CHAPTER 51

RUN-UP OF IRREGULAR WAVES ON GENTLY SLOPING BEACH

YOSHIMICHI YAMAMOTO[1], KATSUTOSHI TANIMOTO[2], KARUNARATHNA G. HARSHINIE[3]

ABSTRACT

Irregular wave run-up on beaches has been studied on the basis of laboratory and field data, which confirmed that long-term wave run-up corresponding to surf beat (or infragravity waves) appears in case of a sea bottom slope gentler than about 1/20. Moreover analytical and numerical models to calculate surf beat caused by wave groups are investigated, and empirical and numerical models to predict the long period wave run-up are proposed.

1. INTRODUCTION

Various patterns of wave run-up on beaches due to irregular incident waves with surf beat are observed in laboratories and fields. Figure 1 shows schematic patterns of time profiles of run-up height for incident wave groups.

1) Pattern-(a): When a sea bottom slope is steep, the state that individual run-up heights almost correspond to individual incident waves is dominant. This pattern is called "predominant incident wave type".
2) Pattern-(b): When the bottom slope is gentler than that of pattern-(a), because the width of the surf zone with swash zone is wider than that of pattern-(a), the disappearance of small waves by run-down, and the capturing and overtaking of waves predominates in the swash zone. This pattern is called "intermediate type".
3) Pattern-(c): When the bottom slope is sufficiently gentle, because the width of the surf zone is wide enough, short period waves are almost eliminated by wave breaking, and the reflection coefficient of short period waves becomes very small. Thus the run-up of long period waves (surf beat or infragravity waves) is dominant. This

[1] Dr.-Eng., Coastal Engineer, INA Corporation, 1-44-10 Sekiguchi, Bunkyo-ku, Tokyo, 112, Japan.
[2] Dr.-Eng., Professor, Department of Civil & Environmental Engineering, The University of Saitama, Urawa, Japan.
[3] Graduate Student, Doctoral Course, Ditto.

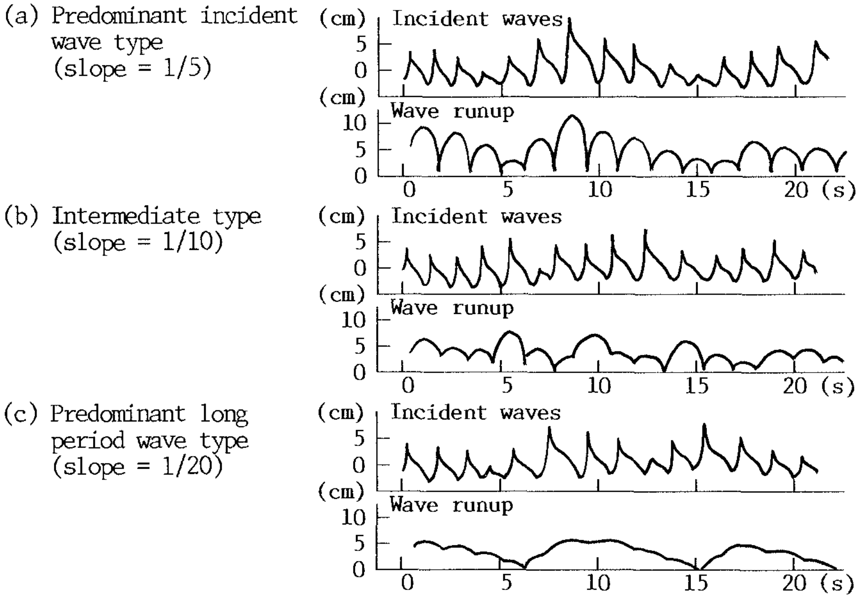


Fig.1 Type of wave run-up profiles.

pattern is called "predominant long period wave type" and the periods of these correspond to those of wave groups.

Mase and Iwagaki (1984) showed that the number of run-up waves becomes smaller as the surf similarity parameter, ξ [= the bottom slope ╱(the wave steepness)$^{1/2}$], becomes smaller. Since the distribution width of bottom slopes is wider than that of the square root of wave steepness, and the groupiness of incident waves can be expressed by the total run length , it is possible to arrange available data by using the mean bottom slope in the surf zone i and the mean total run length j_{2m}. Figure 2 shows the relation between i and j_{2m} for different types of run-up (Yamamoto and Tanimoto, 1994 : by experimental and

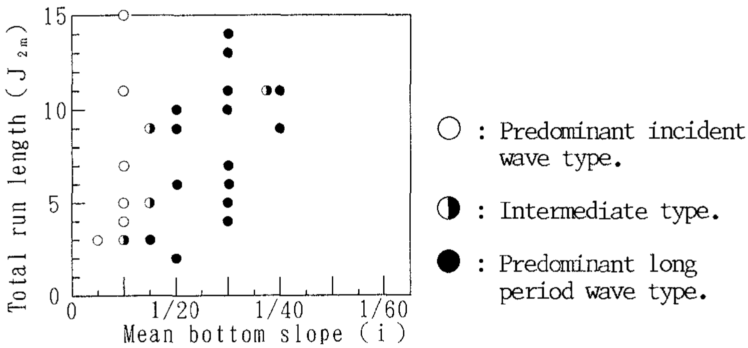


Fig.2 Relation between i and j_{2m} for different run-up types.

field data). As shown in Fig.2, it can be stated that the predominant incident wave type appears mostly when i is steeper than about 1/10, while the predominant long period wave type appears when i is gentler than 1/20. As reasons why data at the slope i=1/37 is peculiar, unsuitable estimation of the slope i and disregard of wave steepness are considered.

In this paper, the predominant long period wave run-up type is investigated. First, the generation mechanism of two-dimensional surf beat is discussed using analytical and numerical models. Then, empirical and numerical models to predict the run-up height are proposed.

2. GENERATION MECHANISM OF SURF BEAT

2.1 Synopsis of Generation Theories

Various theories have been presented concerning the mechanism of generation of surf beat. Since there is a strong correlation between surf beat and run length of offshore waves, the contribution of the long period waves due to wave groupiness under two-dimensional conditions is considered to be very significant.

Two theories have been presented on the generation of long period waves due to wave groupiness.

a) Bound Long Wave (BLW) Theory

The BLW theory was presented by Longuet-Higgins and Stewart(1962). According to this theory, long period waves, whose velocity is constrained by the group velocity, are generated by the periodic variation of the mean water level due to wave groupiness outside the breaker zone. Since the velocity of these waves becomes equal to the velocity of long waves near the breaking point, it is possible to consider that the long period waves propagate landwards as free long waves after breaking of incident waves.

b) Breakpoint-Forced Long Wave (BFLW) Theory

The BFLW theory was presented by Symonds et al.(1982). According to this theory, long period waves are generated by the periodic variation of the mean water level in the breaker zone, caused by the periodic movement of the breakpoint due to wave groupiness. Because the discrepancy between calculated values by their model and measured values was not negligible, Nakamura and Katoh (1992) was later modified model of Symonds et al.

A numerical model embracing both these theories has recently been developed by Goda (1990) and List (1992a,b). The numerical results given by List indicate that, in the case of a uniformly sloping sea bottom, BLW will predominate when the period ratio of the long period waves to incident waves falls below around 10, while BFLW will predominate when the above ratio is greater than 10.

The characteristic of surf beat in surf zone is investigated in the following.

2.2 Treatment of Irregular Waves with Wave Groupiness

In an attempt to simplify the treatment of irregular waves, the wave height H_{off} at the offshore boundary is expressed as follows :

$$H_{off} = H_{offm} - a_H \times \sin(2\pi t / T_{Lm}) \qquad (1)$$

where, H_{offm} is the mean wave height, a_H is the amplitude of the wave height fluctuation, T_{Lm} is the mean period of surf beat.

By Substituting Eq.(1) into the theoretical equation for BLW given by Longuet-Higgins and Stewart, the wave height of BLW can be expressed as follows :

$$\frac{H_L}{H_{offm}} \fallingdotseq \frac{1}{2} \left[\frac{(2c_g/c) - 1/2}{1 - (c_g^2/gh)} \right] \frac{a_H}{h} \qquad (2)$$

where, c_g is the group velocity, c is the wave velocity, g is the acceleration due to gravity, and h is the water depth.

By assuming that the period distribution of irregular waves is narrow, the distribution of H_L corresponds to the distribution of a_H. Thus the incident wave height corresponding to the statistical value of the wave height of surf beat is given by Eq.(1) with specified a_H.

The amplitude a_H corresponding to the significant wave height of surf beat can be expressed (Nakamura and Katoh, 1992) as follows :

$$a_H \fallingdotseq 0.4714\ H_{off1/3} \qquad (3)$$

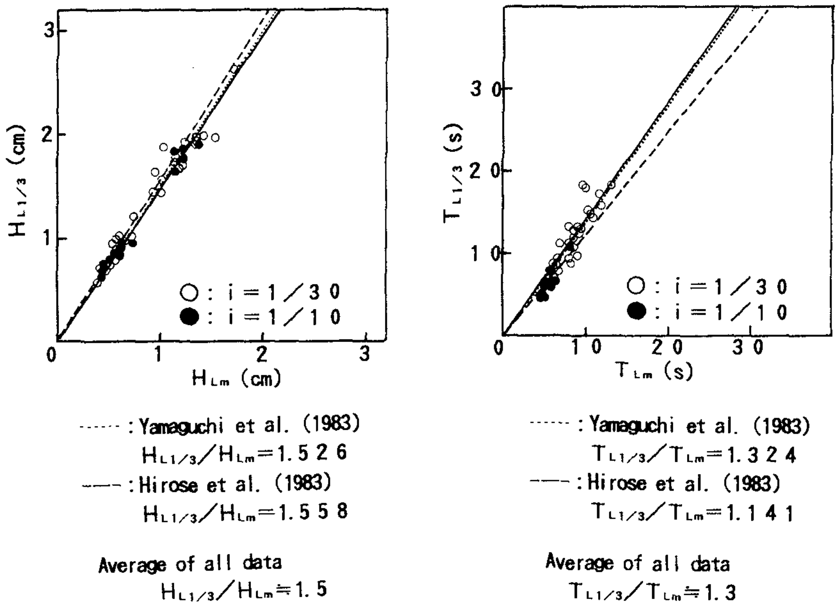


Fig.3 Relation between mean and significant wave heights, and that of wave periods on surf beat respectively (Yamamoto and Tanimoto, 1994).

where, $H_{off1/3}$ is the significant wave height.

Moreover, by experimental and field data as shown in Fig.3, the relationship between the mean wave height and the significant wave height of surf beat (H_{Lm}, $H_{L1/3}$) is expressed as follows :

$$H_{Lm} \fallingdotseq H_{L1/3}/1.5 \tag{4}$$

Thus the amplitude a_H corresponding to the mean wave height of surf beat can be expressed as follows :

$$a_H \fallingdotseq 0.4714\, H_{off1/3}/1.5 \fallingdotseq 0.50\, H_{offm} \tag{5}$$

2.3 Investigation by Analytical Model

Nakamura and Katoh have shown that it is possible to provide a fairly accurate representation of the surf beat in the breaker zone by considering the time lag of breaking accompanying the propagation of waves according to theory of Symonds et al.

The on-offshore distribution of wave heights of surf beat obtained by their model is indicated by the broken lines in Fig.4. In the top three graphs, the calculated values are compared with field data taken at Hazaki Beach (Ibaraki Prefecture in Japan) by Katoh et al. (1991). The bottom two graphs show a comparison between experimental data (at the University of Saitama) and calculated results. In all cases, the surf beat to incident wave period ratio is around 10:1.

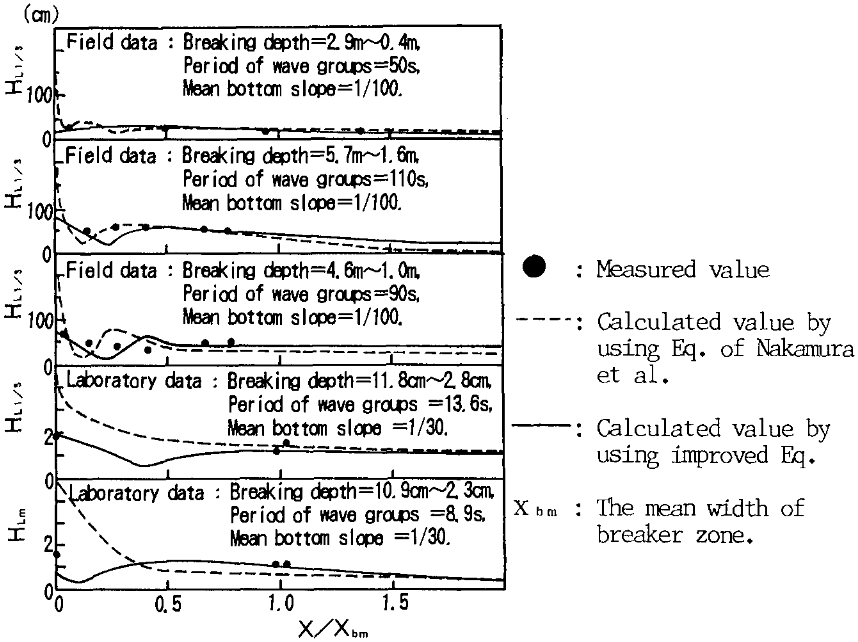


Fig.4 Comparison between observed data and predicted curve.

As can be understood from Fig.4, however, the results obtained by method of Nakamura et al. are abnormally large values near the shoreline. The following improvements were made here, as this study is also concerned also with wave heights at the shoreline.

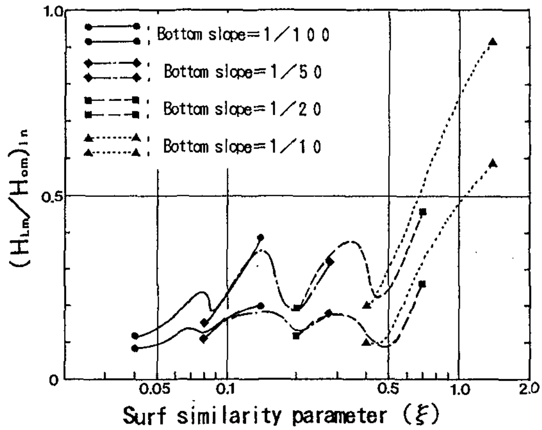
One of the reasons for the abnormally large values near the shoreline could be the absence of considerations for the water level rise in the values used for the water depth in the basic equation proposed by Katoh et al. An improvement can be made for this by solving the following basic equation :

$$\left.\begin{aligned} &\frac{\partial U}{\partial t} + g\frac{\partial \zeta}{\partial x} = -\frac{1}{\rho D}\frac{\partial S_{xx}}{\partial x} \\ &\frac{\partial \zeta}{\partial t} + \frac{\partial (DU)}{\partial x} = 0, \quad D = h + \zeta_0 \end{aligned}\right\} \tag{6}$$

where, U and ζ are the horizontal velocity and the water level of long period waves respectively, t is the time, x is the horizontal coordinate whose origin is the point of intersection between the bottom slope and the still water level, ρ is seawater density, D is the water depth including mean water level rise (ζ_0), and S_{xx} is radiation stress.

The details of the solution are described in the appendix.

The variables relating to BFLW in the breaker zone include the minimum and the maximum widths of the breaker zone (x_{b1} and x_{b2}), the period of the surf beat (T_L), sea bottom slope in the surf zone (i), and the wave height - water depth ratio (2γ) in Eqs.(A.2), (A.5) and (A.6) in the appendix. γ can be ignored as it shows little variation on a gently sloping sea bottom. T_L can be related to the period of short waves (T) using the expression $T_{L1/3} \propto T_{1/3}$ given by Nakamura and Katoh (1992). Furthermore, $L_o = (g/2\pi)T^2$; hence $T_L^2 \propto L_o$. There is a proportionality between x_b and H_o. Thus H_o/L_o and i can be used as approximate indices for the



The subscript "in" stands for mean values inside the breaker zone.

Fig.5 Relation between ξ [$= i/(H_o/L_o)^{1/2}$] and H_{Lm}/H_{om}.

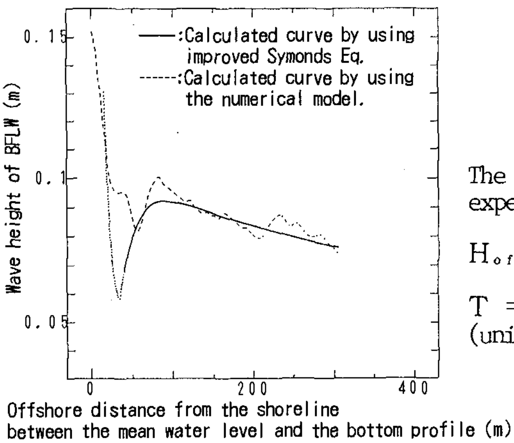
determination of BFLW in the breaker zone.

In Fig.5, the mean wave heights of surf beat (H_{Lm}) are rendered dimensionless and averaged out within the breaker zone, and then arranged by the surf similarity parameter ξ. The values for the cases falling within the range 0.01 < wave steepness < 0.06 have been selected, and ranges in which these values fall are shown by double curves.

2.4 Investigation by Numerical Model

Since real surf beat composed of BFLW and BLW, List's numerical model is improved by taking the change of depth due to the long period waves into account [$D=h+\zeta$ is used instead of $D=h+\zeta_o$ of Eq. (6)]. In this model, incident waves propagate with group velocity. Radiation stress is calculated by the small amplitude wave theory. Breaking points are determined by using Goda's breaker index (1975). BFLW is calculated by giving the radiation stress only in the breaking zone.

The example of calculated wave heights given in Fig.6 are for the BFLW on a beach with a bottom slope of 1/40, caused by wave groups with a mean wave height of 0.8m at a water depth of about 8m, a period of 8.0s and a long wave period of 61.6s. The solid lines in the figure indicate the results obtained using the modified version of the model of Symonds et al., while the dotted lines in the figure indicate the results obtained with the numerical model. This example indicates that the numerical model will give values for BFLW which are more or less the same as those obtained by the theory of Symonds et al. Since the numerical model does not allow the water level to fall below the ground level, the discrepancy between calculated values of both models becomes great near the shoreline.



The same conditions as List's experiment are used.

$H_{off} = 0.8 - 0.4 \times \sin(2\pi\, t/61.6)$,
$T = 8.0$, $T_L = 61.6$,
(units in m and s).

Fig.6 Wave height of BFLW (i=1/40).

Given in Fig.7 are the calculated results for the waves generated on a beach with a relatively steep slope at points shallower than 0.8m and a gentle slope of around 1/70 at points further offshore. Incident wave groups are taken to have mean offshore wave height 0.36m, period 10.9s and long wave period 70s. Since the shallow area where the group velocity is close to the phase velocity of long waves extends widely outside the surf zone, BLW can develop sufficiently.

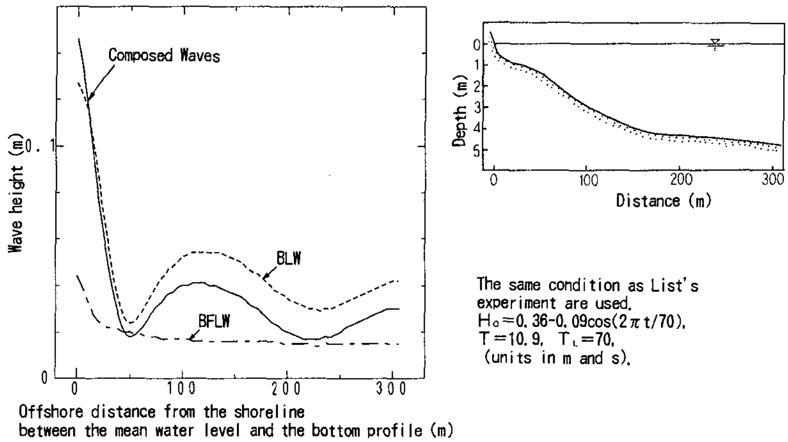


Fig.7 Wave height of surf beat (a concave type coast).

If this assumption is correct, one can expect BFLW to predominate on a beach with opposite slope characteristics. Given in Fig.8 are the calculated results for the waves generated on a beach with a horizontal reef with a water depth of about 2m extending for approximately 500m from the shoreline. Offshore incident waves in this case are considered to have a mean wave height of 2.88m, period of 13.8s, and

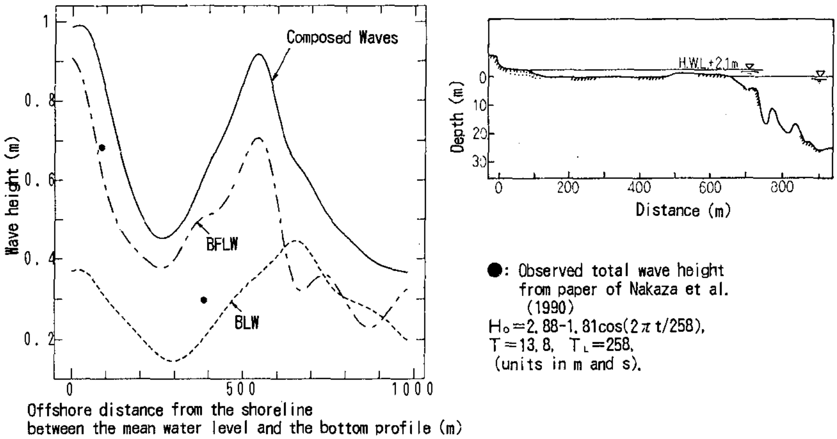


Fig.8 Wave height of surf beat (a convex type coast).

long wave period of 258s. After comparison of calculated results with actual field data, it can be stated that the numerical model has sufficient accuracy.

3. EMPIRICAL AND NUMERICAL MODELS OF WAVE RUNUP

3.1 Empirical Method

By improving Goda's equation (1975) with field and experimental data, the significant wave height of surf beat can be expressed by the following relation (Yamamoto and Tanimoto, 1994) :

$$\left.\begin{aligned} \frac{H_{L1/3}}{H_{01/3}} &\fallingdotseq \frac{0.066\, i^{1/6}}{[(H_{01/3}/L_{01/3}) \times (1+h/H_{01/3})]^{1/2}} \\ \frac{H_{Lm}}{H_{0m}} &\fallingdotseq \frac{0.067\, i^{1/6}}{[(H_{0m}/L_{0m}) \times (1+h/H_{0m})]^{1/2}} \\ &(1/10 \geqq i \geqq 1/70) \end{aligned}\right\} \quad (7)$$

where, $L_{01/3}$ and L_{0m} is significant, mean wavelengths in deep-water respectively, i is the mean bottom slope in the surf zone, and h is the water depth below the still water level.

Assuming the run-up oscillation of surf beat is a parabolic motion, the wave run-up length along the slope can be expressed as follows :

$$y_r \fallingdotseq C_1 U_s t - C_2 g i t^2 / 2 \quad (8)$$

where, U_s is a maximum velocity at shore line [$\fallingdotseq C_3 (g H_{Ls})^{1/2}$], C_1, C_2 and C_3 are empirical coefficients, and H_{Ls} is the wave height of surf beat at the shoreline.

Substitution of Eq.(7) into Eq.(8), and the determination of coefficients C_1, C_2 and C_3 by using field and laboratory data, the period and the run-up height of the surf beat can be expressed by the following equations (Yamamoto and Tanimoto, 1994) :

$$\left.\begin{aligned} (T_{Lm})_{in}^2 &\fallingdotseq 288.0^2 (H_{Lm}/g)_{in} \\ (T_{L1/3})_{in}^2 &\fallingdotseq 305.7^2 (H_{L1/3}/g)_{in} \\ &(1/20 > i \geqq 1/60 \text{ and } H_{om}/L_{om} \geqq 0.015) \end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned} R_{Lm}/H_{0m} &\fallingdotseq 1.52\, i/(H_{0m}/L_{0m})^{1/2} \\ R_{L1/3}/H_{01/3} &\fallingdotseq 1.50\, i/(H_{01/3}/L_{01/3})^{1/2} \\ &(1/20 \geqq i \geqq 1/60) \end{aligned}\right\} \quad (10)$$

where, T_{Lm}, $T_{L1/3}$ are the mean, significant periods of the surf beat respectively, R_{Lm} and $R_{L1/3}$ are the mean, significant run-up heights of the surf beat respectively, and the subscript "in" stands for the mean value inside the surf zone.

Substitution of Eq.(7) and $T_{1/3} \fallingdotseq 3.86(H_{1/3})^{1/2}$ (units in s and m, Bretshneider, 1954) into Eq.(9), and by assuming $H_{01/3}/h \sim 1$ in the surf zone, the same relationship that Nakamura and Katoh (1992) obtained by analysis of field data can be derived ($T_{L1/3} \propto T_{1/3}$).

The agreement between R_{Lm} calculated by Eq.(10) and experimental

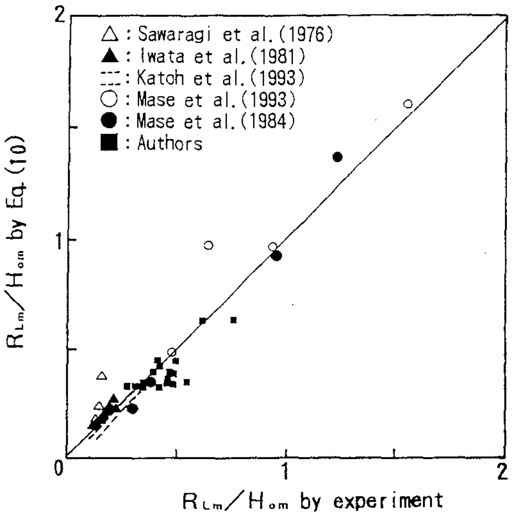


Fig.9 Comparison of R_{Lm} calculated by Eq.(10) with measured data.

and field data are good as shown in Fig.9.

3.2 Numerical Model

Considering the convection and friction terms, the basic equations averaged over the wave period are expressed as follows :

$$\left.\begin{aligned} &\frac{\partial \zeta}{\partial t} + \frac{\partial (DU)}{\partial x} = 0 \\ &\frac{\partial (DU)}{\partial t} + \frac{\partial}{\partial x}\left(\frac{(DU)^2}{D}\right) + gD\frac{\partial \zeta}{\partial x} \\ &= -\frac{1}{\rho}\frac{\partial S_{60}}{\partial x} - \frac{f}{D^2}(DU)\,|DU| \\ &D = h + \zeta \end{aligned}\right\} \tag{11}$$

where, S_{60} is the 60% value of the radiation stress [Mase et al. (1986) states that the equation by the small amplitude wave theory gives excessively large values for the radiation stress and recommends the use of values corresponding to 60% of the obtained value.], and f is the mean bottom friction coefficient, which is obtained by using the following equation based on Freeman and LeMehaute's wave run-up height equation (1964) :

$$f \fallingdotseq \left\{ \frac{(1+C_V)(1+2C_V)}{C_L} - 1 \right\} i\, C_V^2 \tag{12}$$

where, C_V is the wave verocity - particle velocity ratio, and C_L is the loss between potential energy of wave run-up and kinetic energy of waves on a shoreline. According to Yamamoto et al.(1994), these parameters for sand beach are expressed as follows :

$$\left.\begin{array}{l} C_V \fallingdotseq 24.2\, i^{19/30}\, (H_{om}/L_{om})^{1/4} \\ C_L \fallingdotseq 2221\, i^{22/15}\, (H_{om}/L_{om})^{1/4} \\ \qquad (i \leq 1/30) \end{array}\right\} \tag{13}$$

Some calculations on the run-up of the surf beat have been conducted by using the numerical model based on Eq.(11). An example [the incident wave height is $0.8-0.4\times\sin(2\pi\, t/61.6)$ (units in m and s), the wave period is 8.0s, the bottom slope is 1/40] is shown in Fig.10. A relatively close agreement is found between the result obtained by the numerical model and that by the empirical Eq.(10).

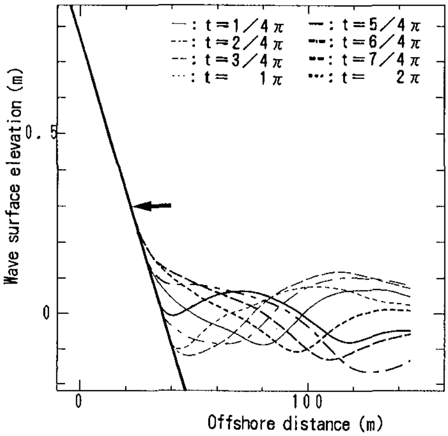


Fig.10 An example of long period wave runup.
[The arrow shows R_{Lm} calculated by Eq.(10)]

A comparison between wave run-up heights computed by the numerical model for beaches with bottom slopes ranging from 1/30 to 1/60 and those obtained by empirical Eq.(10) is shown in Fig.11. Relatively close agreement is found between the model and Eq.(10).

4. CONCLUSIONS

1) Patterns of irregular wave run-up can be classified into three types : predominant incident wave type, intermediate type and predominant long period wave type. The predominant long period wave type appears when the mean bottom slope in the surf zone is gentler than 1/20.
2) Model of Symonds et al. and List's model were modified to allow more accurate calculation of surf beat near the shoreline. The tests conducted with the numerical model show that BLW can develop when the shallow area having a group velocity close to the phase velocity of long waves extends widely outside the surf zone, on the other hand, BFLW can develop when a wide shallow area exists within

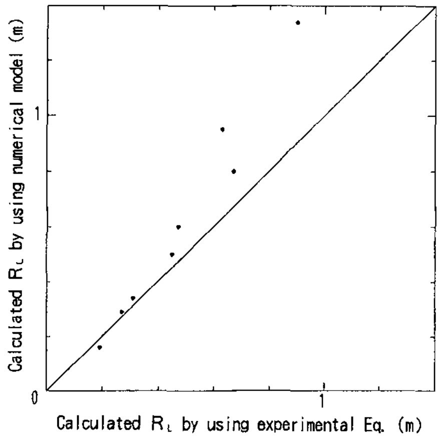


Fig.11 Relation between wave runup height using the numerical model and those using the empirical Eq.(10).

the surf zone such as a reef.

3) Run-up height of long period waves caused by wave groupiness in incident irregular waves can be predicted by both empirical and numerical models.

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APPENDIX : SOLUTION WITH CONSIDERATIONS FOR MEAN WATER LEVEL RISE

The radiation stress in Eq.(6) may be expressed as follows in terms of its relationship to the amplitude [$A_{mp}=\gamma\ (x i+\zeta_o)$] of the incident waves :

$$S_{xx}=(3/4)\rho g\ (A_{mp})^2 \tag{A.1}$$

Eq.(6) is then made dimensionless by using the following dimensionless parameters :

$$\left.\begin{aligned} &A=\frac{a}{x_{bm}},\ X=\frac{x}{x_{bm}},\ \tau=t\frac{2\pi}{T_L},\ X_{b1}=\frac{x_{b1}}{x_{bm}},\ X_{b2}=\frac{x_{b2}}{x_{bm}} \\ &U(X,\tau)=\frac{3}{2}\gamma^2\frac{2\pi}{T_L}Xu,\ Z(X,\tau)=\frac{\zeta(x,t)}{1.5\gamma^2 x_{bm} i} \end{aligned}\right\} \tag{A.2}$$

where, a is $(x_{b1}-x_{b2})/2$, x_{bm} is $(x_{b1}+x_{b2})/2$, γ is (wave height - water depth ratio at breaking limit) $/2$, and i is bottom slope.

The dimensionless horizontal flow velocity U is then eliminated from the dimensionless form of the basic equation, and the differential equation for the dimensionless water level Z is obtained. If the term for the radiation stress in this differential equation is expressed by Fourier series, the following equation will be given :

$$X' \frac{\partial^2 Z}{\partial \tau^2} - \frac{dX''}{dX}\frac{\partial Z}{\partial X} - X''\frac{\partial^2 Z}{\partial X^2} = \frac{d}{dX}\left[X''\frac{dX''}{dX}\left(a_0+2\sum_{n=1}^{\infty} a_n \cos(n\tau)+2\sum_{n=1}^{\infty} b_n \sin(n\tau)\right)\right] \tag{A.3}$$

where, $X' = \left(\frac{2\pi}{T_L}\right)^2 \frac{X}{gi}$, $X'' = X + \frac{3}{2}\gamma^2 Z_0(X)$,
Z_o is dimensionless mean water level rise, and a_o, a_n, b_n is Fourier coefficients (n : term number) - The methods for calculation of these coefficients are given in Nakamura and Katoh (1992).

The solution to Eq.(A.3) may be expressed as follows.

$$Z(X,\tau) = Z_0(X) + \sum_{n=1}^{\infty} Z_n(X,\tau) \tag{A.4}$$

Z_o can easily be obtained from the dimensionless equation of motion, as shown in the first expression in Eq.(A.5) below. For areas further offshore than X_{b1}, however, the values given by Symonds et al. for Z_o are used without alteration, assuming that the effects of the mean water level rise on the water depth here are negligibly small.

Z_n can be obtained in a similar manner to Symonds et al. by means of approximated expression, $dX''/dX \fallingdotseq 1$ (found to be a valid approximation in investigations conducted by substituting actual physical values) during the calculation of the particular solutions between X_{b1} and X_{b2}.

a) From shore-edge to X_{b1}

$$\left.\begin{aligned} Z_0(X) &= (1-X+1.5\gamma^2 A)/(1+1.5\gamma^2) \\ Z_n(X,\tau) &= -(I_{bJ}+I_{aN})\,J_0(z)\cos(n\tau) \\ &\quad - I_{bN}\,J_0(z)\sin(n\tau) \\ z &= 2n(2\pi/T_L)[x_{bm}/(g\,i)]^{1/2} \\ &\quad \times(1+1.5\gamma^2)(X+1.5\gamma^2 Z_0)^{1/2} \end{aligned}\right\} \tag{A.5}$$

Here, $J_o(z)$ is zero-degree Bessel function.

b) From X_{b1} to X_{b2}

$$\begin{aligned} Z_0(X) &\fallingdotseq \{(1-X)\cos^{-1}[(X-1)/A] \\ &\quad - [A^2-(X-1)^2]^{1/2}\}/\pi \\ Z_n(X,\tau) &= [-(I_{bJ}+I_{aN})\,J_0(z) \\ &\quad + C_n N_0(z) + \eta_{pa}]\cos(n\tau) \end{aligned}$$

$$\left.\begin{aligned}
&\quad + [- I_{bN}\, J_0(z) + \eta_{pb}]\, \sin(n\tau) \\
z &= 2n(2\pi / T_L)[X_{bm} / (g\, i)]^{1/2} \\
&\quad \times \{1 + 1.5\gamma^2 \cos^{-1}((X-1)/A)/\pi\} \\
&\quad \times (X + 1.5\gamma^2 Z_0)^{1/2} \\
\eta_{pa} &\fallingdotseq 2\pi [\textstyle\int_{X_{b1}}^{X} X_{an}\, N_0(z) dX\, J_0(z) \\
&\quad - \textstyle\int_{X_{b1}}^{X} X_{an}\, J_0(z) dX\, N_0(z)] \\
\eta_{pb} &\fallingdotseq 2\pi [\textstyle\int_{X_{b1}}^{X} X_{bn}\, N_0(z) dX\, J_0(z) \\
&\quad - \textstyle\int_{X_{b1}}^{X} X_{bn}\, J_0(z) dX\, N_0(z)] \\
X_{an} &= d[(X + 1.5\gamma^2 Z_0) a_n] / dX \\
X_{bn} &= d[(X + 1.5\gamma^2 Z_0) b_n] / dX
\end{aligned}\right\} \tag{A.6}$$

Here, $C_n = 0$ (onshore), $C_n = I_{aJ}$ (offshore), and $N_0(z)$ is zero-degree Neumann function.

c) Offshore from X_{b2}

$$\left.\begin{aligned}
Z_0(X) &= 0 \\
Z_n(X, \tau) &= - I_{bJ}\, J_0(z) \cos(n\tau) \\
&\quad - I_{bJ}\, N_0(z) \sin(n\tau) \\
z &= 2n(2\pi / T_L)[X_{bm} / (g\, i)]^{1/2}\, X^{1/2}
\end{aligned}\right\} \tag{A.7}$$

I_{aJ}, I_{aN}, I_{bJ}, and I_{bN} in Eqs.(A.5) to (A.7) can be obtained as follows using z, X_{an}, and X_{bn} in Eq.(A.6) :

$$\left.\begin{aligned}
I_{aJ} &\fallingdotseq 2\pi \textstyle\int_{X_{b1}}^{X_{b2}} X_{an}\, J_0(z)\, dX \\
I_{aN} &\fallingdotseq 2\pi \textstyle\int_{X_{b1}}^{X_{b2}} X_{an}\, N_0(z)\, dX \\
I_{bJ} &\fallingdotseq 2\pi \textstyle\int_{X_{b1}}^{X_{b2}} X_{bn}\, J_0(z)\, dX \\
I_{bN} &\fallingdotseq 2\pi \textstyle\int_{X_{b1}}^{X_{b2}} X_{bn}\, N_0(z)\, dX
\end{aligned}\right\} \tag{A.8}$$

It should be noted here that partial integration was used for the integration of the above equations.