CHAPTER 40

Velocity Field Measurements and Theoretical Comparisons For Non-Linear Waves on Mild Slopes.

Paul A. Quinn,[1] Marco Petti,[2] Michele Drago[3] & Clive A. Greated[4]

Abstract

The way in which the Boussinesq and Serre models deal with the internal kinematics of waves on mildly sloping beaches is examined. The equation normally used on the depth-averaged horizontal velocity to impose a parabolic profile vertically up through the wave is studied using experimentally and theoretically obtained values. In general, it is found that for near-bed regions the equation provides satisfactory comparisons, however, as one approaches the surface the theoretical values can substantially exceed the experimental ones, particularly in the crest of the wave.

Introduction

A great deal of emphasis has been placed recently on the use of the Boussinesq and Serre models in determining near-shore wave motion (Dingemans, 94a, 94b), mainly due to its recently improved frequency dispersion capabilities. It has some shortcomings, however, in its treatment of the internal wave kinematics based mainly on the fact that it only predicts explicitly the depth-averaged horizontal velocity and relies on an equation to provide a vertical profile of velocity up through the wave, and the divergence of that equation to provide a distribution of the vertical component of velocity.

A description of the Boussinesq and Serre models used in this comparison can be found in Brocchini et al, (1992). They also give the equation for the parabolic profile of velocity as:

---

[1]Research Associate, Fluid Dynamics Unit, Department of Physics & Astronomy, The University of Edinburgh, Edinburgh, EH9 3JZ, U.K.

[2]Lecturer, Department of Civil Engineering, The University of Florence, Via di S. Marta, 3, I-50139 Florence, Italy.

[3]ENVI, Snamprogetti SpA., Fano, Italy.

[4]Professor of Fluid Dynamics, Department of Physics & Astronomy, The University of Edinburgh, Edinburgh, EH9 3JZ, U.K.

$$u(z) = \bar{u} - (h/2)(h\bar{u})_{xx} + (h^2/6)\bar{u}_{xx} - z(h\bar{u})_{xx} - (z^2/2)\bar{u}_{xx} \quad (1)$$

Where u is the horizontal velocity component, $\bar{u}$ is its depth-average, $\bar{u}_{xx}$ is its second derivative with respect to x and h is the local depth from the still water level (SWL). If the bottom topography is known then all that is required to calculate $u(z)$ is a spatial distribution of $\bar{u}$. As Particle Image Velocimetry provides a 2-D spatial distribution of the velocity field it is ideally suited for use in Equation 1.

The first part of this paper uses the values for $\bar{u}$ and $\bar{u}_{xx}$ from PIV results in Equation 1 to calculate the vertical profile of velocity, $u(z)$, and then compares this to the actual profile measured in the experiments. This is based upon the supposition that the best fit to the actual velocity profile will be obtained using the experimental values of $\bar{u}$ and $\bar{u}_{xx}$ in Equation 1. Alternatively, if the model predicted the depth-averaged velocity distribution perfectly will it then provide the correct vertical profile of velocity?

The second part of the paper is then to compare the distribution of the depth-averaged horizontal velocity predicted by the Boussinesq and Serre models with the experiments and then calculate their vertical profiles of velocity.

Particle Image Velocimetry (PIV)

Although there are a number of institutions around the world who now use PIV as a matter of course, it is still relatively new in the field of coastal engineering when compared to techniques like Laser Doppler Anemometry (LDA), and as such warrants a brief description. A comprehensive review of PIV is given by Adrian (1991) and an introduction to the process in a coastal engineering context is given by Greated et al (1992). Quinn et al (1993) gives an account of the errors inherent in the technique and uses as an example the measurement of waves breaking on a 1:30 plane sloping beach. Powell et al (1992) gives an account of the specific application to waves breaking on non-uniform beach slopes.

The first stage of PIV is to photograph the flow, which is seeded with tiny particles and illuminated with a pulsing laser light sheet. The idea is that the camera shutter is held open for several, say four, pulses of the light sheet so that on the photographic negative there will appear four multiple images of each of the seeding particles in the flow. Measuring the separation of the multiple images at any point will yield the velocity at that point, when coupled with the time separation of the light pulses. The seeding particles used in these experiments was Conifer Pollen which has a diameter of 50-70 μm and is almost exactly neutrally buoyant in water. Typical illumination pulse intervals are about 5ms, and the shutter speed of the camera is usually set at 1/60s for this type of flow.

The second stage of the technique is to analyse the developed negative. The analysis is carried out on an automated rig described in some detail in Greated et al (1992) and Quinn et al (1993). The process involves probing the negative on a regular grid (normally 1mm x 1mm) with the beam from a low power Helium-Neon laser. The multiple images of the seeding particles illuminated in the 1mm diameter laser beam cause an interference fringe pattern which is captured by a CCD camera and passed to a Personal Computer (PC). The separation and orientation of the fringes is calculated which gives the average displacement of the multiple images in the small interrogation area of the negative and hence the velocity vector at that point. This process is carried out for every point in the flow field and results in a 2-D velocity array.

Experiments

Beach slopes of 1:30 and 1:100 were used in this study. The experiments were carried out in the Universities of Edinburgh and Florence respectively.

Considering the 1:30 slope first, the experiments were conducted in a 10m wave flume with a SWL of 0.75 m. The wave maker is a hinged, absorbing paddle (Salter, 1982). Due to the restricted length of the flume a 1:30 beach does not reach the bottom of the tank. As we have a hinged paddle rather than a piston-type wave maker, the water depth cannot be altered significantly, so a ramp was installed from the bottom of the wave maker to the foot of the beach, 3m away. The water depth at the foot of the beach was 0.19m and as the waves were so small, (T = 1s, H = 0.032m) the set-up was deemed, if not ideal, then at least satisfactory. A section of the tank showing the beach and the PIV illumination system is shown in Figure 1.

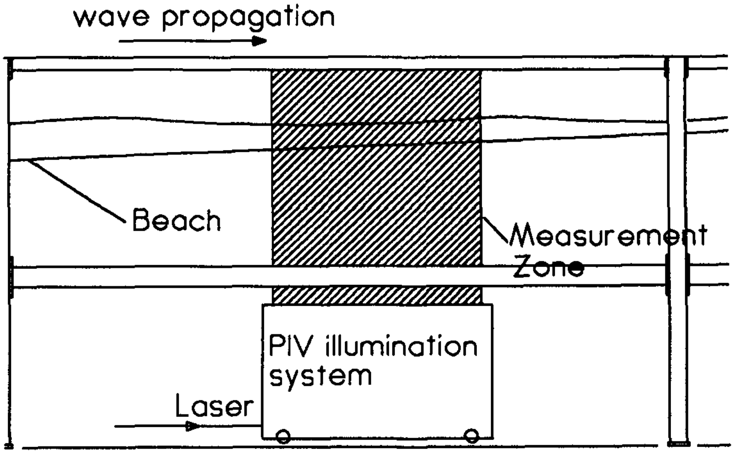

Figure 1: The Flume and PIV Illumination Section.

Regular waves with a period of 1s and a wave height of 0.032m were used. PIV measurements were made at five adjoining positions along the beach, each position being 0.6m in length. Four phases of the wave were recorded at each position.

As the 1:100 beach slope was too mild for the wave flume in Edinburgh the experiments on this slope were carried out in the 50m flume at The University of Florence, Italy. The laser and PIV illumination system were taken over to Florence, from Edinburgh, for these collaborative experiments. As the flume in Florence does not have a glass bottom a special section of the beach had to be built which allowed the PIV laser light sheet in through the side of the tank, below the level of the beach, and reflected it off an underwater mirror up through the beach and into the flow. This is shown in Figure 2.

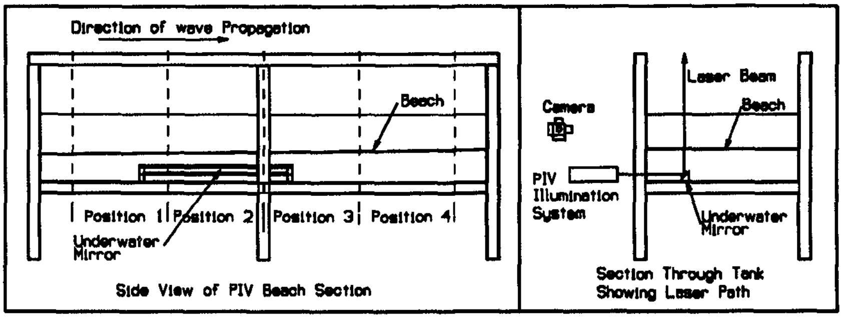

Figure 2: The PIV Beach Section in Florence.

Once again regular waves were used, this time with a period of 3s and a height of 0.1m. The waves were generated in a water depth of 0.42m and propagated about 25m up the beach before they broke. Eight positions were measured around the breaking zone and four phases of the wave were measured at each position.

The wave flume in Florence is fitted with a piston-type wave maker which does have an absorbing system, however, for this experimental set-up with an extremely mild beach slope and no structure to provide a significant reflection, the wave repeatability, as measured by wave-gauge analysis, was worse with the absorbing system activated. For this reason the absorbing system was switched off for these experiments. In order to minimise the effect of the unrepeatability of the waves the first PIV measurement was taken 40s after the start of the wave maker and the subsequent phases were taken from successive waves. The tank was allowed to settle for about 5 to 10 minutes between runs.

Velocity Profile Analysis

Returning to the main aim of the paper, to examine the equation providing a vertical profile of velocity from the depth-averaged velocity component given by the Boussinesq and Serre models. All that is required to calculate the vertical profile of the horizontal velocity component, $u(z)$, from Equation 1, is a spatial distribution of the depth-averaged horizontal velocity. This can easily be calculated from PIV measurements. If one considers Figure 3, which shows the velocity field of the crest phase of a 1s wave on a 1:30 slope, the depth-averaged velocity can be calculated for each column of vectors. Figure 4 shows the resulting spatial distribution of the depth-averaged velocity.

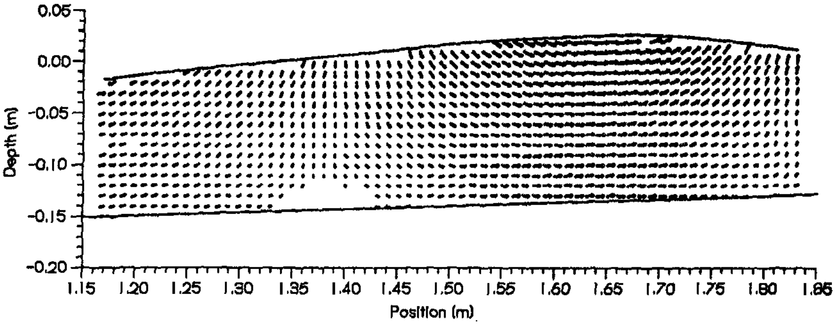

Figure 3: Velocity Vector map for a 1s wave on a 1:30 slope.

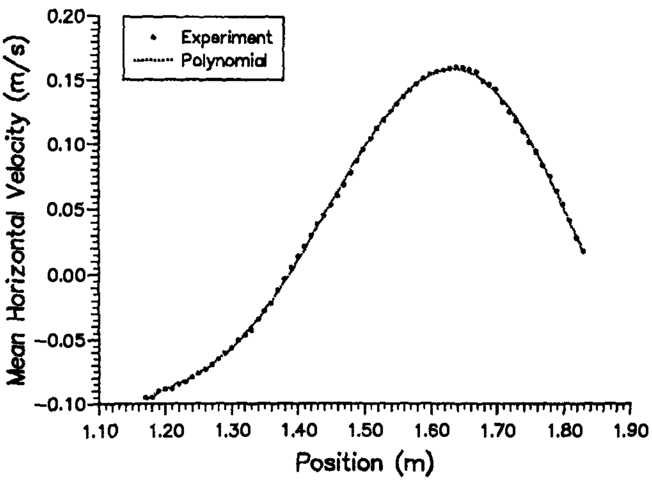

Figure 4: Depth-averaged velocity distribution for a 1s wave on a 1:30 slope.

In Figure 4 the points are the depth-average horizontal velocity components calculated from the vector map (Figure 3). The curve is a least squares polynomial and has been used to calculate $\bar{u}_{xx}$ and $(h\bar{u})_{xx}$. It was not possible to calculate these derivatives from the actual values of $\bar{u}$ because the standard deviation in calculating the means was of the same order of magnitude as the difference between the mean values at adjacent positions. This resulted in the second derivatives being wildly inaccurate. The curve has been used to smooth out this essentially statistical variation, and as the fit is so good it is not thought to introduce any significant errors.

One can now look at the parabolic velocity profiles calculated from the experimentally obtained values of $\bar{u}$, $\bar{u}_{xx}$ and $(h\bar{u})_{xx}$ together with the actual measured profiles. Figure 8 shows this at 5cm steps along the wave.

There are several things to notice from Figure 8: firstly the agreement in general is quite good and particularly so near the bed. For x = 1.20cm the poor agreement is due to an error introduced by trying to fit the polynomial approximation, used to calculate $\bar{u}_{xx}$, near the edges of the data. There is a tendency for the parabolic profile to exceed the measured values in the near-surface region of the crest. This effect is even more noticeable further up the beach where the wave is more non-linear. This is shown in Figure 9 for a 3s wave approaching breaking on a 1:100 slope, the vectorplot of which is shown in Figure 5. The same effect also occurs on the 1:30 slope.

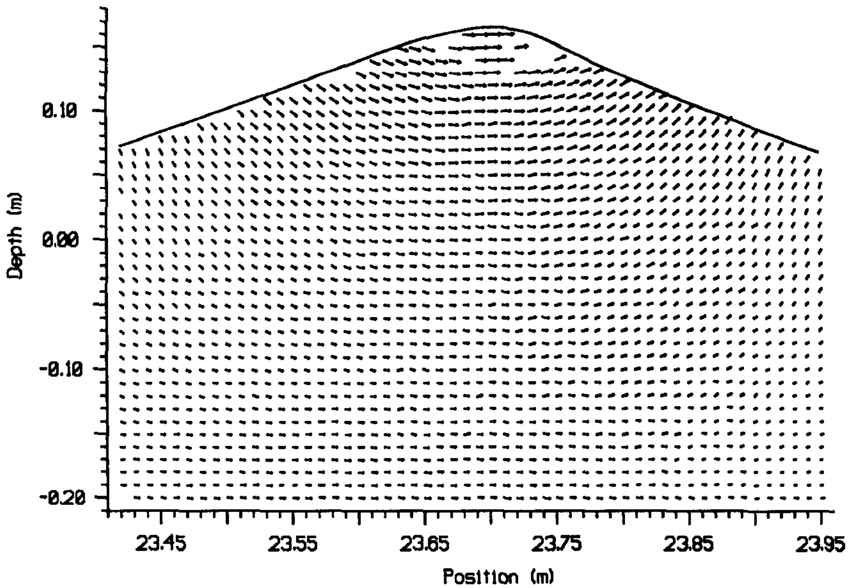

Figure 5: Vector map for a 3s wave on a 1:100 beach

The next step is to see how well the Boussinesq and Serre models can predict the depth-averaged horizontal velocities. The models were given the same wave input parameters, the bottom slope and initial depth for the 1:30 beach. Figure 6 shows the comparison of the distribution of the depth-averaged horizontal velocity given by the Boussinesq and Serre models with the measured values from the vector map in Figure 3.

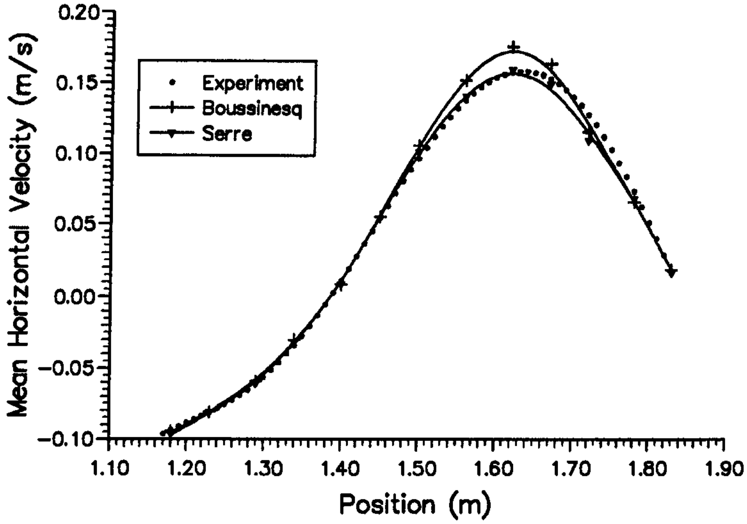

Figure 6: Comparison with the Boussinesq and Serre Models on a 1:30 Beach

In order to calculate the velocity profiles from the models' predictions a least squares polynomial was used, once again. This is also shown in Figure 6. The velocity profiles, calculated in the same way as before are now shown in Figure 10.

Doing the same for a position further up the beach we get the comparison shown in Figure 7 and the velocity profiles shown in Figure 11.

Conclusions

PIV results have been used to test the equation used by the Boussinesq and Serre models to provide a profile of velocity up through the wave.

In general the comparison of a parabolic profile calculated from experimental values agreed fairly well with the measured profiles. The agreement was particularly good near the bed.

There was a consistent overestimate of the measured velocity by the parabolic profile in the near-surface region of the crest of the wave, which appears to get worse as the wave steepens. The degree to which this discrepancy increases with non-linearity has yet to be quantified. One can briefly envisage one of the problems; Equation 1 is derived assuming the same range of validity as the

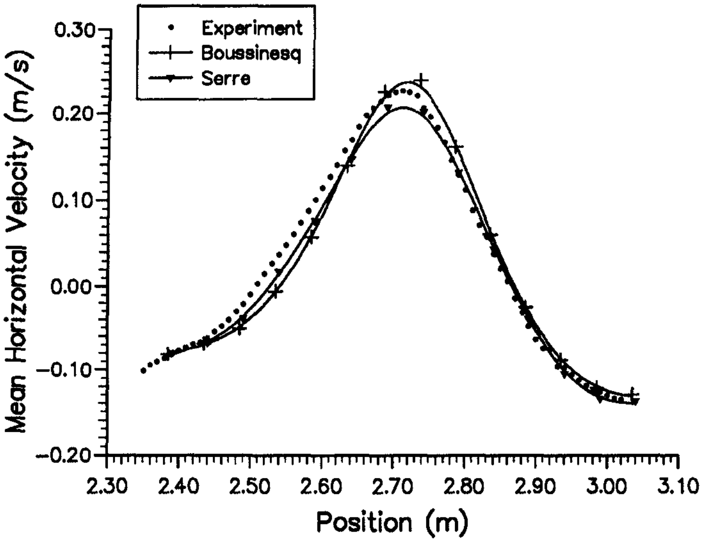

Figure 7: Comparison with the Boussinesq and Serre Models on a 1:30 Beach

Boussinesq and Serre models ie., they include terms of order $(kh)^2$, but they ignore those of order $(a/h)(kh)^2$, where a is the wave amplitude; however, as one approaches breaking $a/h = O(1)$ and so the neglected terms are of the same order of magnitude as the included ones.

The comparison of the Boussinesq and Serre model predictions of the depth-averaged horizontal velocity with the measured values shows an overestimate of the peak velocity by the Boussinesq model and a very close estimate for the Serre model at the first position shown. In the second position, further up the beach, the Boussinesq still overestimates the peak value but the Serre now underestimates it. In both positions the models provided a slightly sharper mean velocity distribution than that given by the measurements.

The velocity profiles calculated from the Boussinesq and Serre model predictions did not show particularly good agreement. This is due to the difference between the predicted and measured spatial distributions of the depth-averaged horizontal velocity, and manifests itself by a shift along the x-axis of the models' profiles with respect to the experiment's. The shape of the curves predicted by the models is similar to that of the experimental profiles, but it still tends to overestimate the curvature of the parabola leading to an overestimate of the near-surface velocity in the crest region. This indicates the sensitivity of the variable $\bar{u}_{xx}$ because small differences in the predicted and measured distributions of $\bar{u}$ can lead to large differences in the parabolic profile. Only the near-bed velocities were adequately modelled on a consistent basis.

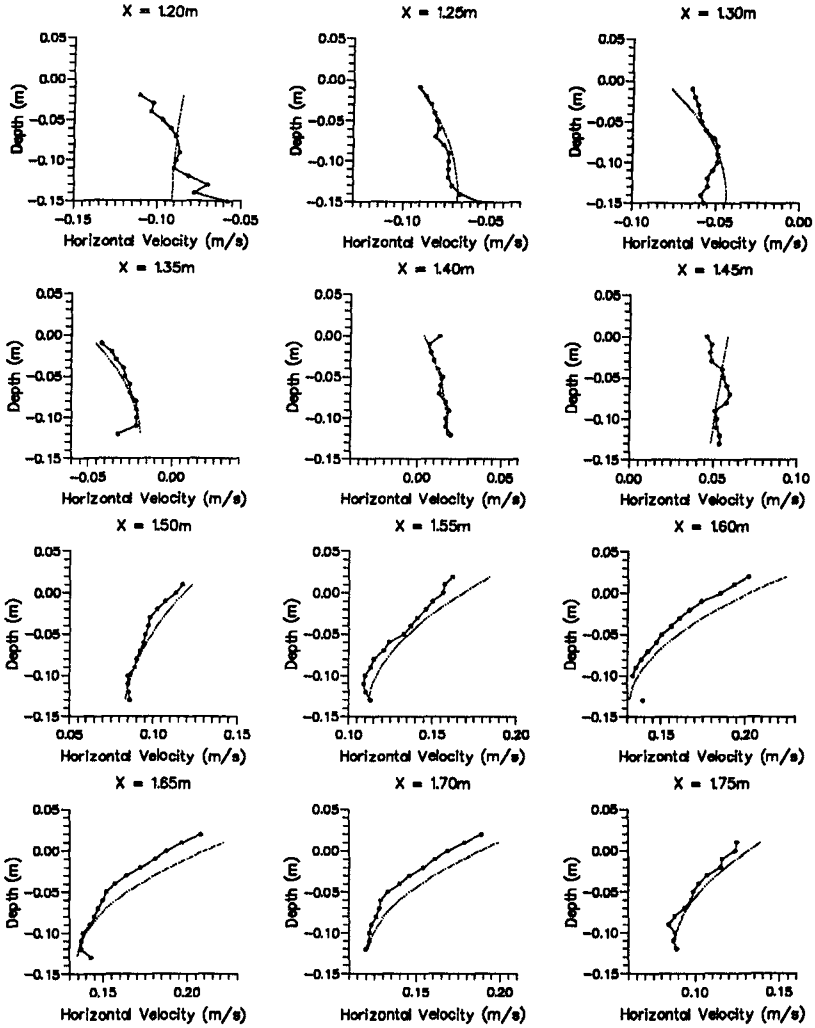

Figure 8: Calculated and Measured Velocity Profiles at Different Horizontal Positions (x) in the 1s wave on a 1:30 Beach.

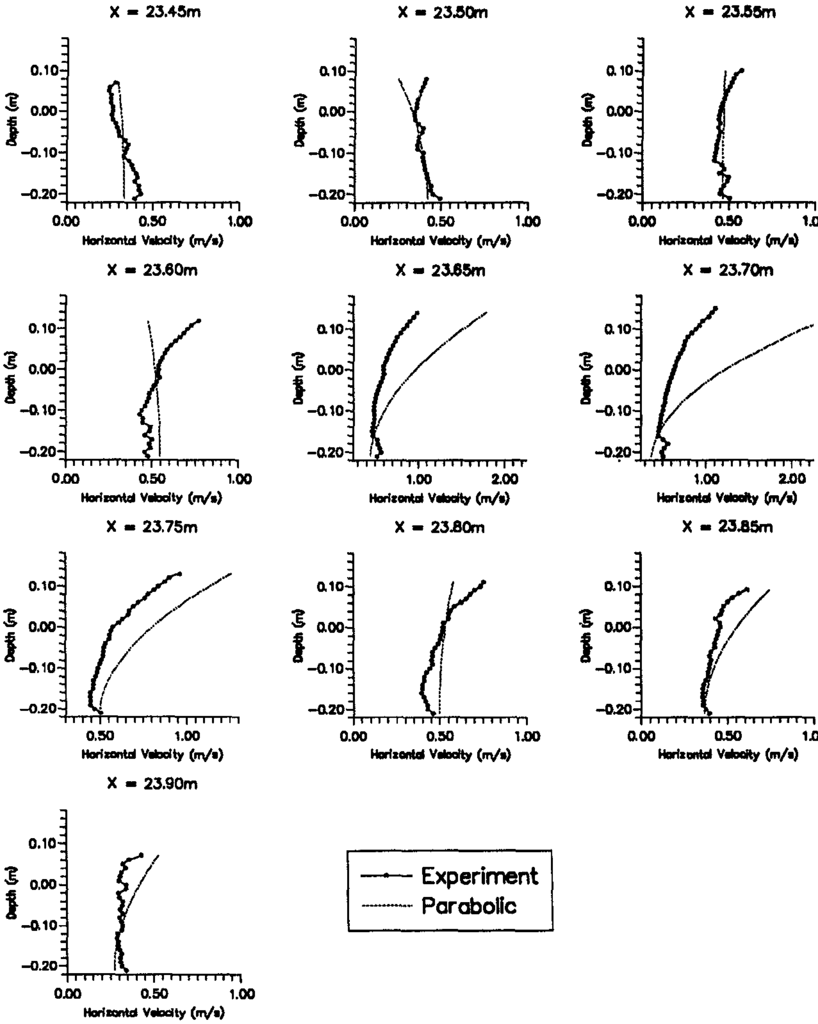

Figure 9: Calculated and Measured Velocity Profiles at Different Horizontal Positions (x) in the 3s wave on a 1:100 Beach.

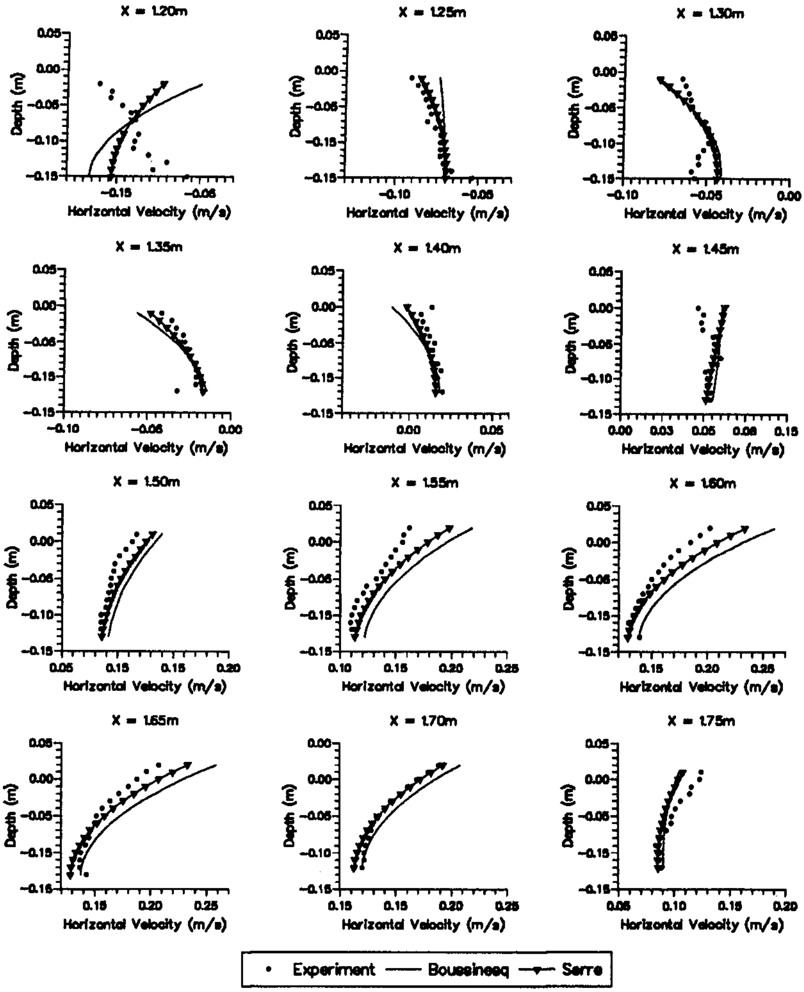

Figure 10: Modelled and Measured Velocity Profiles at Different Horizontal Positions (x) in the 1s wave on a 1:30 Beach.

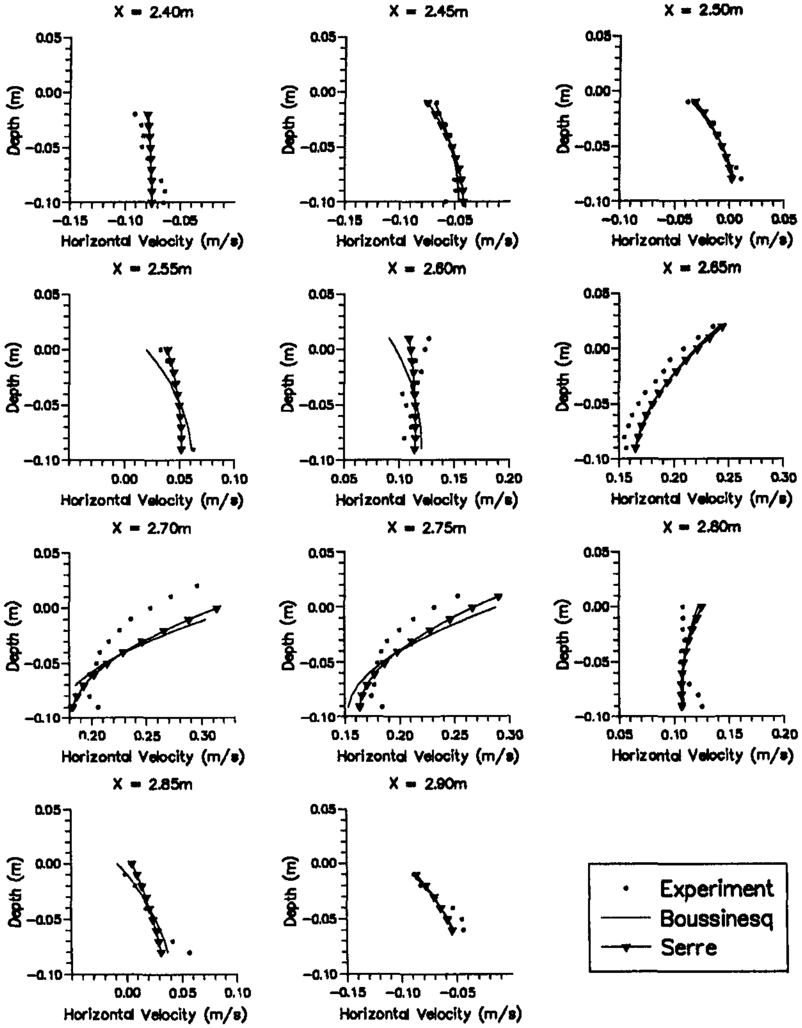

Figure 11: Modelled and Measured Velocity Profiles at Different Horizontal Positions (x) in the 1s wave on a 1:30 Beach.

Acknowledgements

This work was undertaken as part of the MAST G8-M Coastal Morphodynamics research programme. It was funded by the Commission of the European Communities Directorate General for Science, Research and Development under contract N^o. EC MAST-II 0CT 92 0027; their support is greatly appreciated.

The authors also wish to extend their gratitude to the technicians, Frank Morris, Mauro Gioli and Muzio Mascherini, who played an essential role in conducting the experiments in Italy with such a tight time limit.

References

Adrian, R.J. (1991) *Particle Imaging Techniques for Experimental Fluid Dynamics.* Ann. Rev. Fluid Mechanics, 23:261-304.

Brocchini, M., Drago, M. & Iovenitti, L., (1992) *The modelling of short waves in Shallow Waters. Comparison of numerical models based on Boussinesq and Serre equations.* Proc. 23rd Int. Conf. Coastal Eng. 4:76-88.

Dingemans, M.W. (1994a) *Boussinesq Approximations* Proc. EC MAST G8-M Workshop, Gregynog, Wales. (Abstract in Depth)

Dingemans, M.W. (1994b) *Water Wave Propagation over Uneven Bottoms.* World Scientific, Singapore. (To be published)

Greated, C.A., Skyner, D.J. and Bruce, T., (1992) *Particle Image Velocimetry (PIV) in the Coastal Engineering Laboratory.* Proc. 23rd Int. Conf. Coastal Eng. 15:212-225.

Powell, K.A., Quinn, P.A. and Greated, C.A., (1992) *Shingle Beach Profiles and Wave Kinematics.* Proc. 23rd Int. Conf. Coastal Eng. 181:2358-2369.

Quinn, P.A., Skyner, D.J., Gray, C., Greated, C.A. and Easson, W.J., (1993) *A Critical Analysis of the Particle Image Velocimetry Technique as applied to Water Waves.* In *Flow Visualization and Image Analysis*, Ed. F.T.M. Nieuwstadt. Kluwer Academic Publishers, Dordrecht, The Netherlands.

Salter, S.H., (1982) *Absorbing Wave Makers and Wide Tanks.* Proc. Conf. Directional Wave Spectra Applications, ASCE, 185-200.