CHAPTER 81

WAVE FORCES ACTING ON A VORTEX EXCITED VIBRATING CYLINDER IN WAVES

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ABSTRACT

An experimental investigations into the wave forces acting on the vortex-excited vibrating vertical circular cylinder in regular waves have been performed with emphasis being placed on the amplification of the wave forces caused by the fluid-structure interaction. The cylinder is vibrating only in the transverse, cross-flow, direction by means of the restriction of the vibration of the in-line direction. The results indicate that the existence of amplification of the lift forces acting on the vortex-excited vibrating cylinder in comparison with the stationary cylinder is a function of the ratio of wave frequency f_w to the natural frequency of the cylinder in water f_{nw} and Keulegan-Carpenter number at still water level CKC. The in-line forces are also amplified in the range of CKC where large amplitude of oscillation in the transverse direction occurs.

INTRODUCTION

The wave forces acting on a small diameter offshore structure are usually resolved into two components. One, the inline force, acts in the direction of wave propagation and the other , the lift force or transverse force, acts in the transverse direction of it. The predominant frequency of lift force caused by vortex shedding is a multiple of that of the inline force. Therefore, the

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structure's dynamic response to the lift forces "Vortex-excited vibration" must be considered more significantly.

A great number of studies for the vortex-excited vibration of a circular cylinder in steady flow have been made. An important phenomenon of this vibration is that of "lock-in" between the frequency of the vortex shedding and the frequency of the vibrating cylinder. Under "lock-in" condition, large resonant vibration occurs and the lift forces acting on this vibrating cylinder are amplified by the fluid-structure interaction, Blevins (1977).

A similar phenomenon may occur under certain conditions if a flexible cylinder is placed in planar oscillatory flow or in waves . However, this has not been sufficiently understood and relatively little work has been carried out into this interesting problem in hparmnic flow and in waves.

The results for the amplification of the forces acting on a vortex-excited vibrating cylinder in harmonic flows have been reported by Sarpkaya and Rajabi (1979), Hayashi et al.(1990) and Sumer et al.(1994). Sarpkaya and Rajabi(1979) show that at perfect resonance, the lift forces acting on a vortex-excited vibrating cylinder in plannar oscillating flow are amplified nearly two times compared to that of rigidly mounted cylinder. Hayashi et al.(1990) show that the lift and in-line forces acting on a vortex-excited cylinder vibrating only in cross flow direction being resonant with the second harmonic component of the lift are amplified in the ranges of Keulegan-Carpenter number 4<KC<8 and 6<KC<16 respectively. The maximum increase in the lift force is about 200% at around KC=6 and that in the in-line force is about 100% at around KC=8. Sumer et al.(1994) have studied the influence of KC, reduced velocity V_r, amplitude of cylinder vibration and wall-proximity effect to the forces acting on a cylinder vibrating only in cross-flow direction. Their results show that the increase in the drag coefficient is about 50-200% for around KC=10 and nearly negligible for the range 60<KC and the increase in lift force is up to 200%.

The results for the amplification of the forces acting on a vortex-excited vibrating vertical cylinder in waves have been reported by Isaacson and Maull(1981), Zedan and Rajabi(1981), Angrilli and Cossalter(1982). They show that the increases of lift forces acting on a vortex-excited vertical cylinder are 70-290% for f_w/f_{nw}=1/2, CKC=10-12 and kd=1-3.9, 60% for f_w/f_{nw}=1/3, CKC=17.8, and kd=0.88, and 90% for f_w/f_{nw}=1/4, CKC=35.9, and kd=0.62, where f_w and f_{nw} are the incident wave frequency and the natural frequency of cylinder in water, k is the wave number, and d is the mean water depth.

Hayashi(1984) shows that the lift forces acting on a vortex-excited vibrating vertical cylinder in waves are amplified in the range of 5<CKC<15 for $f_w/f_{nw}=1/2$ and kd=1.83 and in the range of 18<CKC<30 for $f_w/f_{nw}=1/3$ and kd=1.01. Maull and Kaye(1988) shows similarly that when f_w/f_{nw} is fixed at 1/2, the lift forces are amplified in the range of 5<CKC<12 for kd=1.32 and 1.73.

The forces acting on a partial parts of a flexibly mounted vertical cylinder in waves have been measured by Bearman(1988) and Borthwick and Herbert(1988). Force coefficients of in-line and lift forces from the flexibly mounted cylinder are found to be larger than those for the same cylinder but with a rigid mounting.

In the present paper, experimental investigations into the wave forces acting on the vortex-excited vibrating vertical cylinder in regular waves have been described with emphasis being placed on the amplification of the forces acting on the cylinder caused by the fluid-structure interaction.

EXPERIMENTS

Laboratory experiments were carried out in a wave flume 40m long, 0.8m wide, and 1m deep. The general arrangement of the test cylinder is shown in Figure 1. In order to give a high degree of rigidity to the test cylinder, both ends of it are connected to the core cylinder. The flange weights are attached to the core cylinder above the test cylinder to adjust the equivalent mass m_e of the test cylinder.

The support plate is attached to the holder flag at the bottom end of the test cylinder. Both end in the inline direction of the support plate are pivoted on the floor of the flume to prevent vibration in the inline direction. The upper end of the core cylinder, above the water level, is mounted with springs only in the transverse direction. Each spring is connected to a strain-gauged steel canti-

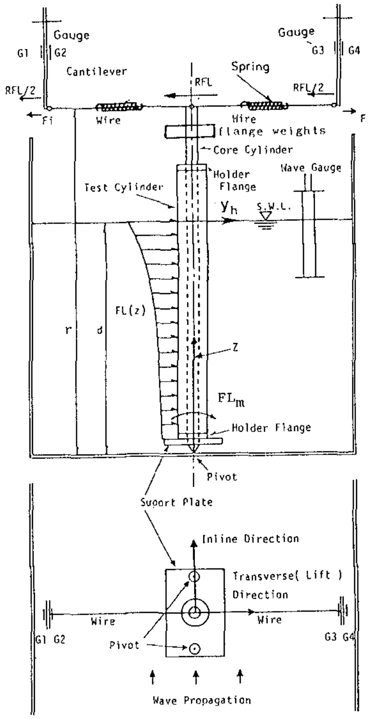


Fig. 1 General Arrangement of Test Cylinder

lever by wire. Two strain gauges were fixed on each cantilever to measure its bending moment produced by the force acting on the end of it through the springs and wires. Strain gauges were connected into a wheatstone bridge circuit in the bridge conditioner to produce the out put signal corresponding to the displacement of the top end of the core cylinder in the transverse direction.

In order to obtain the relationship between the output voltage signals from the bridge circuit and the top end displacements of the core cylinder, known loads are applied horizontally to the top end of the core cylinder by weights hung over a pulley.

Free vibration tests are performed in air and in still water. The damping factor, obtained from the logarithmic decrement, and the natural frequency of the test cylinder are measured by releasing its top end from an initial displacement and recording amplitude decay of the transient oscillation of the cylinder. The equivalent mass per unit length of the test cylinder in water, m_e, is calculated from the measurements of natural frequency and stiffness of test cylinder in still water, Hayashi(1984). Thus the value of m_e includes the mass of structure and the entrained fluid.

Two kinds of experiments, Tests-A and Tests-B, are carried out. The experimental conditions in Tests-A are shown in Table 1. Where $k(=2\pi/L$, L=wave length) is the wave number , f_{na} and f_{nw} are the natural frequencies of the test cylinder in air and in water , h_{ta} and h_{tw} are damping factor of the cylinder in air and in water, ρ is the density of water, and $\beta\,(=D^2 f_w/\nu$, ν =kinematic viscosity of water) is the viscous-frequency parameter.

The still water depth d is 40cm. The test cylinder, of outside diameter D=1.9cm and 73.7cm length, is at-

Table 1 Experimental Conditions in Tests-A

Case	D (cm)	d (cm)	f_w (Hz)	f_w/f_{nw}	kd (2π.d/L)	CKC	β
V1	1.9	40	1.07	1/2	1.92	5.2-26.4	386
V2	1.9	40	0.7	1/3	1.03	5.6-38.0	253
V3	1.9	40	0.53	1/4	0.73	4.5-35.1	191
	f_{na}=2.17Hz, f_{nw}=2.12Hz h_{ta}=0.002, h_{tw}=0.005 Mass ratio : $m_e/(\rho_w D^2)$=14.9 Reduced damping : $K_s=2m_e(2\pi h_{ta}/(\rho_w D^2))$=0.37						
S1	1.9	40	1.07		1.92	5.5-25.5	386
S2	1.9	40	0.70		1.03	2.8-39.9	253
S3	1.9	40	0.53		0.73	2.1-36.4	191

tached to the support plate(25x10x0.2cm) with the core cylinder of 0.9cm diameter and 78.5cm length.

In Case V1-V3, the relationship between the vortex-excited vibration of the test cylinder and the Keulegan-Carpenter number at still water level $CKC=U_{ms}\cdot T/D$, is measured. Where U_{ms} is the maximum horizontal water particle velocity at still water level and T is the wave period. In each of these Cases, the frequency ratio, f_w/f_{nw}, is fixed at around one value of the values of the resonance frequency ratios, $f_w/f_{nw}=1/2$, 1/3, and 1/4.

In Case S1, S2, and S3, the base bending moment FL_m(=the moments about the bottom of the test cylinder) due to the lift forces acting on the test cylinder rigidly mounted with strings replacing the springs in the transverse direction are measured in the similar waves used in the Case V1, Case V2 and Case V3.

In Tests-B, in-line and transverse forces acting on a partial part of the test cylinder are measured by using a force sleeve supported by two components small load cell installed in the test cylinder as shown in Figure 2. The force sleeve of outside diameter D=3cm and 3cm length is positioned 15.9 cm below still water level with water depth d=80cm. The test cylinder of outside diameter D=3cm and 98.5cm length is attached to the support plate(20x5x0.6cm) with the core cylinder of 1cm diameter and 104.6cm length. The conditions are shown in Table 2. The measurements are made for both cases of vortex-excited cylinder being resonant with the second to sixth harmonic components of lift forces($f_w/f_{nw}=1/2-1/6$) and rigidly mounted cylinder for comparison.

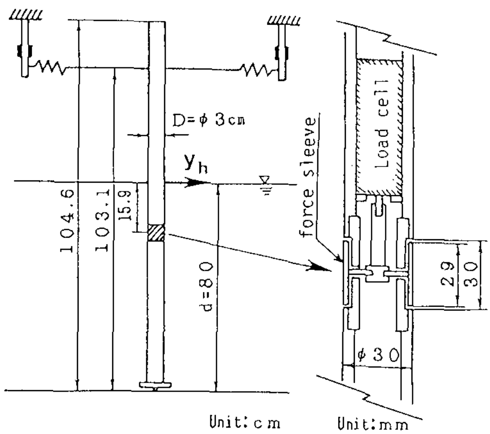


Fig. 2 Experimental Set-up (Force Sleeve and Load Cell)

Table 2 Experimental Conditions in Tests-B

Case	D (cm)	d (cm)	f_w (Hz)	f_w/f_{nw}	kd (2π.d/L)	CKC	β
LV2	3.0	80	0.85	1/2	2.41	6.0-20.6	655
LV3	3.0	80	0.56	1/3	1.20	6.9-30.5	426
LV4	3.0	80	0.42	1/4	0.90	4.4-29.7	342
LV5	3.0	80	0.34	1/5	0.69	9.3-29.4	274
LV6	3.0	80	0.28	1/6	0.56	5.2-40.7	228

f_{na}=1.79Hz, f_{nw}=1.69Hz
h_{ta}=0.0035, h_{tw}=0.014
Mass ratio, : $m_e/(\rho_w D^2)=8.34$
Reduced damping : $K_s=2m_e(2\pi h_{ta}/(\rho_w D^2))=0.37$

MODEL FOR VIBRATION

The definition sketch of the test cylinder, which illustrates the transverse response of the cylinder which is rod pivoted on the bottom of flume and supported by spring in the transverse direction, is shown in Figure 3. The dynamic response of the cylinder to the lift forces may be described by using the equation of motion as

$$M_{mt}\cdot\ddot{y}_h + C_{mt}\cdot\dot{y}_h + K_{mt}\cdot y_h = FL_m, \qquad ---(1)$$

where $\ddot{y}_h$, $\dot{y}_h$ and y_h are the transverse displacement, velocity and acceleration of the cylinder at still water level, M_{mt}, C_{mt} and K_{mt} are the effective mass , damping and stiffness of system. M_{mt} includes the mass of cylinder and added mass in water. C_{mt} includes the structural damping and fluid damping. K_{mt} includes the stiffness due to the spring force, and the buoyancy and distributed weight when the cylinder is in a deflected position from the vertical. FL_m is the bending moments around the pivot which is produced by the lift force acting on the cylinder.

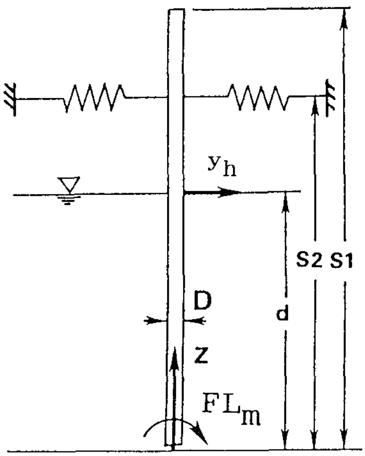


Fig. 3 Coordinate System

$M_{mt}\cdot\ddot{y}_h$, $C_{mt}\cdot\dot{y}_h$ and $K_{mt}\cdot y_h$ show the moments produced by the inertia forces, the damping forces and the stiffness of the structure respectively. Thus, Eq.(1) shows the equivalence of the moments taken about the pivot of the base, Hayashi(1984). FL_m may be expressed in a series form as

$$FL_m = FL_m(n)*\sin[2\pi.n.f_w.t + \phi(n)], \qquad ---(2)$$

where $FL_m(n)$, n=1,2,3---, is the nth frequency component of FL_m and $\phi(n)$ is the phase lag. The amplitude $Y_h(n)$ of response vibration y_h to $FL_m(n)$ may be given as the solution of Eq.(1) as

$$Y_h(n)=FL_m(n)/K_{mt}*[\{1-(n.f_w/f_{nw})^2\}^2+(2h_{tw}.n.f_w/f_{nw})^2]^{-1/2} \qquad ---(3)$$

, where h_{tw} is the total damping factor of the cylinder in water and may be expressed as the sum of the structural damping factor h_s and fluid damping factor h_f, Hayashi and Chaplin (1991).

RESULTS AND DISCUSSION FOR TESTS-A

Figure 4 shows the decay of free oscillations y_h of the test cylinder in air and in water, which is used in the experiments of Tests-A. Figure 5 shows the variation of the damping factor h_{tai} in air and h_{twi} in still water, d=40cm, with the non dimensional amplitude Y_{hi}/D. Where Y_{hi} is the amplitude of i-th oscillation of the cylinder y_h at still water level. The values of h_{tai} and h_{twi} for each amplitude of Y_{hi}/D are defined as

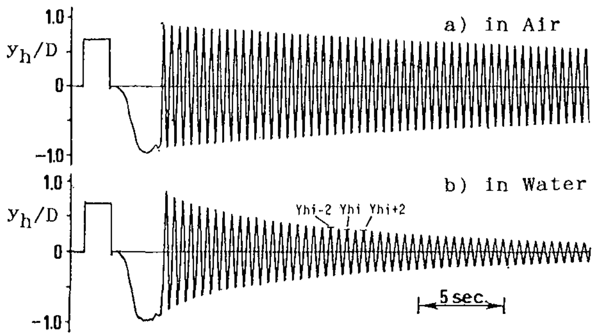


Fig. 4 Decay of Free Oscillation

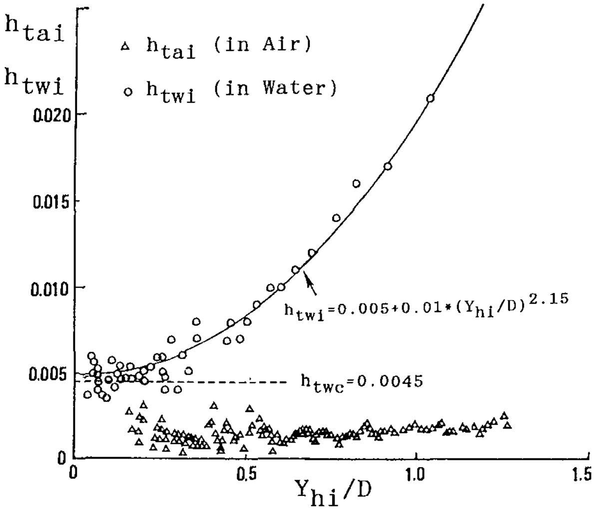


Fig. 5 Variation of Damping Factor with Y_h/D

$$h_{tai} \text{ or } h_{twi} = 1/(2\pi)*\{\ln(Y_{hi-2}/Y_{hi+2})\}/4, \qquad ---(4)$$

where Y_{hi-2} and Y_{hi+2} are the amplitudes of the (i-2)th and (i+2)th periods respectively (see Figure 4).

The value of h_{tai}, which shows the structural damping factor, is nearly independent of the value of Y_{hi}/D. The constant value of h_{ta}=0.002 is written in Table 1. On the other hand, the value of h_{twi} , which is composed of structural damping and fluid damping, is independent of amplitude effect only for low values of Y_{hi}/D and it becomes amplitude dependent at higher value of Y_{hi}/D owing to the characteristics of the fluid damping. The constant value of h_{twi}, $Y_{hi}/D<0.3$, is nearly corresponding to the theoretical value of h_{twc}=0.0045 which is derived from the Stokes's theory, Stokes (1851), for the forces on a cylindrical pendulum bobs oscillating at low KC number, Hayashi and Chaplin (1991). The increase of h_{twi} in $Y_{hi}/D>0.3$ may be due to the appearance of vortex-sheddings. The variation of the h_{twi} with Y_{hi}/D is approximated by the regression equation as

$$h_{twi} = 0.005 + 0.01*(Y_{hi}/D)^{2.15}. \qquad ---(5)$$

The variation of $Y_{hm}(n)/D$ and $Y_{hc}(n)/D$, n=2,3,4 are shown in Figure 6 (a),(b),(c) where $Y_{hm}(n)$ is the Fourier amplitude of measured displacement y_h and $Y_{hc}(n)$ is a calculated value obtained by substituting the measured moments $FL_m(n)$ acting on the rigidly mounted cylinder into Eq.(3). Here $FL_m(n)$, n=1-4, is the first four harmonics of FL_m, which are measured for the three values of kd=1.92, 1.03 and 0.73 in Case S1, S2 and S3. In calculation, Eq.(3) is coupled to Eq.(5) by means of $Y_h(n)=Y_h=Y_{hi}$.

The measured response $Y_{hm}(n)$, n=1, 2,3, is above the predicted response $Y_{hc}(n)$ in the range of 8<CKC<15 for the case of f_w/f_{nw}=1/2, in the range of 12<CKC<23 for the case of f_w/f_{nw}=1/3, and 25<CKC for the case of fw/fnw=1/4 respectively. These phenomena may be due to the amplification of the lift acting on the vortex-excited vibrating cylinder caused by the fluid-structure interaction. Similar phenomena are obtained by Maull and Kaye (1988) for the case of kd=1.73 and 1.32.

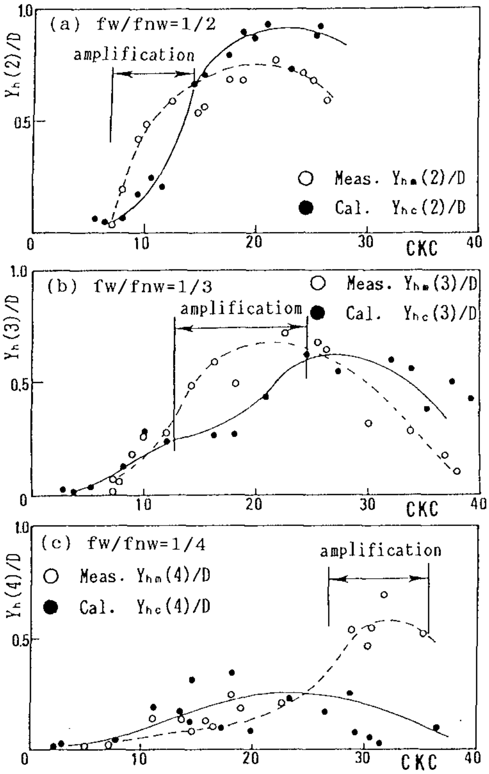


Fig. 6 $Y_{hm}(n)$ and $Y_{hc}(n)$

RESULTS AND DISCUSSION FOR TESTS-B

The traces for free decay of oscillations y_h and of the forces dF_Y in air and in water are shown in Figure 7 (a) and (b), where dF_Y is the transverse, Y-direction, forces acting on the unit length of the force sleeve, which is measured by the load cell in the test cylinder used in Tests-B. The relationship between these dF_Y and y_h/D are shown in Figure 7 (c). From this figure, dF_Y in air and in water may be expressed as

$$dF_Y,\text{(in air)} = dF_{Y1a} = m_{sa} * d^2y_h/dt^2 = 2.37 * y_h/D, \quad ---(6)$$

$$dF_Y,\text{(in water)} = dF_{Y1w} = m_{sw} * d^2y_h/dt^2 = 5.17 * y_h/D, \quad ---(7)$$

where dF_{YIa} and dF_{YIw} are inertia forces acting on the unit length of force sleeve in air and in water, and m_{sa} and m_{sw} are reduced mass per unit length of the force sleeve in air and in water.

The variations of η, y_h/D, dF_Y, dF_{Ya}, dF_{Yw}, dF_X with time for both cases of vortex-excited cylinder and rigidly mounted cylinder for comparison for the case of $f_w/f_{nw}=1/2$, CKC=12, kd=2.41 are shown in Figure 8 (a),(b) and (c). Here η is the water surface elevation. The Keulegan-Carpenter number at the level of force sleeve, 16cm below the still water level is LKC=7.5. dF_X and dF_Y are in-line and lift force acting on the unit length of force

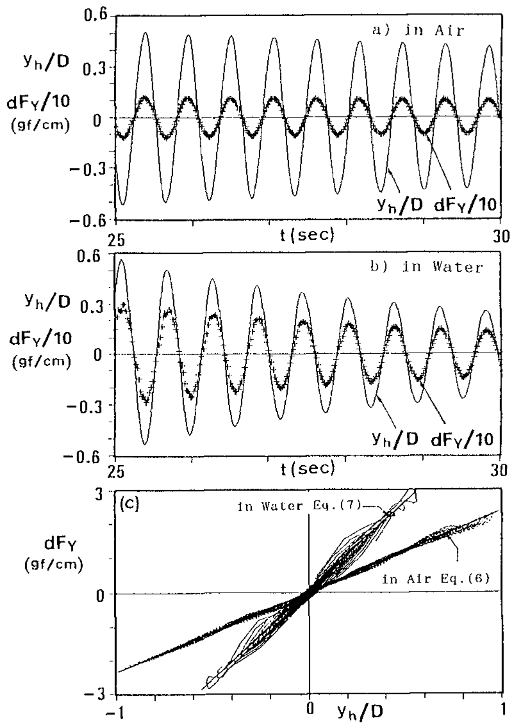
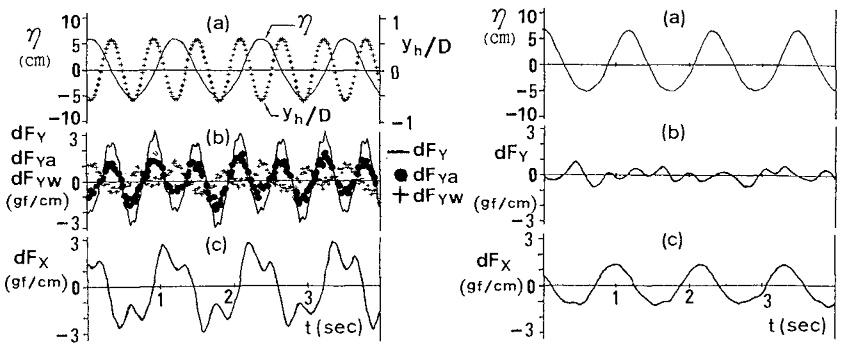


Fig. 7 Decay of Free oscillation



(A) Vibrating Cylinder (B) Rigid Cylinder

Fig. 8 η, y_h/D, dF_Y, dF_{Ya}, dF_{Yw}, dF_X versus time sleeve, which are measured by the two components small load cell. dF_{Ya} and dF_{Yw} are defined as

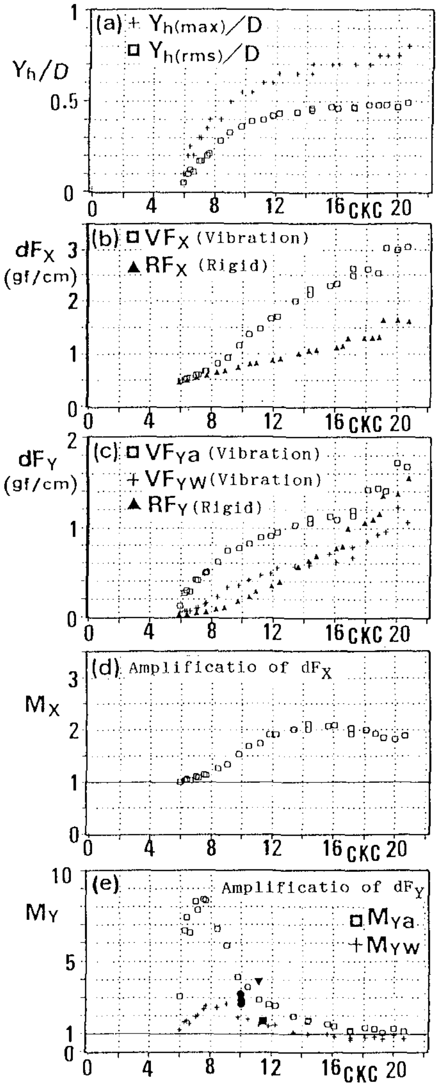
$$dF_{Ya} = dF_Y - dF_{YIa} = dF_Y - 2.37.y_h/D, \qquad ---(8)$$

$$dF_{Yw} = dF_Y - dF_{YIw} = dF_Y - 5.17.y_h/D. \qquad ---(9)$$

dF_{Ya} shows the transverse fluid force acting on the unit length of the force sleeve. dF_{Ya} may be composed of the inertia force due to the added mass of water, the fluid damping force and the loading force per unit length of force sleeve. Thus dF_{Yw} shows the transverse force which is composed of the damping and the loading force per unit length of force sleeve in water. When the test cylinder is vibrating in resonance condition, f_w/f_{nw} = 1/2, 1/3, 1/4, 1/5, 1/6, dFY_{Ia} and dF_{YIw} may be estimated by using Eq.(6) and Eq.(7).

dF_X shows the in-line fluid force acting on a unit length of force sleeve in water because the vibration of the in-line direction is restricted.

We can recognize that the forces dF_X, dF_{Ya} and dF_{Yw} acting on the



● Isaacson and Mall
[fw/fnw=1/2 Ks=3.54]
▼ Zedan and Rajabi
[fw/fnw=1/2 Ks=0.83]
■ Anglilli and Cossalter
[fw/fnw=1/2 Ks=0.98]
[fw/fnw=1/3 Ks=0.98]

Fig. 9 Amplification versus CKC for fw/fnw=1/2 vibrating cylinder are large compare to the forces dF_X and dF_Y acting on the rigidly mounted cylinder.

Fig. 9 (a)-(e) show the variation of the non-dimensional cylinder vibration (Y_h/D), wave forces (dF_X, dF_Y), and amplification factor (M_X, M_Y) with CKC for the case of $f_w/f_{nw}=1/2$. Here $Y_{h(max)}$ and $Y_{h(rms)}$ are the maximum half-amplitude and the root mean square value of the cylinder displacement y_h at still water level. VF_X, VF_{Ya} and VF_{Yw} are the root mean square values of in-line and transverse forces, dF_X, dF_{Ya} and dF_{Yw}, acting on the force sleeve of vibrating cylinder. RF_X RF_{Ya} and RF_{Yw} are the root mean square values of those of the rigidly mounted cylinder. The amplification factors M_X, M_{Ya} and M_{Yw} are defined as

$$M_X = VF_X / RF_X, \qquad \text{---(10)}$$

$$M_{Ya} = VF_{Ya}/RF_Y, \qquad \text{---(11)}$$

$$M_{Yw} = VF_{Yw}/RF_Y. \qquad \text{---(12)}$$

Figure 9 (d) and (e) shows that M_X increases with increasing Y_h/D and M_{Ya} is little affected with increasing Y_h/D. M_{Ya} is large in the range of $6<CKC<13$ where Y_h/D increases rapidly with increasing CKC. This range of CKC, where the amplification of lift force is large, is nearly consistent with the results obtained for $f_w/f_{nw}=1/2$ in Tests-A, see Figure 6 (a). The amplification factor M_Y obtained by Isaacson and Maull(1981), Zedan and Rajabi(1981), and Anglilli and Cossalter(1982) for the case of $f_w/f_{nw}=1/2$ are plotted in Figure 9 (e) only for reference. It should be noted that their results are obtained for the total lift forces acting on the vertical cylinder in waves.

The variations of the cylinder vibration Y_h/D and the amplification factors with CKC for f_w/f_{nw}= 1/3, 1/4, and 1/6 are shown in Figure 10 (A),(B) and (C) respectively. We can also recognize that M_X increases with increasing Y_h/D , which is nearly consistent with the results obtained in the case of steady flow, Griffin et al.(1975) and Sumer et al. (1994), and M_{Ya} is a little affected with increasing Y_h/D. M_{Ya} is a function of f_w/f_{nw} and CKC. The amplification factor M_Y obtained for the total lift acting on the vertical cylinder by Anglilli and Cossalter (1982) for the case of $f_w/f_{nw}=1/3$ is also plotted in Figure 11.

The value of M_{Yw} is bigger than 1 in the range of $6<CKC<13$ for $f_w/f_{nw}=1/2$. On the other hand , it is less than 1 for $f_w/f_{nw}=1/3$, 1/4 and 1/6. We need more consideration to the characteristics of M_{Yw}.

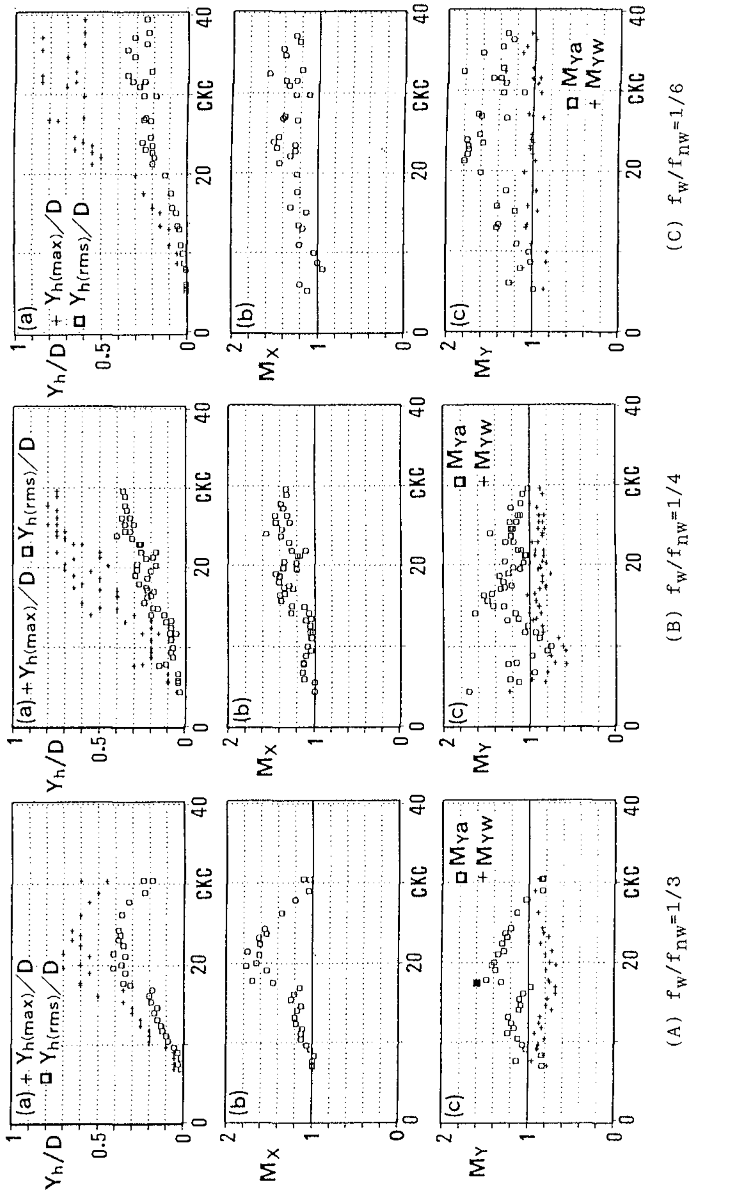


Fig. 10 Cylinder Vibration and Amplification Factor

CONCLUSIONS

The main conclusions obtained in this study are summarized as follows:

1). The existence of the amplification of lift force acting on the vortex-excited vibrating cylinder in comparison with the rigidly mounted cylinder is a function of the ratio of wave frequency f_w to the natural frequency f_{nw} of the cylinder in water and Keulegan-Carpenter number at still water level. When f_w/f_{nw} is fixed at about 2, large amplitude of transverse, cross-flow, vibration Y_h/D of cylinder occurs in the wide range of CKC, but the amplification of lift occurs only in the range of 6<CKC<13.

2). The in-line force acting on the partial part of the vortex-excited vibrating cylinder is amplified with increase of cross-flow vibration Y_h/D.

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