CHAPTER 54

The Directional Wave Spectrum in the Bohai Sea*

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Abstract

The directional spectrum is one of the basic characteristics of sea waves. The observations of directional spectrum of sea waves are successfully conducted at platform Bohai 8 during 1991 and 1992 using a wave gage array and it is the first time in China. Based on the field data, the directional spectrum which depends on the wave growth is given in this paper. Before observations, the effects of the figure of gage array, the distance between the gages and the platform itself on the measured results and the precision of some methods for estimating the directional spectrum were investigated and compared with the methods of numerical simulations and model tests of multi-directional irregular waves. This ensures the quality of the observations and estimations of the directional spectrum.

Introduction

One of the main practical problem for research of the sea wave theory and its application is to seek the directional spectrum because it is the key for studying the generating mechanism of the wind waves and the wave deformation as it propagate into shallow water. It is also the important reference for the design of maritime structures. But there is less research work on the directional spectrum due to the complex of the sea wave phenomena and the difficulty of the observation and the analysis of directional spectrum.

At present, the main method for obtaining directional spectrum is observations in field. The commonly used direct measurement methods may be of three types. Early in 1954, Barber(1954) had used wave gage array to measure the directional

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spectrum and gave proposals concerning the wave gage number and the distance between gages for a linear array. After that, some researchers studied several types of gage array with numerical and physical simulation. Longuet-Higgins et al.(1963) used pitch-and-roll buoy to measure the water surface elevation and the orthogonal components of wave surface slopes, and estimated the directional spectrum with Fourier series method. Mitsuyasu et al.(1975) designed a cloverleaf buoy which can measure three components of surface curvature besides the water surface elevation and two components of wave slopes. Therefore, the directional resolution of buoy measurement was enhanced. Also, he gave a formula of directional spectrum which was called Mitusuyasu type spectrum according to the measurement results in the sea of Japan. The another commonly used method is to measure the two horizontal components of orbital velocities by a two-axis current meter and the wave surface which was first proposed by Nagata(1964). This method is simpler and has a higher directional resolution. Besides, the remote sensing such as the stereo wave observation and the microwave technique is also used to measure directional spectrum.

The observations of directional spectrum in China were carried out later. After the ENDECO 956 buoy was introduced from U.S.A. in 1982, it is used to measure the waves for general engineering design. Because there are some disputes about their measured directional spectrum, so less systematic study has been done on it. The Ocean University of Qingdao tried observing directional spectrum with stereophotogrametry several years ago. In this paper, the observation and estimation methods of directional spectrum is mainly studied. Based on the analysis of existing observations of the directional spectrum, and the existing facilities(equipments) and the condition of experiment, wave gage array method is used to measure the directional spectrum for two times. The advance research works were conducted in laboratory and then the available methods of measurement and analysis were set up. Based on the observation results, the directional spectrum of the wind-waves is given in the paper.

Measurement Program

Observation method

At present, there are several measurement and analysis methods of directional spectrum. But their results have a big difference each other. So the measurement and estimation of directional spectrum themselves are an important research problem and they are studied with the numerical simulation and the laboratory experiments(Yu and Liu, 1992, 1993). Considering the present conditions of the equipments and laboratory in China, the wave gage array method which is more reliable and easily carried out on platform is selected to measure the directional waves.

Type and dimension of array

Platform Bohai-8 is almost situated in the center area of the Bohai Sea(39°09'N, 119°42'E). The water depth d is 27.0 m. It consists of a life platform and a working platform and they are linked with a bridge of 50m long. The orientation of the bridge is west-east. The number of wave gages must be greater than 3. 5 wave gages of BCS vertical line type (made by the Institute of Oceanology, Academia Sinica) are used in the observation and they can be arranged in many types. The effects of the array type including linear array, T-type array and pentagon array and the spacing of wave gages on the directional resolution are investigated through numerical simulation and model tests in 3-dimensional basin respectively(Yu and Liu, 1992). The results show that the linear array is only suitable for the waves which have a narrow directional spreading and whose main direction has a large angle with the axis of the array and the gage number must be greater than 5. Both T-type and pentagon array have higher resolutions and the imposing restrictions on gage spacing are not so strict. Considering the situation of the Platform Bohai 8, T-type array is selected.

The wave gage spacing in an array has a big effect on the directional resolution. The results of the numerical and physical simulation show that the minimum spacing should be less than the half of the measured wave length. Considering the irregularity of the sea waves, the ratio between the minimum spacing and the wave length corresponding to the peak frequency of the spectrum can be about equal to 0.3. The vector distance of each pair of gage should be distributed as uniformly as possible in a range as wide as possible and no pair of wave gages should have the same vector distance. According to above guidelines and the real situation of the platform, the T-type array is arranged as Fig.1, in which the minimum distance between wave gages is 4.9m and the maximum value is 21.2m. It can be available for the waves with the period larger than 2.5s.

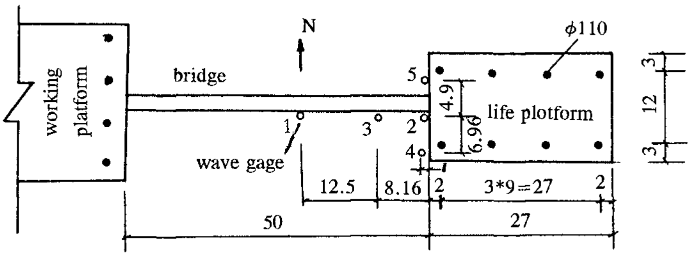

Fig.1 The arrangement of the wave gage array

Effect of the platform on the results

The life platform is supported by 8 steel cylinders and there are some horizontal and inclined cylinders between them. To check the effect of the platform on the waves from all directions, the model tests with the multi-directional irregular waves are carried out in the basin of the State Key Laboratory of Coastal and

Offshore Engineering in Dalian University of Technology, the model scale is 1:40. The test results show that the effects of the platform structure on the directional spreading are small.

The Observations and Data Analysis

Situation of the measurement

The T-type array consisted of 5 gages is arranged as Fig.1. The wave data are acquired simultaneously and automatically with computer for 1200 seconds every hour. The time interval is 0.25s. At the circumstance of strong wind, the continuous wave data are acquired by artificial control. The velocity and direction of wind are recorded simultaneously by wind-velocity-direction meter. At the same time, according to the request of the Specification of Ocean Inspection, the sea state, the wave pattern and the wave direction are observed visually in the daytime. During October 20 to November 4, 1992, 271 runs of 4 wind histories are recorded. There are 144 runs whose significant wave height are larger than 0.5m. The measured maximum wind velocity is 21m/s and the maximum significant wave height is 2.58m.

The Selection of the data

The Bohai sea is basically a closed inner sea and can not be affected by the ocean swell. The directional spectrum formed by a single wind is emphatically investigated in this paper and the wave data are selected according to following principles: (1) The frequency spectra have only one peak, (2) the frequency spectra measured by the 5 gages are almost identical each other and (3) the directional spreading function also basically has one peak and the directional range of the wave energy is narrower. So the 51 runs of data (Table 1) are analyzed. In the table, U is the wind velocity at 10m elevation above the sea level. Cp is the wave velocity corresponding to the peak frequency. $H_{1/3}$, $T_{H1/3}$ and Ls are the significant wave height, period and length respectively. Because U is used in most of the present formula of the directional spectrum, 32 runs of wave data with the angle between the wind direction and the main wave direction being less than 45^0 are used as the observation results being compared with the formulas.

Analysis method of directional spectrum

There are several methods for analyzing directional spectrum. For selecting the optimum method, the single direction per frequency method(Yu and Liu 1991) is used to simulate the multi-directional waves on computer and in 3-dimensional basin and then the waves are measured by a T-type array including 5 wave gages. Afterwards, the directional spreading is analyzed by the parameterized method, the direct Fourier transform method, the extension maximum likelihood method and the Bayesian approach and their results are compared with the input one. It is

Table. 1 The main parameter of the selected data

Date	Runs	Time	U/Cp	$H_{1/3}/L_s$	$H_{1/3}$	$T_{H1/3}$
Oct. 22	8	11:00~19:00	0.76~1.17	0.0282~0 .0374	0.90~ 1.45	4.50~5.07
Oct. 23	8	3:00~17:00	1.01~1.32	0.0296~0 .0366	0.93~ 1.23	4.06~4.92
Oct. 24	2	9:00~10:00	1.20~1.44	0.0286~0 .0302	0.63~ 0.72	3.64~4.02
Oct. 25	2	7:00~10:00	0.38~0.49	0.0159~0 .0202	0.50~ 0.66	4.49~4.58
Oct. 26	1	13:00	0.55	0.0157	0.55	4.74
Oct. 28	7	7:00~22:00	0.76~1.31	0.0235~ 0.0335	0.60~ 1.36	3.92~5.26
Oct. 29	3	5:00~13:00	0.50~0.84	0.0264~ 0.0298	1.01~ 1.07	4.66~5.10
Oct. 30	2	19:00~20:00	1.03~1.35	0.0390~ 0.0397	0.86~ 0.87	3.75~3.76
Oct. 31	12	4:00~23:00	0.96~2.52	0.0247~ 0.0447	0.63~ 2.58	3.83~6.53
Nov. 1	6	0:00~5:00	0.68~1.14	0.0249~ 0.0346	1.48~ 2.12	5.98~6.27
Sum	51		0.38~2.52	0.0157~ 0.0447	0.50~ 2.58	3.66~6.40

shown(Yu and Liu 1993) that the Bayesian approach(Hashimoto, et. al., 1987) has the highest directional resolution and small effect by the wave gage number(greater than 4 only), the distance between wave gages and the width of the directional spreading. So the Bayesian approach is used.

The Frequency Spectrum of the Measured Waves

There are many types of frequency spectrum. The commonly used one is JONSWAP spectrum. In the Technical Specification for Port Engineering of China, the wind-wave frequency spectrum proposed by Wen et al(1989) is adopted. To the equilibrium ranges at high frequency of wave spectra, the slope, p of -5 is adopted by the former spectrum and the latter one use $p=-(2-0.626Hs/d)$(where d is the water depth) i.e., $|p|<4$. Figure.2 show the measured results of the slopes and the mean value can be taken as $p=-4.125$.

For comparing each other, the frequency spectra are nondimensionalized. Comparing the above two spectra with the measured average spectrum shows that the shape of the JONSWAP spectrum is wider and that of the Wen's spectrum is narrower. The spectrum shape proposed by Hong and Yong(1980) is consisted of two parts and can fit the measured one easily(Fig.3), i.e.

$$s(f)=0.117H_s^2T_p(f/f_p)^{-4.125}\exp[-1.171(f/f_p)^{-11}][1+3.893(f/f_p)^{-11}] \tag{1}$$

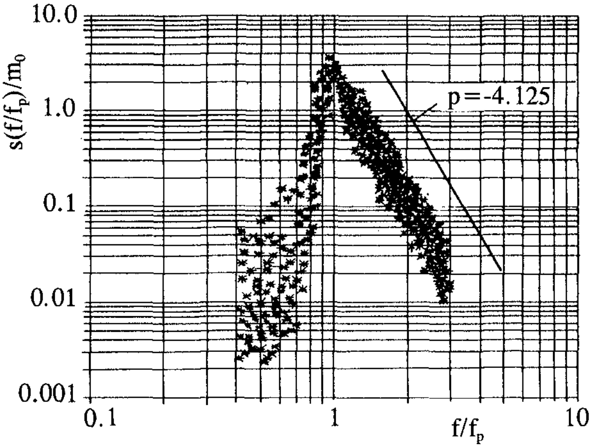

Fig.2 The slope of the equilibrium ranges of the frequency spectrum

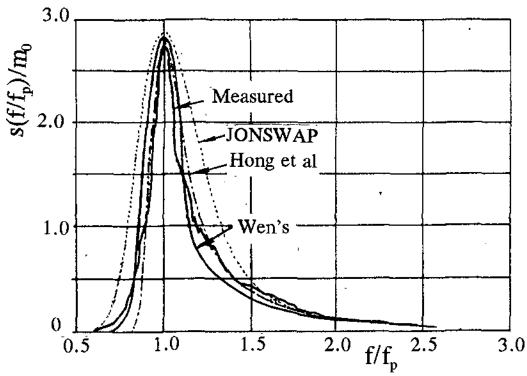

Fig.3 Comparation of the measured frequency spectrum with the fitted one

The Directional Spectrum of Sea Waves

The main aim of the observations is to obtain the directional spectrum of sea waves. The directional spectrum is generally expressed as the product of the frequency spectrum and the directional spreading function $G(f,\theta)$, i.e.,

$$\left.\begin{aligned} &S(f,\theta)=S(f)\cdot G(f,\theta) \\ &\int_{-\pi}^{\pi} G(f,\theta)d\theta=1 \end{aligned}\right\} \tag{2}$$

The measured directional spreading functions of the 51 runs data are obtained by the Bayesian approach and compared with 5 existing typical models.

The present used directional spectrum model

Equation (2) is used in most models, but the spreading function in each model is different.

1. Simple empirical formula. Assuming the directional spreading is independent of wave frequency.

$$G(f,\theta)=G(\theta)=C(n)\cos^{2n}\theta \tag{3}$$

The coefficient C(n) can be determined by Eq.(2). When n=1, C(n)=1/π,; n=2, C(n)=8/3π.

2. Mitsuyasu-type spreading. Longuet-Higgins et al(1963) expressed the directional spreading function as

$$\left.\begin{aligned} &G(f,\theta)=G_0(s)\cos^{2s}\frac{\theta-\theta_0}{2} \\ &G_0(s)=\frac{1}{\pi}2^{2s-1}\frac{\Gamma^2(s+1)}{\Gamma(2s+1)} \end{aligned}\right\} \tag{4}$$

where θ_0 is the main direction of waves, s is the spreading parameter. According to the data measured by a cloverleaf buoy, Mitsuyasu et al(1975) gave the relationship between s and frequency and wind velocity as following

$$\left.\begin{aligned} &s=s_{max}(f/f_p)^5 \qquad f\le f_p \\ &s=s_{max}(f/f_p)^{-2.5} \qquad f>f_p \\ &s_{max}=11.5(2\pi f_p U/g)^{-2.5}=11.5(C_p/U)^{2.5} \end{aligned}\right\} \tag{5}$$

where f_p is the peak frequency of the spectrum and Goda(1985) proposed that f_p can be estimated by $f_p=1/(1.05T_{H1/3})$. When $f=f_p$, $s=s_{max}$, it means that the spreading at the peak frequency is the narrowest. In Eq.(5), Cp/U is the wave age and the less is Cp/U, the younger is the waves. The mean value of Cp/U is around 1.0 for Mitsuyasu's data and it corresponds to fully developed sea waves.

3. Hasselmann directional spreading. Hasselmann et al(1980) used Eq.(4) for

spreading function but gave the following parameters according to the data of JONSWAP:

$$s = s_{\max}(f/f_p)^{\mu} \tag{6}$$

$$When\ f/f_p \geq 1.0 \quad \begin{cases} s_{\max} = 9.77 \pm 0.43 \\ \mu = -(2.33 \pm 0.06) - (1.4 \pm 0.45)(\dfrac{U}{C_p} - 1.17) \end{cases} \tag{7}$$

$$When\ f/f_p < 1.0 \quad \begin{cases} s_{\max} = 6.97 \pm 0.83 \\ \mu = 4.06 \pm 0.22 \end{cases} \tag{8}$$

This model is similar to the Mitsuyasu-type spreading, but s_{max} and the power are different and only at the high frequency side, the spreading function is dependent on the wind velocity. In Eq.(7), U/Cp should be greater than 1.0 because of their measured wave data $U/C_p = 1.0 \sim 1.8$.

4. Donelan's spreading function. Donelan et al(1985) measured the directional spectrum systematically using an array consisted of 14 wave gages at the Lake Ontario in Canada. U/Cp of the data is $0.83 \sim 4.6$. The directional spreading function was given as

$$G(f,\theta) = \frac{1}{2}\beta sech^2 \beta\theta \tag{9}$$

$$\left.\begin{array}{ll} \beta = 2.61(f/f_p)^{1.3}; & 0.56 \leq f/f_p \leq 0.95 \\ \beta = 2.28(f/f_p)^{-1.3}; & 0.95 < f/f_p < 1.6 \\ \beta = 1.24 & others\ f/f_p \end{array}\right\} \tag{10}$$

This model does not include the parameter which expresses the wave growing. When $f/f_p = 0.95$, $\beta_{\max} = 2.44$ and the spreading is the narrowest. When $f/f_p < 0.56$ and $f/f_p \geq 1.6$, the spreading is a constant.

5. Wen's directional spectrum. Wen et al obtained the directional spectrum as follows from the frequency spectrum of each wave propagating direction θ which is derived analytically.

$$\left.\begin{array}{ll} \tilde{F}(\tilde{\omega},\theta) = \dfrac{k_1 k_3}{k_2} P\cos^n\theta\, \tilde{\omega}_\theta^{-p_\theta} \exp[-\dfrac{p_\theta}{q_\theta}(\tilde{\omega}_\theta^{-q_\theta} - 1)]; & \tilde{\omega} \leq \tilde{\omega}_L \\ \tilde{F}(\tilde{\omega},\theta) = \dfrac{\tilde{F}(\omega_L,\theta)}{\tilde{\omega}^4}\tilde{\omega}_L^4 \ ; & \tilde{\omega} > \tilde{\omega}_L \end{array}\right\} \tag{11}$$

where $\tilde{F}(\tilde{\omega},\theta) = \dfrac{\omega_p F(\omega,\theta)}{m_0}$; $\tilde{\omega}_\theta = \dfrac{1}{k_2 \cos^{n_2}\theta}\tilde{\omega}$, $\tilde{\omega} = \omega/\omega_p$, $\tilde{\omega}_L = \dfrac{\omega_L}{\omega_p}$, $\omega_L = 2.38P^{-0.406}$

$$P=1.69\frac{U}{C_p}=1.69\frac{\omega_p U}{g} \tag{12}$$

P express the peakedness of the frequency spectrum and also the wave growing. Other parameters are related to P.

The selection of the spreading model

Figure 4 is an example of the measured directional spectrum and directional spreading function. It is shown that the main directions of the component waves corresponding to different frequency are close to each other. The wave energy are distributed in a narrow direction range and most directional spreading functions are symmetric approximately. The spreading is the narrowest near the peak frequency. These are basically the same as all above models.

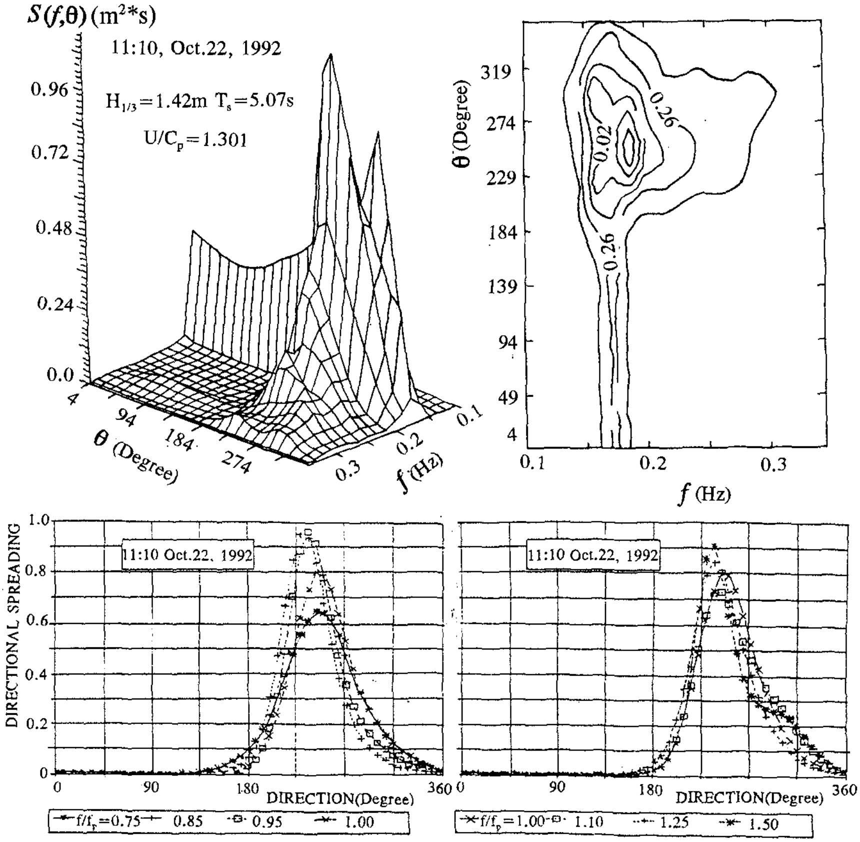

Fig.4 Example of the measured directional spectrum

To check the adaptability of 5 models to the measured results, these results are divided into 2 groups according to that $U/C_p \geq 1.0$ or $U/C_p < 1.0$. The measured results are compared with above 5 models according to the directional spreading curves, the directional cumulative spreading curves(refer to Eqs(13) ~ (15)), the maximum of the directional spreading function and the standard deviation of the spreading respectively. The results show that the Donelan's model conforms to the measured spreading best and the Wen's model takes the second place, but Donelan's model can not consider the effect of the wave growing and Wen's model is too complex. On the other hand, in Mitsuyasu model, the spreading parameter s is dependent on the condition of wave growing and this model has been commonly used in the recent years. So the Mitsuyasu model is used to fit the measured spreading and the Donelan's model will be modified in this paper.

Fitting the measured spreading with Mitsuyasu's Model

Let the cumulative curve of the directional spreading, $G(f,\theta)$ be

$$F(f,\theta)=\int_{-\pi}^{\theta} G(f,\theta)d\theta \tag{13}$$

then the deviations of two directional spreading functions G_X and G_Y, and their cumulative curves are as followings respectively

$$\Delta G_{xy}(f)=\{\int_{-\pi}^{\pi}[G_x(f,\theta)-G_y(f,\theta)]^2 d\theta\}^{1/2} \tag{14}$$

$$\Delta F_{xy}(f)=\{\int_{-\pi}^{\pi}[F_x(f,\theta)-F_y(f,\theta)]^2 d\theta\}^{1/2} \tag{15}$$

When the Mitsuyasu's model X is used to fit the measured spreading, Y, let $\Delta G_{xy}(f)$ and $\Delta F_{xy}(f)$ take the minimum, the parameter s for a given frequency can be calculated and the values of s are varied with f/f_p as shown in Fig.5. In this figure, the Mitsuyasu's spreading of U/C_p=0.7, 1.0 and 1.3 are also given. Figure 5 shows that when $f/fp > 1.0$, the power -2.5 basically conforms to the average measured condition, but when $f/f_p \leq 1.0$ the power 5 is obviously too large and 2.5 may be suitable. s takes the maximum value at $f/f_p = 1.0$. So the parameter s can be determined by

$$\left.\begin{array}{ll} s=s_{\max}(f/f_p)^{2.5}, & f\leq f_p \\ s=s_{\max}(f/f_p)^{-2.5}, & f>f_p \end{array}\right\} \tag{16}$$

Concerning s_{max}, the measured results show that s_{max} is tending to decrease with U/C_p.increasing, but their relationship is not apparent. The growing state of the stable wind-generated waves can be described by U/C_p, but in fact the wind velocity and wind direction are usually changeable and the wave growth much lags

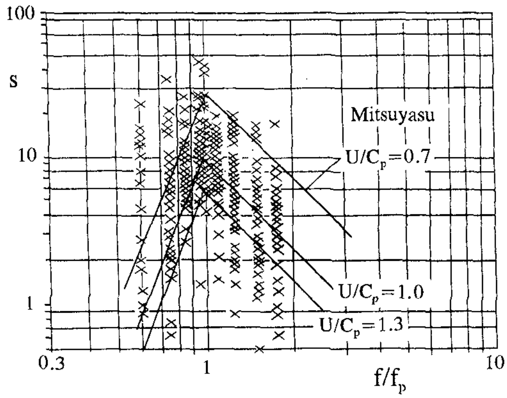

Fig.5 Relation between s and f/f_p

behind the wind changing. Furthermore, it is very difficult to determine the wind velocity corresponding to the design waves for engineering design. So Goda(1985) proposed that $s_{max}=10$, 25 and 75 for wind-waves, swell with short decay and swell with long decay distance respectively, but no observation data yet certify it. In another respect, Sverdrup and Munk had obtained the famous relationship between wave steepness (H/L) and wave age (C/U). When $C/U>0.4$, wave steepness decreases with wave age increasing. The similar results are obtained by this measurement(Fig.6). So the wave steepness is used to express the growing state of the waves, it is convenient for engineering application, and s_{max} can be determined as follows(Fig.7).

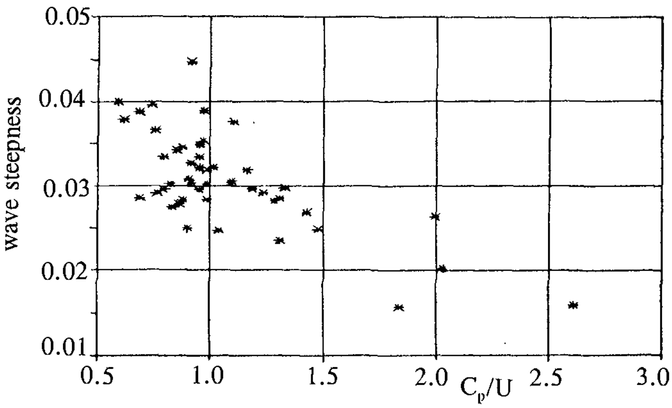

Fig.6 Relation between the significant wave steepness and C_p/U

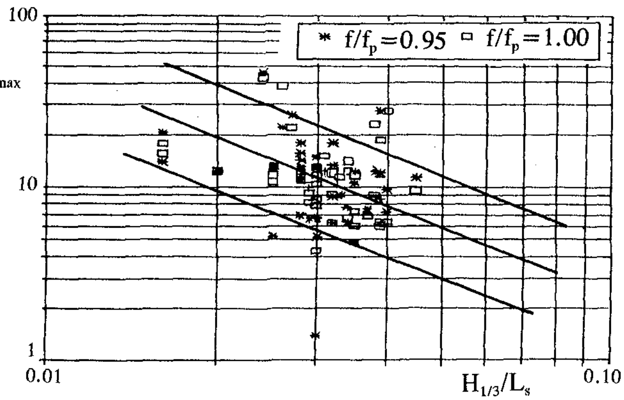

Fig.7 Relation between s_{max} and $H_{1/3}/L_s$

$$\left.\begin{aligned} s_{max} &= 0.13(H_{1/3}/L_S)^{-1.28} \quad (Mean) \\ s_{max} &= 0.26(H_{1/3}/L_S)^{-1.28} \quad (Upper) \\ s_{max} &= 0.065(H_{1/3}/L_S)^{-1.28} \quad (Lowwer) \end{aligned}\right\} \tag{17}$$

The spreading functions with s calculated by Eqs(16) and (17) are compared with the measured. It is shown that they are close each other.

Wang(1992) obtained similar results from the wave data in severe seas during storms that occurred offshore California.

$$\left.\begin{aligned} s_{max} &= 0.11(H_S/L_P)^{-1.28} \quad (Mean) \\ s_{max} &= 0.20(H_S/L_P)^{-1.28} \quad (Upper) \\ H_S &= 4.0\sqrt{m_0} \end{aligned}\right\} \tag{18}$$

Fitting with Donelan model

Similar to the above, let deviations $\Delta G_{xy}(f)$ and $\Delta F_{xy}(f)$ between measured spreading, X and that of Donelan's model, Y take the minimum, the variation of β with f/f_p is shown in Fig.8. Also the relationship between β in Eq.(9) and s in Eq.(4) is shown in Fig.9 Figure 8 shows that Donelan model is basically identical to the measured. But the value of β in Eq.(9) is in the large side. Moreover, for considering the effect of wave growing, β can be determined by Eq.(17) and Fig.9.

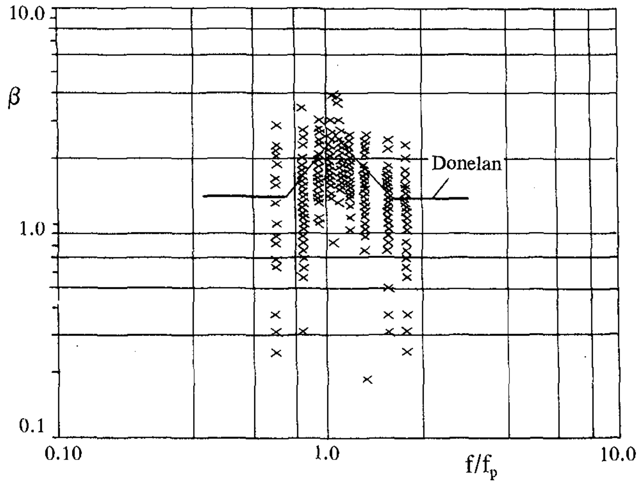

Fig.8 Relation between β and f/f_p

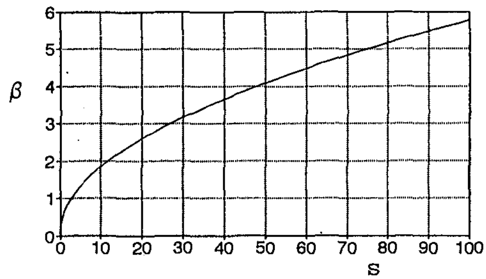

Fig.9 Relation between β and s

Conclusion

1. The observation and estimation methods of directional spectrum are investigated in this paper by combining the numerical simulation and the laboratory experiments with the field observations. The results show that a reliable directional spreading can be obtained using a 5-gage array to measure the waves and the Bayesian approach for the directional spectrum analysis. This method is more suitable for measuring waves in laboratory or on a field platform.

2. The results of the spectral analysis show that the power of the equilibrium range of frequency spectra can be selected as $p=-4.125$. The frequency spectrum can be expressed as Eq.(1).

3. The wave steepness is used to describe the growing state of wind-waves and the relationship between the directional spreading parameter s_{max} and the wave steepness is given in this paper. It is convenient for engineering applications.

4. Using Mitsuyasu model to fit the measured results, the directional spreading function in the Bohai Sea is obtained preliminarily as Eqs(4), (16) and (17). The variation of β in Donelan model with wave steepness is also given in this paper.

5. The process of sea wave growing is very complex and the measured wave data are affected by many factors. In this paper, even if only 51 runs of wave data which generated by a single wind in the center part of the Bohai Sea are used, the datum points obtained from analysis are rather scattered. For obtaining the directional spectrum at coastal and offshore area of China, much more observations with gage array or other methods whose directional resolution is high are needed.

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