CHAPTER 71

Steep wave diffraction by a submerged cylinder

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Abstract

The steep wave diffraction by a fixed circular submerged cylinder is numerically simmulated as a completely nonlinear time domain evolution from an initial condition. Attention is focused on two aspects of the problem. The first is a study of the perturbation introduced by the cylinder on the wave field; the second is a study of the hydrodynamic forces on the cylinder due to the waves.

Introduction

We study the diffraction of steep waves on a fixed, submerged circular cylinder in deep water; being particularly interested in the hydrodynamic forces induced by the waves on the cylinder and the disturbances produced by the cylinder on the wave field.

The problem of a cylinder held fixed beneath waves field was first studied by Dean (1948) in the context of gentle waves in deep water; Dean finds, to the linear approximation, that incoming waves are not reflected and the only disturbance produced by the cylinder on the waves is an uniform phase delay. Ogilvie (1963) extend the method of Ursell(1950) to study some related problems of wave–body interaction and shows, in the linear approximation for the problem of a restrained circular cylinder, that the hydrodynamic force components oscillate in quadrature with the wave period, have the same amplitude and a phase difference of $\frac{\pi}{2}$; a second order,

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steady vertical force component is calculated from the first order potential, but the steady horizontal force component is found to be zero. Mehlum (1980) gives a first order potential solution for a restrained cylinder under waves which is very easy to compute. Salter et al. (1976) measures forces on a submerged circular cylinder and finds that mean horizontal forces are quite small and come to be negative for steeper waves. Chaplin (1984) makes experiments with a submerged cylinder under waves finding no wave reflection, even for the highest waves; it is also found that the phase lag is smaller than predicted by the linear theory for Keulegan Carpenter, K_c, numbers smaller than 2. Vada (1987) calculates the second order oscillatory forces on a cylinder of arbitrary cross section; for the case of circular cylinders and K_c numbers smaller than results agree well with Chaplin's (1984). Palm (1991), working with a high order perturbation scheme, shows that there is no reflection at the order m and frequency $m\omega$, where m is an integer and ω is the frequency, for monochromatic incident waves. Bichromatic waves are shown to have a null second order reflection coefficient; this result is shown to hold for higher order terms. Liu, Dommermuth & Yue (1992) working with a time domain, higher order perturbation scheme calculate, numerically, the wave diffraction by a circular cylinder, giving results for the steady force components on the cylinder, and the transmission coeficients.

Mathematical Formulation

We assume that the flow is incompressible and irrotational being thus described by a velocity potential ϕ. The free surface is described parametrically by $\mathbf{R}(\xi, t) = [X(\xi, t), Y(\xi, t)]$ to allow for overturning. The velocity potential satisfy the nonlinear boundary value problem:

$$\nabla^2\phi = 0 \tag{1}$$

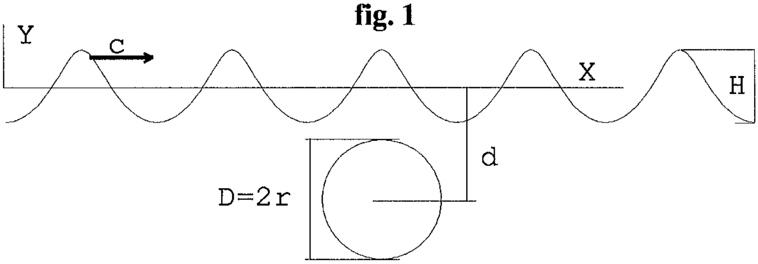
$$\phi_t + \frac{1}{2}(\nabla\phi)^2 + gY = 0 \tag{2}$$

$$\frac{D\mathbf{R}}{Dt} = \nabla\phi \tag{3}$$

$$\frac{\partial\phi}{\partial n} = 0 \tag{4}$$

(1) is to be valid inside the fluid region, (2) and (3) on the free surface and (4) along the cylinder's surface. At some initial time, e. g. $t = 0$, values for $\mathbf{R}$ and $\phi|_{\mathbf{R}}$ must be prescribed; we set a steep nonlinear steady wave, impulsively, above a submerged cylinder in deep–water, see figure 1. The steep steady wave, was computed through a numerical code described in Teles da Silva & Peregrine (1988). The nonsteady evolution of the initial condition is computed by a Boundary Integral code which is an extension of the one developed by Dold & Peregrine (1984) to compute the evolution of surface waves on water of uniform finite depth; in the original scheme the free surface is mapped on a closed curve and the horizontal bed into a circle which is surrounded by the mapped free surface; the mapped free surface is then reflected about the bed to ensure impermeability; in the present case the transformed free surface is reflected about the transformed cylinder contour, instead of about the bed, Peregrine (1989).

fig. 1



Results

Lengths have been scaled by $\frac{1}{k}$, where k is the wave number, accelerations by gravity g, and time by $(gk)^{-\frac{1}{2}}$.

The solutions of the problem depend on three parameters which are: i) wave-height H, ii) cylinder radius R, iii) the depth of the cylinder d. The potential flow modelling, given in the last section, poses strong limitations on the variation of these parameters: first, The Keulegan-Carpenter numbers, K_c, used as measure of the vortex-shedding, must be kept low, smaller than 1.5 or 2.0 according to the depth of the cylinder, Chaplin (1984); second, wave breaking stops the computations; and third, because of the imaging, the cylinder cannot be uncovered.

Most of the computations were made on a Sun Sparc Station 2. The simmulation of five wave crests moving over the submerged cylinder, during five wave periods, with twenty numerical points per wave takes 9.9 CPU seconds.

Figure 2 presents the time evolution of an initial wave, for $r = \frac{4}{3}$, $d = \frac{5}{3}$, which are the data used by Dean in his 1948 paper; the wave must be small, $H = 0.1$, for the cylinder being near the mean level, higher waves would break or uncover it; the centre of the cylinder, is at the start, located below the sixth crest at the position $x = \pi$; the wave length is $\frac{1}{11}\pi$; the numerical simmulation is made during a time of 11 wave periods.

fig. 2

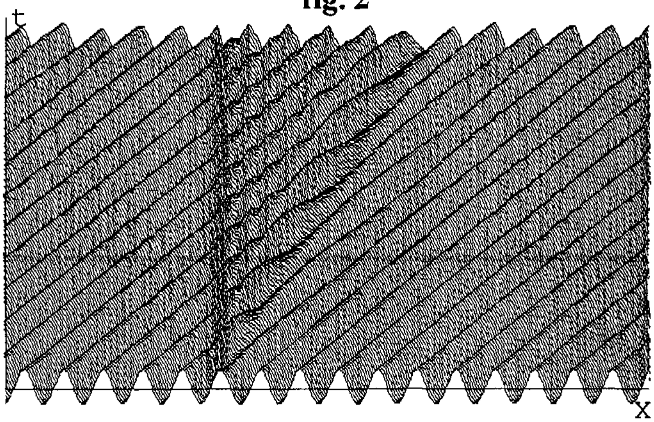


Figure 2 reveals some important aspects of the wave diffraction. With the exception of a faint disturbance starting above the cylinder at time zero, propagating upstream, due to the impulsive start of the numerical experiment, no other disturbances move upstream of the cylinder. This appears to be a nonlinear confirmation of Dean, and Palm's results about the absence of wave reflection in deep water. Following the lines of the crests in the space time diagram, Figure 2, we see that waves lose their steady shape as they pass a region of perturbation shaped like a 'V'; putting a ruler over a crest line we see that as the waves pass the cylinder the crests are over this line meaning that they are delayed; it is also possible to see for some crests, those that start nearer the cylinder, with the ruler, that after they leave the 'V' region of perturbation they get back to the original line, meaning that once outside the 'V' crests recover the delay. Another feature, in Figure 2, is that the wavelength of the perturbation on the 'V' appears to get longer towards its downstream edge as it would be for a wave group; with a ruler it is possible to estimate the slope of the 'V's edge and hence the velocity of propagation of the perturbations; these estimated velocities agree with the group velocity of small deep water waves with a wave-length which is a half of the wave length of the incident waves. The fact that the perturbed region be defined by a second harmonic of the transmitted wave suggests an analysis of the harmonics of the incident and the perturbed wave.

Fig. 3a

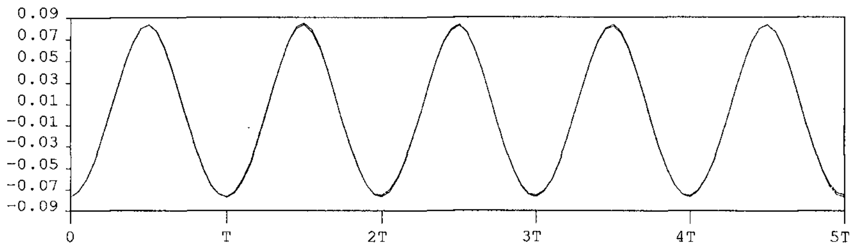
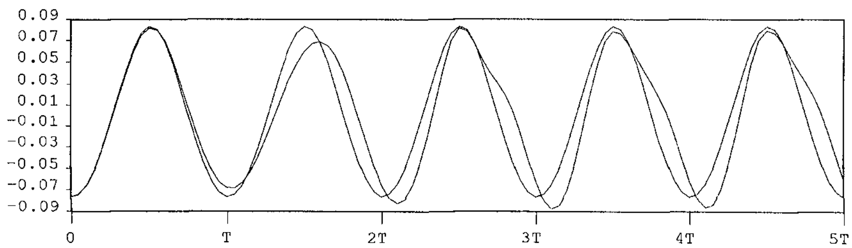


Fig 3b



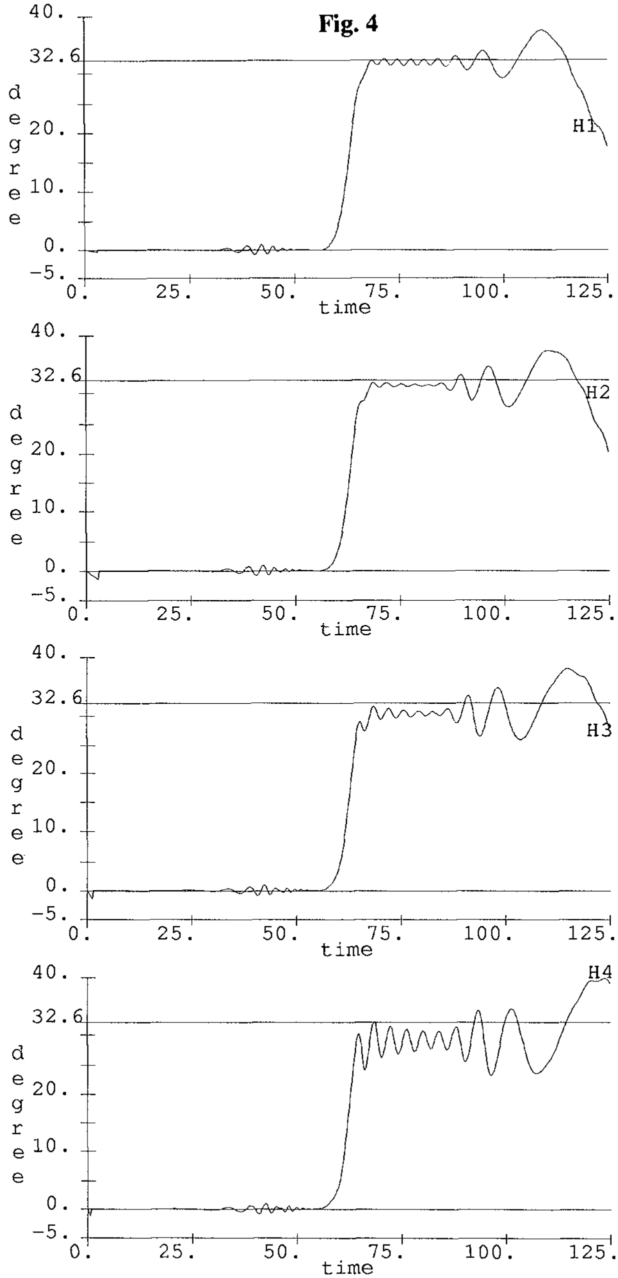
In order to have an idea of the relative magnitude of wave harmonics in simmulations, the time history of the elevation of 2 points on the free surface have been recorded for some cases; the points have been placed at a horizontal distance of eight times the radius of the cylinder, one downstream and the other upstream. A typical case is shown in Figure 3 where, for an incident wave of heigth $H = 0.16$ and a cylinder of radius $r = 0.4$ submerged at the depth of $d = 0.8$, the time histories of the upstream, Figure 3a, and downstream, Figure 3b, surface elevation are depictured; in both cases, for comparison, the time history for the original steady wave has been included. In Figure 3a the time history of the upstream free surface elevation is undiscernible from that of the steady wave supporting the conclusion that waves are not reflected; the some does not happens for the upstream elevation where from the third wave period a conspicuous second harmonic appears in the perturbed wave. To see it more clearly we decomposed the time histories in its Fourier modes in the same way as for the forces in equations 5 and 6 below. The results are shown in Table 1 where the most important features, when we compare the upstream elevation with the elevation of the incident steady wave, appear to be the permanence of the first harmonic and the great increase of the higher harmonics; among the higher harmonics the most important is the second which in fact shapes the 'V'.

Table 1

Harmonic	Steady Wave	Upstream Wave	Downstream Wave
a_0	0.000010	-0.000106	0.000200
r_1	0.079812	0.078808	0.078772
r_2	0.003213	0.003252	0.016048
r_3	0.000213	0.000219	0.003312
r_4	0.000029	0.000003	0.001447
r_5	0.000022	0.000008	0.000195
r_6	0.000004	0.000006	0.000167

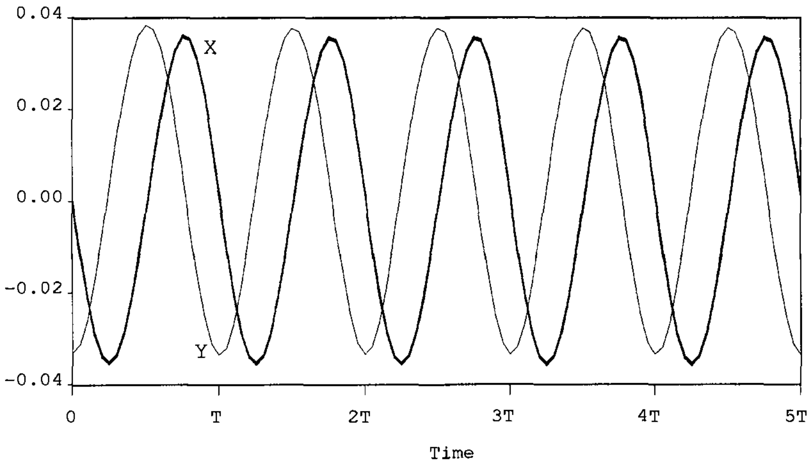
A remarkble fact in Figure 2 is that outside the perturbed 'V' region the waves recover their original shape and steadyness with no traces of the interaction with the cylinder although the interaction is nonlinear; this is the same for the steeper waves provided they do not break. Comparing the position of the crests going past the cylinder with the position they should have in the absence of the

Fig. 4



cylinder, we find the phase lag. Figure 4 shows the time evolution for the phase lag, vertical axis in degrees, for four different waves; namely, wave 1: $H = 0.1$, $T = 6.275$, $c = 1.001$, wave 2: $H = 0.2$, $T = 6.252$, $c = 1.005$, wave 3: $H = 0.3$, $T = 6.213$, $c = 1.011$, wave 4: $H = 0.4$, $T = 6.1588$, $c = 1.0202$; H, T and c are respectively the nondimensional wave-height, period and phase speed. In all this experiments $r = 2\pi$ and $\frac{d}{r} = 1.2$; for this same radius and submergence Mehlum (1980) calculates a phase lag of 32.6 degrees. Note that this is nearly exact for wave 1, but, the phase lag tend to decay with wave-height. A bigger wave with $H = 0.5$ would break. In all the above cases, in given time, the phase lag returns to zero as the tails of the first two waves suggest; a permanent phase lag is an asymptotic result

Fig. 5



A typical time history for the X and Y force components is shown in figure 5 for a wave of heigth $H = 0.5$; the figure shows a remarkble periodicity from a few periods after the begining of the experiment. For small waves the linear theory predicts that the X and Y, horizontal and vertical, force components have the same amplitudes, oscillate in quadrature with the wave period and there is a phase difference of $\frac{1}{2}\pi$ between them. We observed these results to hold for smaller waves; however for the higher waves we find the phase difference between the X and Y force components to become smaller, by as much as 10%, than $\frac{\pi}{2}$. The X and Y force components have been, both, decomposed in Fourier modes:

$$\frac{a_0}{2} + \sum(a_n cos\theta_n t + b_n sin\theta_n t)$$

where $\theta_n = \frac{2n\pi}{T}$, T being the wave period; or simmilarly as

$$\frac{a_0}{2} + \sum r_n cos[\theta_n t - \delta_n] \tag{5}$$

$$r_n = (a_n{}^2 + b_n{}^2)^{\frac{1}{2}} \tag{6}$$

where δ_n is a phase shift. Results for fixed parameters 'R' and 'd' and a set of four waves with increasing wave height are displayed in table 2. We find for all waves a negative horizontal drift, $\frac{a_0}{2}$, which increases with the wave-height.

Despite the fact that the amplitudes of the X and Y force components are the same, irrespective of wave-height, the amplitudes of the Fourier modes of the same frequency increasingly differ for the higher waves. The vertical drift, $\frac{a_0}{2}$, increases with wave height reaching, for the last wave $H = 0.7$, an amplitude greater than any other Fourier mode, except the first.

As should be expected, the force harmonics for the smaller waves behave as in a perturbation expansion in the small parameter $\epsilon = H$, which is the dimensional wave height scaled by $\frac{1}{k}$; representing either the vertical or horizontal force components as:

$$f_1\epsilon + f_2\epsilon^2 f_3\epsilon^3 + ...$$

where the first term in the expansion represents the first harmonic of the vertical or horizontal force component, the second term represents the second harmonic and so on. Observe in Table 2 that: the first harmonic for $H = 0.3$ is roughly three times the first one for $H = 0.1$; the second harmonic for $H = 0.3$ is roughly nine times the second one for $H = 0.1$; the steady components for $H = 0.3$

are roughly nine times the steady components for $H = 0.1$. For the waves with $H = 0.5$ and $H = 0.7$ these relations hold approximately only for the first harmonic. This provides a measure of the range of validity of the linear and weakly nonlinear results and also a way to estimate the force components of waves from the force components of particular wave. This is not new but it is very interesting to know that it holds for the first harmonics of a wave with $H = 0.7$ that is over 80% the height of the highest mathematically, not physically, possible wave.

Table 2

Wave/cylinder	Harmonic	X-component	Y-component
H=0.1	a_0	-0.000002	0.000289
T=6.2753	r_1	0.007285	0.007285
c=1.0012	r_2	0.000029	0.000029
D=0.5,d=1.	r_3	0.000001	0.000001
Kc=0.23	r_4	0.000000	0.000001
H=0.3	a_0	-0.000021	0.002489
T=6.2129	r_1	0.021710	0.021704
c=1.0113	r_2	0.000310	0.000313
D=0.5,d=1.	r_3	0.000003	0.000004
Kc=0.69	r_4	0.000003	0.000003
H=0.5	a_0	-0.000111	0.006417
T=6.0899	r_1	0.035615	0.035549
c=1.0317	r_2	0.001097	0.001102
D=0.5,d=1.	r_3	0.000035	0.000028
Kc=1.08	r_4	0.000005	0.000008
H=0.7	a_0	-0.000200	0.010207
T=5.9107	r_1	0.044369	0.044097
c=1.0630	r_2	0.002526	0.002534
D=0.5,d=1.	r_3	0.000192	0.000183
Kc=1.56	r_4	0.000037	0.000019

Conclusions

A numerical scheme for the time domain simmulation for the nonlinear steep wave diffraction by a submerged cylinder has been successfully implemented. Results have been checked, with very good agreement, with analytic, semi-analytic and experimental results given in the literature, namely Ogilvie (1963), Mehlum (1980), Vada (1987) and Chaplin (1984). In these comparisons an important point appears: many of the analytic and semi-analytic results presented in the literature are for waves that actually break; specially these for small cylinder depth where the diffraction effects are enhanced; by one side when waves break the flow ceases to be a potential flow and results should not be valid, but by another side Chaplin finds with experiments that, in his case E experiment, the mean vertical force agrees well with Ogilvie's results and the total force is smaller than that predicted by Ogilvie's results. This may suggest that approximate results frequently provide a good prediction of phenomena and sometimes give an upper bound for quantities.

Despite the limitations inherent to potential theory, and the computational costs in a time domain simmulations, which preclude their use for a thorough description of the phenomenon, some important aspects of the problem can be studied and complemented with frequency domain and experimental investigation; since computational costs are comparatively very cheap for frequency domain calculations and potential modelling is not a problem with experiments.

With respect to the disturbances on the waves due to the presence of the cylinder we have two main aspects: the phase-lag on the transmitted wave and the transmission and reflection coefficients. With respect to the phase lag, we find that the linear potential given by Mehlum (1980) gives cheap and accurate results for smaller waves and for the higher ones these results provide an upper bound. With respect to reflection our results support the linear prediction of no reflection; regarding the transmitted wave we find that the presence of the cylinder greatly enhances the higher harmonics; but to this stage there is not yet a satisfactory description of the dependence of this phenomenon on the parameters; neither have we yet atempted to quantify it with the help of transmission coefficients.

Regarding the forces on the cylinder due to waves an important result is that for the very steep waves the horizontal and vertical steady force components become evident and the vertical steady component come to be at the same order of magnitude as the first harmonic. Also important is the increase of the magnitude of the higher harmonics which is inherent to the very nonlinear character of steep waves; this increase in magnitude of the harmonics is followed by a loss of symmetry between the horizontal and vertical components.

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Bibliography

AQUIJE, J. T. 1993 Non–linear diffraction of water waves by a submerged body, M.Sc. degree, Prog. Eng. Mech. COPPE/UFRJ

CHAPLIN, J. R. 1984 Nonlinear forces on a horizontal cylinder beneath waves, *J. Fluid Mech.* **147**, 449 – 464.

DEAN, W.R. 1948 On the reflection of surface waves by a submerged cylinder, *Camb. Phil. Soc.* **44**, 483 – 491.

DOLD, J.W. & PEREGRINE, D.H. 1984 Steep unsteady water waves. An efficient computational scheme, *Proc. 19th ICCE*, Houston, Texas, **1**, 995 – 967.

LIU, Y., DOMMERMUTH, D. G. & YUE, D. K.P. 1992 A high–order spectral method for nonlinear wave–body interactions, *J. Fluid Mech.* **245**, 115 – 136.

MEHLUM, E. 1980 A circular cylinder in water waves, *App. Ocean Res.* **2**, 171 – 177.

OGILVIE, T. F. 1963 Fristand second order forces on a cylinder submerged under a free surface, *J. Fluid Mech.* **16**, 451 – 472.

PALM, E. 1991 Non–linear wave reflection from a submerged circular cylinder *J. Fluid Mech.* **233**, 49 – 63.

PEREGRINE, D. H. 1989 private communication

SALTER, S. H., JEFFREY, D.C. & TAYLOR, J.R.M 1976 *The Naval architect*, Jan. 1976, pp.21 – 24.

TELES DA SILVA, A. F. & PEREGRINE, D. H. 1988 Steep, steady finite depht const. vort. water waves, *J. Fluid Mech.* **195**, 281 – 302.

VADA, T. 1987 A numerical solution of the second order wave–diffraction problem for a submerged cylinder of arbitrary shape, *J. Fluid Mech.* **174**, 23 – 37.

URSELL, F. 1950 Surface waves on deep water in the presence of a submerged circular cylinder, I, *Proc. Camb. Phil. Soc.* **46**, 141 – 152.