CHAPTER 61

A Comparative Evaluation of Wave Grouping Measures

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ABSTRACT

This paper describes the limitations associated with some of the wave grouping measures that are currently available in the literature. A brief review of the new concepts that have been proposed recently to overcome these limitations is also included in it.

Extensive analysis of prototype waves is carried out worldwide to establish a relationship between the degree of grouping in waves and their variance spectral density. But some of these analyses have been unable to identify any relationship because of the statistical variability inherent to records of finite length. A brief discussion of this variability is included in this paper.

INTRODUCTION

The occurrence of wave groups and their importance are well recognized by the engineering community. During the last fifteen years, many studies have illustrated the relevance of wave groups to the design of maritime structures. Much research is therefore being carried out in this field and their main objectives can be summarized as follows:

(i) to develop an adequate measure of the degree of grouping;
(ii) to establish a statistical model for the wave groups; and
(iii) to achieve a better understanding of the effect of wave groups on structural response.

Several measures have been proposed to quantify the degree of grouping in a sea state, but none of them seems fully adequate for practical use. Some of the limitations of these measures are reviewed in this paper, using numerical simulation.

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In order to establish statistical models for wave groups, many researchers have advanced the concept that all information about wave groups is contained within the primary spectrum.This implies that, once the shape of the primary spectrum is known, the information about wave groups can easily be deduced. A brief evaluation of this hypothesis is carried out in the later sections.

Attempts are made to achieve a better understanding of the complex physics involved in the interaction of wave groups with coastal structures in order to relate the degree of wave grouping to group-induced response of structures. Some of the techniques used for this purpose are also discussed in this paper.

BRIEF REVIEW OF THE WAVE GROUPING MEASURES

Amongst the large number of concepts that have been proposed to describe wave grouping, the following are some which have been explored in detail.

i) Statistics of Run Lengths;
ii) Wave Grouping from the Square of the Water Surface Elevation;
iii) Correlogram;
iv) Phase Spectrum; and
v) Concepts Based on Hydrodynamics of Wave Motion.

Amongst these concepts, only the first two are used extensively. This paper will focus on them, nevertheless indicating relevant bibliography for the remaining three.

Statistics of Run Lengths

The Run Length is defined as the number of successive waves with wave heights exceeding a specified threshold; the threshold can be the average, the median or the significant wave height of a sea state. The average run length ($\bar{j}_1$) within a wave record is then used as the measure of the degree of grouping. The total run is analogous to a wave group period, and is defined as the number of waves contained in the interval which commences with the first threshold exceedance of one run and ends with the first exceedance of the next run. The average value of the total run within a wave record is usually denoted as $\bar{j}_3$.

Kimura (1980) provides the necessary expressions for predicting the mean and the standard deviation of the run lengths and of the total runs, as a function of a correlation parameter κ derived from wave records. This parameter is related to the correlation coefficient between two successive heights, ρ_{HH}, and can be calculated from the zero crossing-analysis of waves. But if the time series has to be used anyway for determining this correlation coefficient, the run length statistics can be easily inferred at the same time. It appears therefore that there is no distinct advantage in using Kimura's theory except to compare his predictions with observed values in field data. Hence researchers such as Battjes and van Vledder (1984) promote the use of the correlation parameter κ derived from the spectral density of the sea state as follows:

$$\kappa_f^2 = \frac{1}{m_0^2}\left\{\left[\int_0^\infty S(f)\cdot\cos(2\pi f\tau)df\right]^2 + \left[\int_0^\infty S(f)\cdot\sin(2\pi f\tau)df\right]^2\right\} \quad , \qquad (1)$$

where: m_0 is the zeroth moment of the spectral density function S(f) and τ is the mean wave period.

If the run length statistics can be satisfactorily predicted from this spectrum-derived correlation parameter κ_f, it can be concluded that most information regarding wave groups is contained in the spectrum itself.

Wave Grouping From the Square of the Water Surface Elevation

Several measures have been proposed to quantify the degree of grouping present in a sea state based on the concept of the square of the water surface elevation. A brief description of some of these measures is given below.

Smoothed instantaneous wave energy history (SIWEH)

One of the well known measures for characterizing the degree of grouping in a sea state is the Groupiness Factor based on the SIWEH (Funke and Mansard, 1979). The SIWEH is a time series of the low frequency part of the square of the elevation, $\eta_f^2(t)$. It is computed by squaring the instantaneous water surface elevation and then smoothing it by a Bartlett low-pass filter of size $2\times T_p$ (where T_p is the peak period of the spectrum). The Groupiness Factor, GF, which is a measure of the degree of grouping, is defined as the standard deviation of SIWEH normalized with respect to its mean value.

Hilbert transform of wave record

A technique which involves no low-pass filtering to compute the low frequency part of the square of the water surface elevation is the Hilbert transform technique, proposed by Bitner-Gregersen and Gran (1983) and extensively used by Medina & Hudspeth (1988, 1990).

If the water surface elevation and its conjugate are expressed as $\eta(t)$ and $\hat{\eta}(t)$ respectively, it can be shown that they are the Hilbert transform of each other. From these two functions, the envelope of the water surface elevtion, A(t), the wave height function, H(t), and the low frequency part of the square of the water surface elevation, $\eta_f^2(t)$, can be computed easily (see Medina et al., 1990 and Medina & Hudspeth, 1990).

A concept similar to the SIWEH Groupiness Factor can also be used in this case to define the degree of grouping. Medina et al. (1990) propose the use of GF defined as the standard deviation of the square of the wave height function normalized by $8m_0$ (m_0 is the zerth moment of the sea state spectrum). This approach of computing GF is obviously superior to SIWEH analysis since it involves no low-pass filtering. However, it has some limitations which will be discussed later in this paper.

Expected wave grouping spectrum

For a given sea state, the expected spectrum of the low frequency part of the square of the water surface elevation (known also as spectrum of wave groups) can be computed easily from the auto-correlation of the primary spectrum, without having to deal with the time series of water surface elevation and its Hilbert transform (see Pinkster, 1984). Figure 1 shows three JONSWAP spectra with different peak enhancement factors, γ, and their corresponding wave grouping spectra. The cut-off frequency used in the low pass filtering of SIWEH is also indicated in Figure 1b for easy reference.

It should be pointed out that individual realization of finite length wave records, simulated from these JONSWAP spectra, can result in grouping spectra that differ from the expected functions illustrated in Figure 1.

Correlogram

The concept of using the correlogram (i.e. auto-correlation function) to describe wave grouping was attempted by Rye and Lervik (1981) and Sobey & Read (1984) without much success. More recently, Medina and Hudspeth (1990) showed that the correlation parameter κ used in the prediction of run lengths can be derived from the auto-correlation function of the square of the wave height function.

Phase Spectrum

Funke and Mansard (1981) and Sobey and Read (1984) speculate that phase spectra of sea state may contain some information on the degree of grouping. In fact, Funke and Mansard (1979) showed that, by just changing the phase spectrum, wave trains with different degrees of grouping can be simulated from a given variance spectral density, without however establishing any specific relationship between the degree of grouping and the phase spectrum. Goda (1983) analyzed some swell records for the probability distribution of phases, and concluded that the phases were indeed uniformly distributed between $-\pi$ and π but not completely random. Nevertheless, the deviation from randomness was only slight.

Concepts Based on Hydrodynamics of Wave Motions

All the wave grouping concepts presented so far are based only on statistical wave properties such as wave energy spectrum or wave energy envelope. But other approaches, based on the hydrodynamics of wave motions, have also been explored in the literature.

Mase and Iwagaki (1986a and b) carried out investigations using modulational instability theory to describe the wave grouping process. They suggest that a wind wave field could be considered as a modulated non-linear wave train with a single carrier wave. The other approach used to describe wave grouping is the envelope soliton model. Yasuda et al. (1986) postulate that all wave groups observed in waves behave as envelope solitons and can be defined by the multi-envelope soliton solutions of the plural non-linear Schrödinger equations. More research is required in this field for practical applications.

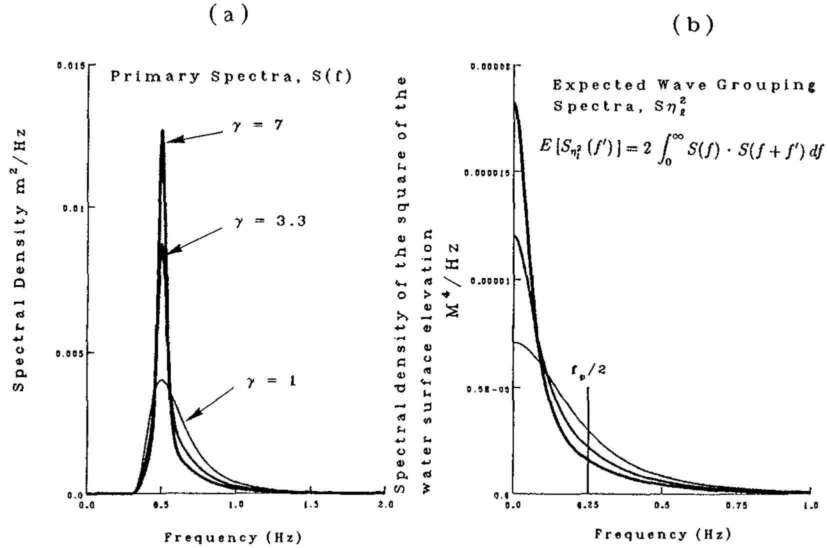


Figure 1. Expected spectra of wave groups for JONSWAP sea states

ANALYSIS OF WAVE GROUPING AND THEIR STATISTICAL VARIABILITY

As indicated above, only the run length statistics and Groupiness Factor are extensively used for quantifying the degree of grouping in a sea state. Hence the detailed evaluation presented below discusses only these two measures.

Many prototype waves have been analyzed in different parts of the world, in order to relate the degree of grouping to the spectral width of the sea state, or to the correlation parameter κ, using the measures of run length and Groupiness Factor. However, some of these analyses have not been too successful because of the various limitations associated with the two wave grouping measures. In order to get a better understanding of these limitations, some investigations were undertaken in this study, using numerical simulation.

Statistical Variability of Wave Grouping Measures

It can be shown that, for records of finite length, it is not easy to establish relationship between the measures of grouping such as GF and j_1 and spectral width parameters such as 'Q_p' or correlation parameter κ. The statistical variability associated with sea states of finite record length obscures any relationship that might exist between these parameters. To illustrate this point, the following investigations were undertaken.

JONSWAP spectra with three different peak enhancement factors γ = 1, 3.3 and 7 were first formulated. From each one of these three spectra, 200 different realizations of wave trains (N_R = 200) were simulated numerically using the random phase spectrum method. This random phase spectrum method is a technique that pairs a given spectral density with a randomly selected phase spectrum. By varying the random numbers used in this phase spectrum, wave records with different time domain characteristics, and therefore with different Groupiness Factors and run lengths can be generated (Funke et al, 1988). The record length of each of these wave trains, T_R, was chosen arbitrarily to be 200s (model scale). At a scale of 1:36, this would represent a 20 minute record in prototype. The peak frequency of the spectrum was made equal to 0.55 Hz, and thus the total number of waves contained in each of these time series was about 120 to 150 waves.

The synthesized time series were subjected to wave grouping analysis, and the 200 values of Groupiness Factor obtained from each one of these spectra, were then subjected to statistical analysis. For all three JONSWAP spectra, the estimated Groupiness Factors exhibited a large variability. It was therefore considered relevant to determine if there is an optimum record length that would ensure a distinct relationship between GF and γ. For this purpose, the numerical simulation described earlier was extended to include several other record lengths during the synthesis process. For instance, instead of simulating 200 time series each 200 s long, 100 time series each 400 s long were simulated and then subjected to wave grouping analysis. In a similar fashion, the record lengths were increased to 800 s, 1600 s, 3333 s, 5000 s and 6667 s respectively. The corresponding number of time series used in the analysis were therefore equal to 50, 25, 12, 8 and 6 respectively. The results of all these analyses are summarized in Figure 2a for the three types of spectra under consideration. This figure shows the mean values of the Groupiness Factors, their maxima and minima and the mean values $\mp$ one standard deviation obtained from the statistical analysis. Figure 2b illustrates the run length statistics obtained from similar numerical simulations. These statistics correspond to the average run length, $\bar{j}_1$, exceeding the significant wave height. Two conclusions are evident from these figures 2a and 2b:

Mean values of GF(i.e $\overline{GF}$) and $\bar{j}_1$ (average of j_1) exhibit an increase with narrower spectra. The statistical variability of GF and j_1 decrease as the record length increases. However, even with a record length of 111.12 minutes, there is a small variability in the values of GF and j_1. For instance, the standard deviation of GF which is estimated to be 2 to 3% of the mean value, for record length T_R = 55.56 minutes, does not seem to decrease with more increase in the record length. If the scale factor of 1:36, used earlier to represent 200 s of model time to full scale 20 minutes, is adopted here, the record length of 55.56 minute would correspond to 5.56 hours prototype.

Since natural wave records are often of short duration, it is believed that this figure would be useful for a judicious interpretation of wave group statistics.

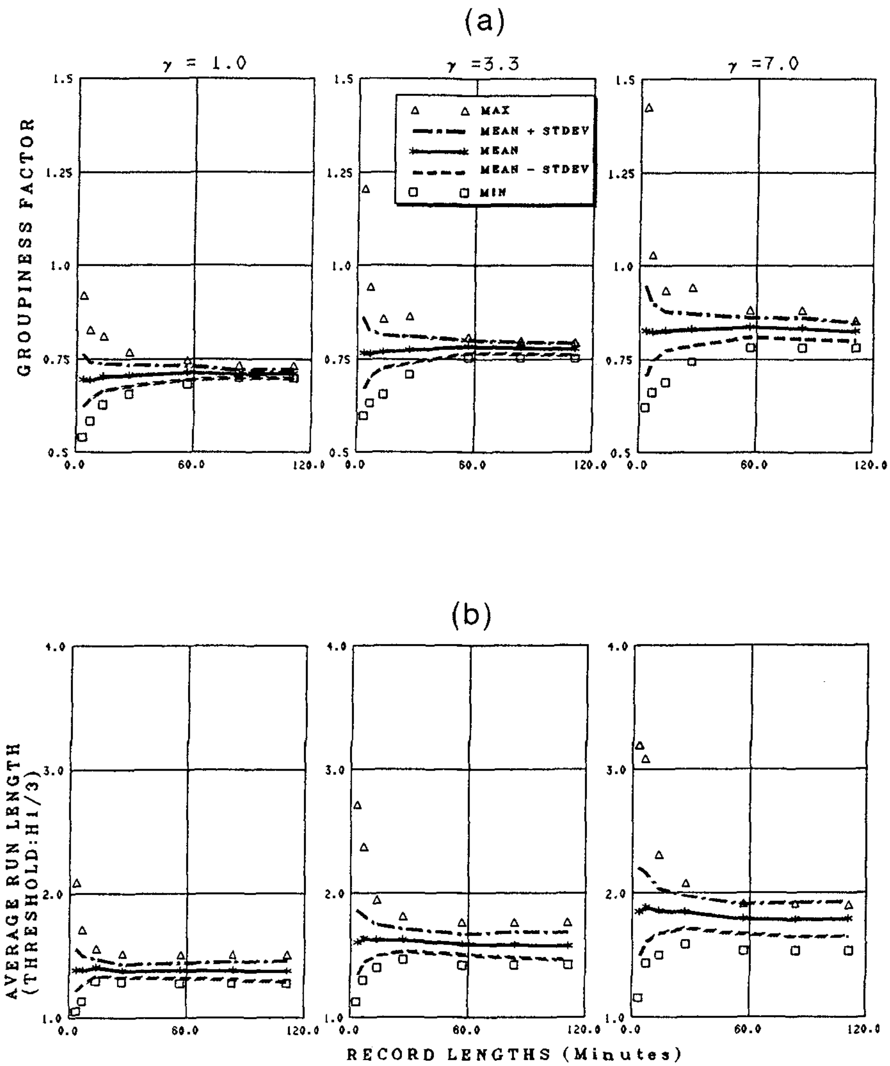


Figure 2. Statistical variability in Groupiness Factors and Run Length statistics.

COMPARISON BETWEEN VARIOUS WAVE GROUPING MEASURES

In spite of the statistical variabilities described above, GF and run length statistics are being commonly used to provide prototype information on wave groups, for purposes of model testing in laboratory basins. But these two measures do not characterize the degree of grouping in the same fashion, i.e. a large Groupiness Factor does not necessarily mean a long run length.

A discussion on the lack of relationship that exists between these two measures and on the advantages of the Hilbert Transform are given below.

Comparison Between Run Length Statistics and Groupiness Factors Derived from SIWEH

The measures of GF and average run length $\bar{j}_l$, are based on two different approaches: The Groupiness Factor is based on the square of the water surface elevation while the run length is based on the count of successive waves which exceed a certain threshold. A large GF implies the existence of many distinct energy packets of large waves but a long run length does not say much about the wave heights contained in the group except that they exceed the given threshold. It is therefore possible for a wave train with a small GF to have long run lengths and vice versa. A relationship can however be found between the average period of SIWEH and the total run (see Goda, 1983).

Between these two measures, GF is perhaps the parameter which is more commonly used in numerical and physical model studies, since simulation techniques exist to control the value of GF without having to change the spectral characteristics of the sea state. A similar technique is not available for a control of run lengths. Furthermore, the GF provides a measure of the low frequency energy contained in a sea state and therefore could be related to structural response, such as the slow drift oscillations of floating structures, seiches in harbours etc. The run length statistics do not readily provide any indication on the low frequency energy but they may be more appealing for the study of fixed structures.

Comparison Between the Concepts of SIWEH and Hilbert Transform

The main criticism that the SIWEH based Groupiness Factor is subjected to is the necessity to perform filtering operations and the arbitrary choice of the filter width. Because of this, researchers such as Medina and Hudspeth (1988, 1990) have been promoting the concept of Hilbert transform to compute η^2_ℓ without any low pass filtering. Figure 3 illustrates the difference between SIWEH and Hilbert Transform approaches. The computation used for obtaining these results was carried out as follows: first a 200s long time series of water surface elevation was synthesized from the JONSWAP spectrum, shown in Figure 3a as the primary spectrum. This time series was then subjected to wave grouping analysis using the SIWEH and Hilbert transform analysis.The time series of the low frequency part of the square of the water surface elevations, η^2_ℓ, derived from these two analyses were then subjected to spectral analysis. It can be seen from Figure 3a that the spectrum of wave groups derived from SIWEH goes nearly to zero at $f_p/2$ (i.e.

0.25 Hz), while the spectrum from the Hilbert Transform analysis overlaps the primary spectrum. Since the Groupiness Factor is a global measure of the variance contained in the entire low frequency part of the square of the water surface elevation, its value derived from the results of Hilbert transform is larger than the one provided by SIWEH analysis (see Figure 3b). However, neither of these values is directly applicable to predict the structural response of floating structures for the reason indicated below.

The response of a floating structure depends only on the variance contained within the range of its frequency response. Hence the Groupiness Factor which characterizes the total variance of η_t^2 is not suitable for the prediction of structural response. To overcome this problem, Mansard and Sand (1992) suggest the use of some alternative concepts.

Care must also be exercized in using the concept of Groupiness Factors for evaluating the sensitivity of fixed structures to the degree of grouping. By using the SIWEH concept, Vidal et al. (1995) and Mansard et al (1994) show that there is a correlation between the degree of grouping defined by the Groupiness Factor and the statistics of large wave heights. (This correlation can be expected, to a certain extent, since the Groupiness Factors are defined by the standard deviation of instantaneous wave energy history in the time domain). Unlike floating structures whose frequency response of horizontal oscillations are in the range of wave group frequency, stability of fixed structures such as breakwaters is highly sensitive to the statistics of large wave heights. Hence higher damage in breakwaters with larger values of GF does not necessarily mean that the degree of grouping is the main cause of damage: it could be induced by the large wave heights associated with high GF. Hence, when evaluating sensitivity of fixed structures to wave grouping, the time series that are selected for testing purposes should exhibit comparable wave height statistics, but different values of GF (see Mansard et al., 1994). Researchers such as Johnson et al. (1978) and Galand and Manoha (1991), who illustrated the influence of wave grouping on breakwater stability, did not specifically evoke the aspects related to wave height statistics, possibly because their intent was to illustrate the unsuitability of qualifying a sea state solely by its frequency domain characteristics. Medina et al. (1990) have recently proposed a new concept which can overcome the difficulty described above. This concept is included below in the discussion on new concepts.

NEW CONCEPTS OF WAVE GROUPING MEASURES

Improved Predictor of Run Length Statistics

In recent years, the prediction of run length statistics from the spectrum-derived correlation parameter, κ_f, has undergone substantial advances. Earlier investigations using the spectrum-derived correlation parameter (see Equation 1) resulted in the underprediction of run lengths since the correlation coefficient between successive wave heights computed by zero-crossing analysis was larger than the value derived using κ_f. The main reason for this discrepancy is in the implicit assumption of the narrow-banded process used in defining κ_f. Recently van Vledder using the investigations of Tayfun (1990) on broad-banded spectrum, proposed a modified expression for the spectrum-derived correlation coefficient between successive wave heights $\rho_{HH,f}$ that correlates well with the estimation made in the time domain, $\rho_{HH,t}$. The modified expression is:

$$\rho_{HH,f,new} = \frac{\rho_{HH,f}(\frac{1}{2}\hat{T}) + 2\rho_{HH,f}(\hat{T}) + \rho_{HH,f}(\frac{3}{2}\hat{T})}{2 + 2\rho_{HH,f}(\frac{1}{2}\hat{T})} \tag{2}$$

where :

$$\hat{T} = T_{m02}\,(1 - \frac{1}{2}\nu^2) \tag{3}$$

T_{m02} is the average period based on the second spectral moment and ν is the spectral width parameter proposed by Longuet-Higgins (1975). The relationship between $\rho_{HH,f}$ and the κ_f parameter given in Equation 1 can be found in van Vledder (1992).

Motion Equivalent Groupiness Factor

Mansard and Sand (1992) claim that a large Groupiness Factor, derived either from SIWEH or from the Hilbert transform, does not necessarily lead to higher structural response for all test structures. It is possible that the large variance of η_t^2 which results in high value of GF, may be outside the range of frequency response of the structures. Mansard and Sand (1992) suggest therefore a new concept of Motion Equivalent Groupiness Factor, which provides an integral measure of the degree of grouping in waves and the frequency response of test structures. As an example of this concept, a new expression was developed relating the surge motion of a simple floating structure to the degree of grouping in waves.

Groupiness Factor Distribution Function

Often, the frequency response of structures may be difficult to formulate and will vary from one structure to another. To account for this, Mansard and Sand (1992) suggest the use of a Groupiness Factor Distribution Function, defined by the following equation:

$$GFDF\,(f_c) = \frac{1}{m_0} * \sqrt{\int_0^{f_c} S_{\eta_t^2}(f) \cdot df} \qquad for \quad 0<f_c<\infty \tag{4}$$

Instead of using a single value of GF based on the total variance of the low frequency part of the square of the water surface elevation, it is proposed here to take into account the distribution of this variance over different frequency ranges. (Hilbert transform technique is recommended for this application). Although this concept does not provide a unique parameter to characterize the degree of grouping, it is believed that it would find wider application because of its suitability to any arbitrary frequency response.

Envelope Exceedance Coefficient

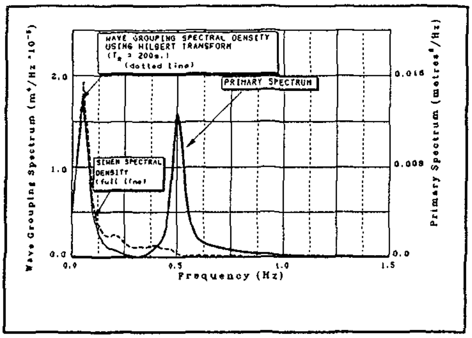
Recently, Medina et al (1990) advanced a new groupiness measure called Envelope Exceedance Coefficient. It is based on the concept of the wave height function computed using the Hilbert transform technique and can be expressed as follows :

$$\alpha = \frac{\alpha'}{E(\alpha')}, \qquad \alpha' = \frac{1}{N}\sum_{n=1}^{N}\left[\frac{H(n\Delta t)-H_{1/10}}{H_{1/10}}\right]^2 \cdot \delta(n) \tag{5}$$

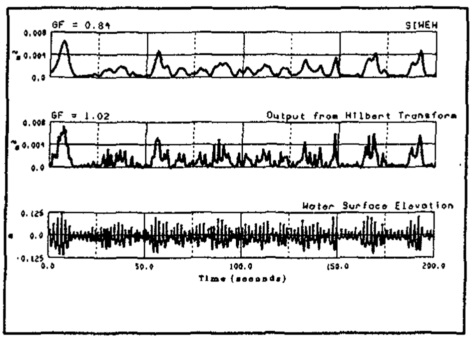
$$\delta(n) = 1 \quad if\ H(n\Delta t) > H_{1/10}, \qquad \delta(n) = 0 \quad if\ H(n\Delta t) < H_{1/10}$$

where:

$H_{1/10}$ = 1.27 Hm_0 (Hm_0 is the estimate of significant wave height derived from the spectrum); and N is the number of data points in the wave height function sampled at Δt intervals.



(a)



(b)

Figure 3. Comparison between SIWEH and Hilbert Transform concepts.

The main assumption used in this concept is that $H_{1/10}$ is the relevant parameter for breakwater stability as suggested in the Shore Protection Manual of 1984. However, this concept can easily be adapted to account for other wave height parameters that may be considered more relevant to structural response. Its main limitation is that it does not lend itself easily to the control of wave group frequency for evaluating the response of floating structures. The SIWEH concept may still be the most useful technique for this particular application, because it gives the user the flexibility of varying the peak frequency and the spectral width of wave grouping spectrum.

As the envelope exceedance coefficient promises to be a useful tool for testing of fixed structures, it was considered relevant to provide a comparative evaluation of this concept with other well known measures through numerical simulation described below.

Comparison of Envelope Exceedance Coefficient with Groupiness Factor and Run Lengths by Numerical Simulation

From a JONSWAP spectrum characterized by its γ value, 100 different realizations of time records were synthesized, using the well known random phase spectrum method. Each wave record, having approximately 650 waves, was then analyzed to provide the values of Envelope Exceedance Coefficient, Groupiness Factors and Run Lengths. For the value of $H_{1/10}$, it was decided to use, as Medina et al. (1990) did, the relationship $H_{1/10} = 1.27\ Hm_0$. In the run length analysis, the threshold was chosen to be $H_{1/10}$. Two Groupiness Factors, one based on SIWEH and another on the expression $\sigma[H^2(t)]/8m_0$ proposed by Medina et al. (1990) were computed.

The above simulation and analysis were carried out for three different values of peak enhancement factor γ = 1, 3.3 and 7. In order to establish the correlation that exists between the exceedance coefficient α and other measures, α values were sorted in the ascending order making sure that their corresponding values of Groupiness Factors and run lengths were also properly re-arranged. A linear polynomial fit was then applied to the various combinations of data sets.

The complete results of these simulations are presented in Figure 4 and they lead to the following conclusions:

The Groupiness Factors, GF, derived from SIWEH increase with γ values. The reasons for this increase can be easily inferred from Figure 1. The goodness of fit between α and GF in the order of 70%.

The GF values from the wave height function are not sensitive to the spectral width. When α=1, the GF value is equal to 1. There is a distinct relationship between these two parameters with a goodness of fit in the order of 78%. However, they are not interchangeable.

There is no correlation between α and run length (goodness of fit less than 10%). The average run length increases with narrower spectral width.

CONCLUSIONS

All the concepts of wave grouping measures proposed in the literature have some limitations in terms of their practical applications.

Statistical variability of groupiness measures increases as the record length decreases. This statistical variability does not permit an easy evaluation of the relationship that exists between wave grouping and spectral width of prototype sea states.

Expected values of wave grouping measures can be deduced from the spectral information of the primary sea state.

The two commonly used wave measures, Groupiness Factor and run length, do not characterize a given sea state in the same fashion.

The Hilbert transform technique of computing the square of the water surface elevation, eliminates the necessity of smoothing it with a Bartlett filter, but needs some improvements in terms of its application to predict structural response of floating structures.

The three new concepts, Envelope Exceedance Coefficient, Motion Equivalent Groupiness Factor, Groupiness Factor Distribution Function, proposed in the literature, provide promising improvements to the existing concepts and deserve further research in terms of their broader applicability.

REFERENCES

Battjes, J.A. and van Vledder, G.Ph. 1984. Verification of Kimura's Theory for Wave Group Statistics. Proc. 19th Int. Conf. on Coast. Engg., Houston, USA.

Bitner-Gregersen, E.M. and Gran, S. 1983. Local Properties of Sea Waves Derived from a Wave Record. Applied Ocean Research, Vol. 5, No. 4.

Funke, E.R. and Mansard, E.P.D. 1979. On the Synthesis of Realistic Sea States in a Laboratory Flume. Tech. Report, Hydraulics Laboratory, National Research Council of Canada LTR-HY-66.

Funke, E.R. and Mansard E.P.D. 1981. On the Meaning of Phase Spectra in the Fourier Transform of Random Waves. Proc. Int. Conf. Hydrodynamics in Ocean Eng., Trondheim, Norway.

Funke, E.R., Mansard, E.P.D. and Dai, G. 1988. Realizable Wave Parameters in a Laboratory Flume. Proc. 21st Int. Conf. on Coast. Engg.,Malaga, Spain.

Galand, J.C. and Manoha, B. 1991. Influence of Wave Grouping on the Stability of Rubblemound Breakwaters, Proc. XXIV IAHR Congress, Vol. B, Madrid, Spain.

Goda, Y. 1983. Analysis of Wave Grouping and Spectra of Long Travelled Swell. Rept of the Port and Harbour Res. Inst. Japan, Vol. 22, No. 1.

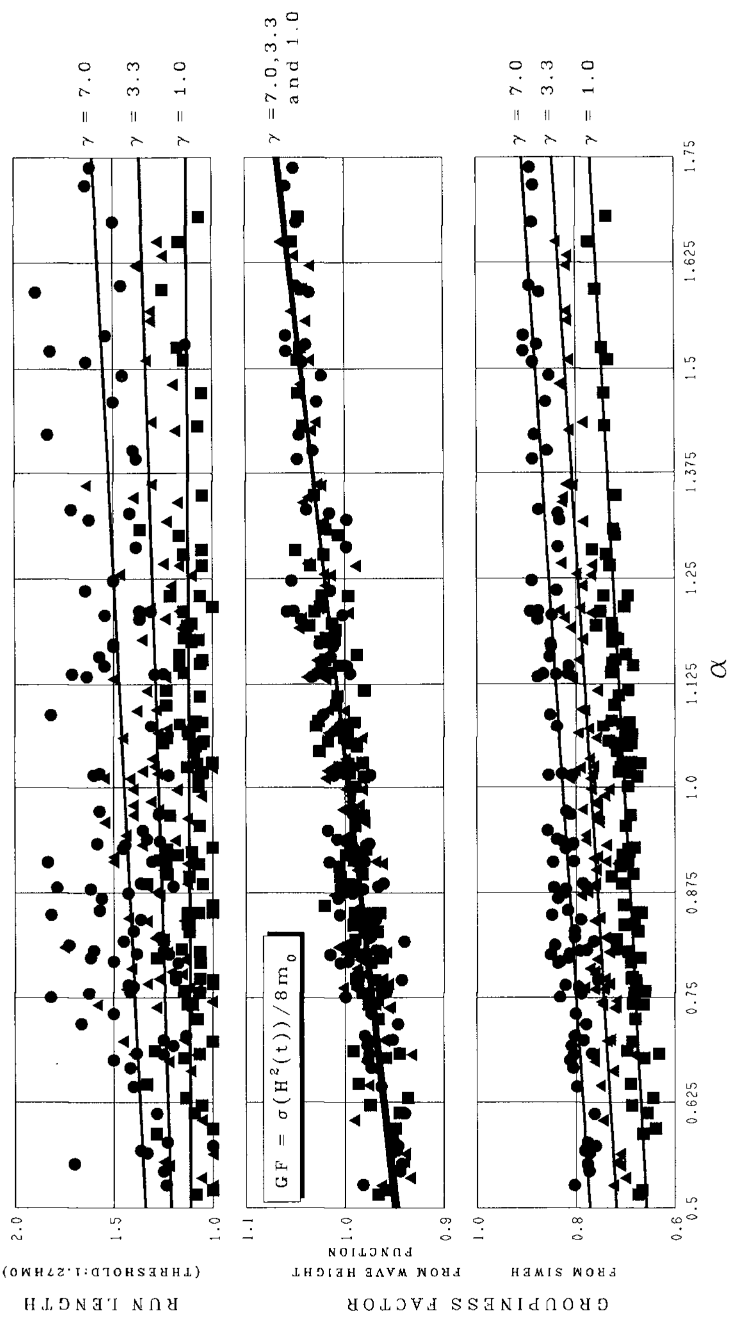


Figure 4. Comparison of four different wave grouping measures

Johnson, R.R., Mansard, E.P.D. and Ploeg, J. 1978. Effects of Wave Grouping on Breakwater Stability. Proc. 16th Int. Conf. on Coast. Engg.,Hamburg, Germany.

Kimura, A. 1980. Statistical Properties of Random Wave Groups. Proc. 17th Int. Conf. Coast. Eng., Sydney, Australia.

Longuet-Higgins, M.S. 1975. On the Distribution of Wave Periods and Amplitudes of Sea Waves. J. of Geophysical Research, Vol. 86 (5).

Mansard, E.P.D., and Sand, S.E. 1992. Towards Improved Concepts for the Description of Wave Grouping. Proc. OMAE'92 Conference Calgary, Canada.

Mansard, E.P.D., Miles, M.D. and Nwogu, O.G. (1994). Laboratory Simulation of Waves: An Outlineof Wave Generation and Analysis Procedures, Proc. Int. Symp. on Waves-Physical and Numerical Modelling, Vancouver, Canada.

Mase, H. and Iwagaki, Y. 1986a. Wave Group Analysis of Natural Wind Waves Based on Modulational Instability Theory. Coastal Eng., Vol. 10, pp. 341-354.

Mase, H. and Iwagaki, Y. 1986b. Wave Group Property of Wind Waves from Modulational Instability. Coastal Engineering in Japan, pp. 565-577.

Medina, J.R., Fassardi, C. and Hudspeth, R.T. 1990. Effects of Wave Groups on the Stability of Rubble-Mound Breakwaters. Proc. 22nd Int. Conf. on Coastal Engineering, Delft, The Netherlands.

Medina, J.R. and Hudspeth, R.T. 1988. Analysis of Wave Groups in Random Fields, Proc., Ocean Structural Dynamics Symposium '88, Corvallis, Oregon

Medina, J.R. and Hudspeth, R.T. 1990. A Review of the Analyses of Wave Groups. Coastal Engineering, Vol. 14, pp. 515-542.

Pinkster, J.A. 1984. Numerical Modelling of Directional Seas. Symp. on Description and Modelling of Directional Seas. Copenhagen, Denmark.

Rye, H. and Lervik, E. 1981. Wave Grouping Studies by Means of Correlation Techniques. Int. Symp. on Hydrodynamic in Ocean Eng., Trondheim, Norway.

Sobey, R.J. and Read W.W. 1984. Wave Groups in the Frequency and Time Domains. Proc. 19th Int. Conf. in Coast. Eng. Houston, USA.

Tayfun, M.A. 1990. Distribution of Large Wave Heights, ASCE J. Waterway, Port, Coastal and Ocean Engineering, Vol 116, No.6.

Vidal, C., Losada, M., and E.P.D. Mansard, 1995. A Suitable Wave Height Parameter for Stability Formulae of Rubble-Mound Breakwaters, to be published in the Mar/Apr issue of ASCE J. on Waterway, Port, Harbour and Coast. Engg.

Van Vledder, G.P. 1992. Statistics of Wave Group Parameters, Proc. 23rd Int. Conf. on Coastal Engineering, Venice, Italy.

Yasuda, T., Nakashima, N.K. and Tsuchiya, Y. 1986. Grouping of Waves and Their Expression on Asymptotic Envelope Solution Modes. Coast. Engg in Japan, pp. 864-876.