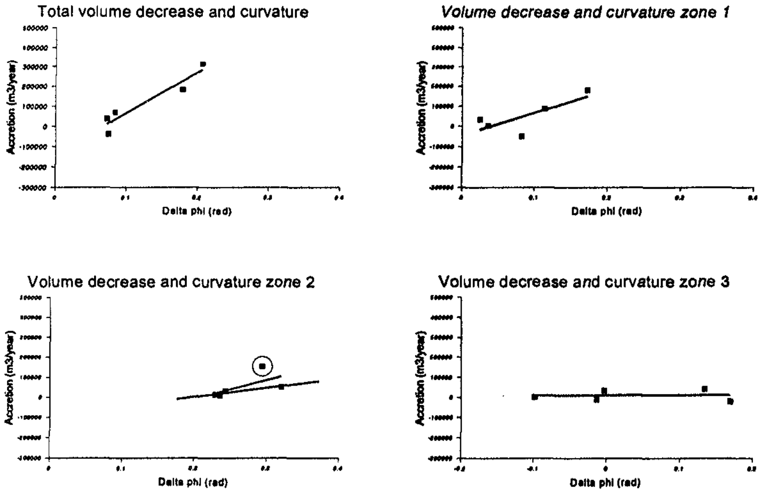


Fig.14a,b,c,d Volume decrease & curvature

3.4.2 Results

In the Norderney case, "$y_{1,2,3}$" will be defined as the surplus volume since the supply, (expressed in m^3/m'/m depth) between 2 successive separation planes (see fig.13). Comparison between measurement and computation show fig.15 ($y_{1,2,3}=f(x);t$ as parameter) and 16 ($y_{1,2,3}=f(t)$; x in the center of the field). For y_1 and y_2 good agreement is found; y_3, which in the model can only change because of cross-shore transport, appears to remain too large in the model. This suggests scour by current in the area below the groynes.

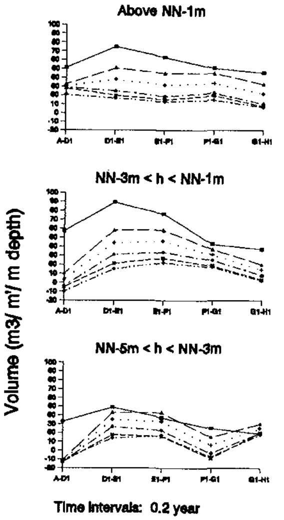


Fig.15a. Interpolated measurements

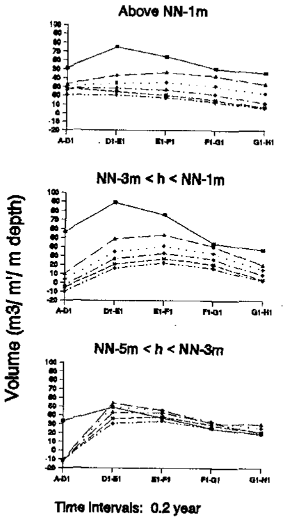


Fig 15b.Computations

Volumes in groyne fields as function of time

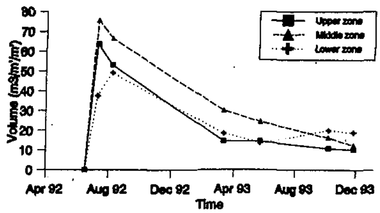


Fig 16a.Measured

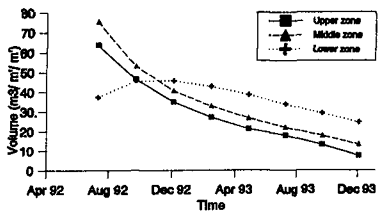


Fig 16b.Computed

Volumes in central groyne section

4 Discussion and conclusions.

4.1 Terschelling

- Depending on the assumptions (of which the extremes are: "only cross-shore diffusitivity" and "cross-shore plus unfavorable high value of longshore diffusion", a "life time" of the supply can be estimated, which varies respectively between 13 and 7 years (estimating the autonomous erosion as 2.75 m/year).

4.2 Norderney

- Concerning Norderney only preliminary conclusions and some indications can be given. Comparing longshore coastal constants derived from transport formulae (sect.3.3) and from evaluation of topographic field data the s_2-value, found from those field data is lower than expected (even considering the effect of the groynes), where the s_1- value is considerably larger. The total of s_1+s_2 is in the expected range.
- The coastal measurements indicate much larger longshore diffusivity on the beach (above NN - 1 m) than on the inshore (NN-3 m<h<NN-1 m).
- In the future effects of currents should be carefully examined, especially concerning the zone seaward of the groynes. For the three-line model (which is able to reproduce the development of beach and inshore rather well) underestimates (without current effects) the erosion of the deeper zone.

4.3 General

- Comparison of coastal constants in general for various stretches of coast is given by Bakker et al. (1988). In that scope, the maximum values of the longshore diffusivity assumed for Terschelling are rather high: values of 1 to $2*10^6$ are more common.
  The cross-shore transport constants $s_{y1,2}$ are (physically) strongly dependent upon the depth of the separation plane, as well on the schematization. Compared to the values, mentioned by Bakker et al.(1988), the cross-shore transport constants assumed for Terschelling are somewhat low. For instance, using Kalman filtering, de Vroeg (1987) finds $s_y \approx .7$ m/year at NAP - .72m ($T_{01} \approx 7$ years) for the Dutch Rijnland-coast. Compared to this, the value of $s_{y1,2}$ of 4 m/year at NAP -3m ($T_{01} = 0.4$ year and $T_{02} = 0.25$ year) seems extremely high. Up to now, T_0-values of the order of 1 year were the smallest ever found (in Cadzand, CUR et al.,1986; s_y=3 m/year at NAP -2m). As in Norderney, in Cadzand a tidal channel is near to the coastal area. In the Norderney case, the wave attack is probably larger than in Cadzand.

Remarkable is the difference in character between the supplies of Terschelling and Norderney:.

- Supplying high on the beach instead of on the inshore would have been favorable for the Terschelling case if the costs of supplying on beach or inshore would have been equal. In Norderney however this is different because (most probably) the groynes largely reduce the longshore diffusitivity of the inshore (compared to the beach).
- Probably the role of the deeper zone in Norderney will show to be more active (current effects near to the Norderneyer Seegatt) than at Terschelling.
- Different as well is the goal of both supplies, which emphasizes the importance of the offshore nourishment in Norderney: here the supply forms an essential part as a maintenance measure for the coastal protection by reducing the damage to the coastal defence works (sea-walls and groynes). This goal was formulated already in 1952 (Arbeitsgruppe Norderney); it has been shown to be very cost effective since then.

Appendix. Smit's solution of the two-dimensional three-line problem.

The dynamic equations for the cross-shore transport S_{yi} between the layers (with $i=1,2$) read:

$$S_{yi} = s_{yi}(y_i - y_{i+1}) \qquad (A1)$$

Continuity ($\Sigma(h_i \cdot y_i) = 0$) with $i = 1..3$ shows, that y_2 is implicitly determined by y_1 and y_3.

Continuity also yields: $S_{y1} = -h_1 dy_1/dt$ $\qquad S_{y2} = +h_3 dy_3/dt$ $\qquad (A2a,b)$

Combination of (A1) and (A2a,b) leads to the vectorial equation: $d\vec{y}/dt = \bar{\bar{A}}.\vec{y}$ $\qquad (A3)$

where $\vec{y}$ is $y_{1,3}$ and the matrix $\bar{\bar{A}}$ is $(a_{11}..a_{22})$, where:

$$a_{11} = -\frac{s_{y1}.(h_1+h_2)}{h_1.h_2} \qquad a_{12} = -\frac{s_{y1}.h_3}{h_1.h_2} \qquad a_{21} = -\frac{s_{y2}.h_1}{h_2.h_3} \qquad a_{22} = -\frac{s_{y2}.(h_2+h_3)}{h_2.h_3} \qquad (A4)$$

Displaying the solution gives a reason to define 5 timescales. Two of those are the time scales T_{01} and T_{02} of the two two-line systems : "beach + inshore" and "inshore + deeper part" (Bakker, 1968), equal to $-1/a_{11}$ and $-1/a_{22}$ respectively. The third one, T_* is related to the other diagonal of the matrix $\bar{\bar{A}}$:

$$T_*^2 = \frac{1}{a_{12}.a_{21}} = \frac{h_2^2}{s_{y1}.s_{y2}} \qquad (A5)$$

The last ones are $1/\lambda_{1,2}$, as occur in the solution of (A3): $\vec{y} = \vec{K}e^{\lambda_1 t} + \vec{M}e^{\lambda_2 t}$ $\qquad (A6)$

where $\vec{K} = (K_1, K_2)$ and $\vec{M} = (M_1, M_2)$, in which:

$$K_1 = -\frac{a_{21}}{\lambda_2 - \lambda_1} y_{30} + \frac{\lambda_2 + \frac{1}{T_{01}}}{\lambda_2 - \lambda_1} y_{10} \qquad M_1 = +\frac{a_{21}}{\lambda_2 - \lambda_1} y_{30} - \frac{\lambda_1 + \frac{1}{T_{01}}}{\lambda_2 - \lambda_1} y_{10} \qquad (A7a,b)$$

$$K_2 = \frac{1}{a_{12}}.\left(\lambda_1 + \frac{1}{T_{01}}\right) K_1 \qquad M_2 = \frac{1}{a_{12}}.\left(\lambda_2 + \frac{1}{T_{01}}\right) M_1 \qquad (A7c,d)$$

where y_{10} and y_{30} are the initial values of y_1 and y_3.

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