CHAPTER 27

Probability of the freak wave appearance in a 3-dimensional sea condition

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Abstract

In this study, the appearance probability of the freak waves is theoretically introduced applying the definitions by Klinting and Sand (1987). Their three conditions are formulated theoretically applying the probability distributions for the run of wave heights (Kimura,1980) and the distance of a mean point of the zero-crossing wave crest and trough from mean water level (Kimura and Ohta, 1992b). Its appearance probability in a uni-directional irregular wave condition is studied first, the theory is extended then to the 3-dimensional wave condition, and the definition is discussed in terms of the probability in the last.

1. Introduction

The term "freak waves" may be used to express a huge wave in height. Freak waves have been seen and reported in many places in the world. Many fishing boats, even a man of wars have been destroyed by exceptionally huge waves. And the possibilities have been pointed out that the recent disasters on break waters at port of Sines (Portugal), Bilbao (Spain), (per Bruun, 1985) are also due to the freak waves. Although a common recognitions "what is the freak wave" may not have been established yet, it may have following properties as described by per Bruun (1985).

It is a single "mammoth" short crested wave with, apparently, little relation to its neighboring waves. It has a high crest but not necessarily a corresponding pronounced trough. It does not stay long but break down in small waves.

In 1987, clear definition was made by Klinting and Sand as,

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(1) it has a wave height higher than twice the significant wave height,
(2) its wave height is larger than 2 times of the fore-going and the following wave heights,
(3) its wave crest height is larger than 65% of its wave height.

Recently, only the first condition may be used for the definition (Sand, 1990). However very large appearance probability is given if only the first condition is used. Furthermore the idea that there is a big jump in wave heights, is not realized in the recent definition. The present study applies three all conditions by Klinting and Sand theoretically in the cases of uni-directional and directional random sea conditions. The importance of the conditions is compared and examined through the individual probability.

2. Definition for freak wave

Three conditions given by Klinting and Sand (1987) are as follows.

If we have a following time series of wave height,

$$\ldots\ldots,\ H_{j-1},\ H_j,\ H_{j+1},\ \ldots\ldots$$

and if H_j is the freak wave (Fig.1), the definition by them is expressed as,

(1) $H_j > H_{1/3}$, (Condition 1)
(2) $H_j > 2H_{j-1}$, (Condition 2A)
and $H_j > 2H_{j+1}$, (Condition 2B)
(3) $\eta_j > 0.65H_j$, (Condition 3)

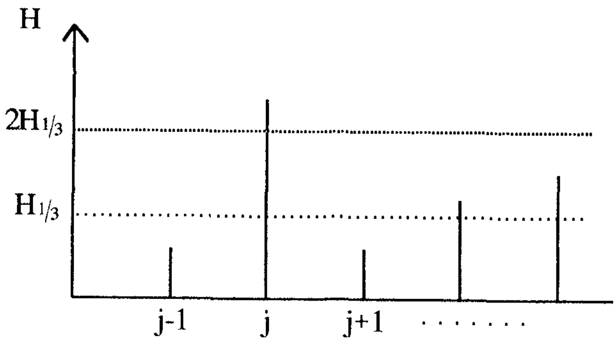

Fig.1 Time series of wave heights. (H_j is the freak wave)

in which $H_{1/3}$ is a significant wave height and η_j is a crest height of H_j.

3. Probability distribution of wave heights

The wave height distribution for the zero-crossing deep water irregular waves agrees well with the Rayleigh distribution. However the agreement of data and distribution in a very large part of wave height has not been sufficiently investigated yet. Goda (1985) reported the measured data show slightly larger probability of appearance than the Rayleigh distribution in a large part of the distribution. Kimura (1981), Mase (1986) reported that increasing non-linearity on wave profiles brings narrower wave height distribution. However Yasuda (1992) showed the non-linearity brings about no significant difference on wave height distribution when the waves are those of fully saturated in a deep sea condition.

If a physical mechanism in which the freak waves are brought about differs from other wind waves, there is a possibility that the freak waves do not follow the statistical law for irregular wave heights. Per Bruun (1985) pointed several phenomena such as orthogonal crossing of waves, overtaking of waves. However numerical simulations for the waves from two separate wind wave sources showed no significant difference in the wave height distribution from the Rayleigh distributions. Therefore we apply the Rayleigh distribution for the wave height distribution in this study. Further assumption used in this study was that waves are those of fully saturated in deep water condition.

4. Formulation of the condition 2A and 2B by Klinting and Sand

If the time series of wave height,

$$\ldots\ldots,\ H_{j-1},\ H_j,\ H_{j+1},\ \ldots\ldots$$

forms a Markov chain and its transition probability is given by the normalized 2-dim. Rayleigh distribution (Kimura, 1980), the probability for the condition 2A is given as follows.

The probability of the first "jump" from H_{j-1} to H_j ($H_j > 2H_{j-1}$) is given as

$$p_{12}(H_j)\,dH = \frac{\int_0^{H_j/2} dH_1 \int_{H_j}^{H_j+dH} p(H_1,H_2)dH_2}{\int_{H_j}^{H_j+dH} p(H_1)\,dH_1}, \tag{1}$$

in which $p(H_1,H_2)$ is the 2-dim. Rayleigh distribution and

$p(H_1)$ is the Rayleigh distribution which are given by

$$p(H_1,H_2) = \frac{\pi^2}{4(1-\kappa^2)} H_1 H_2 \exp\left(-\frac{\pi}{4(1-\kappa^2)}(H_1^2+H_2^2)\right) I_0\left(\frac{\pi\kappa H_1 H_2}{2(1-\kappa^2)}\right) , \qquad (2)$$

and

$$p(H_1) = \frac{\pi}{2} H_1 \exp\left(-\frac{\pi}{4} H_1^2\right) . \qquad (3)$$

H is a normalized wave height by the mean wave height. Correlation parameter (κ) between H_1 and H_2 is correlated to the wave spectrum (Battjes and van Vledder, 1984) as,

$$\kappa = (\rho^2 + \lambda^2)^{1/2} / m_0 , \qquad (4)$$

where

$$\rho = \int_{f_d}^{f_u} S(f) \cos\left(2\pi (f - f_m) T_m\right) df ,$$

$$\lambda = \int_{f_d}^{f_u} S(f) \sin\left(2\pi (f - f_m) T_m\right) df ,$$

$$f_m = m_1 / m_0 ,$$

$$T_m = 1 / f_m ,$$

$$m_n = \int_{f_d}^{f_u} f^n S(f)\, df ,$$

$$f_d = (-0.186 / r + 0.735) f_p \qquad : (4 \le r \le 20)$$

$$f_u = (1.61 / r + 1.62) f_p \qquad : (4 \le r \le 20) \qquad (4)'$$

in which S(f) is a power spectrum, f_p is its peak frequency, r is a shape factor of the spectrum.

$$S(f) = (f/f_p)^{-r} \exp\left[\left(\frac{r}{4}\right)\left\{1 - (f/f_p)^{-4}\right\}\right] . \qquad (5)$$

Narrow integration range from f_d to f_u instead of 0 and ∞ respectively, in the calculations for ρ, λ and m_n is used to improve the value of correlation parameter (Kimura and Ohta, 1992a).

The probability of the second jump from H_j to H_{j+1} ($H_j > 2H_{j+1}$) is also given by

$$p_{21}(H_j)\,dH = \frac{\int_0^{H_j/2} dH_2 \int_{H_j}^{H_j+dH} p(H_1,H_2)\,dH_1}{\int_{H_j}^{H_j+dH} p(H_1)\,dH_1} \tag{6}$$

in which $P(H_1,H_2)$ and $p(H_1)$ are given by eqs.(2), (3) respectively. Combining eqs.(1) and (6), the condition 2A and 2B is given as

$$p_{f1}(H)\,dH = p_{12}(H)\,dH\,p_{21}(H)\,. \tag{7}$$

5. Formulation of the condition 3

Two waves in Fig.2 have the same zero-down-cross wave height and period but different crest heights. To clarify the difference between these two waves, Kimura and Ohta (1992b) introduced the new parameter : the mean point between wave crest and trough (Fig.2). Applying this parameter, the condition 3 is formulated as,

$$d/H_* > 0.15\ , \tag{8}$$

in which d is a distance from the mean water level to the mean point between wave crest and trough, H_* is a wave height.

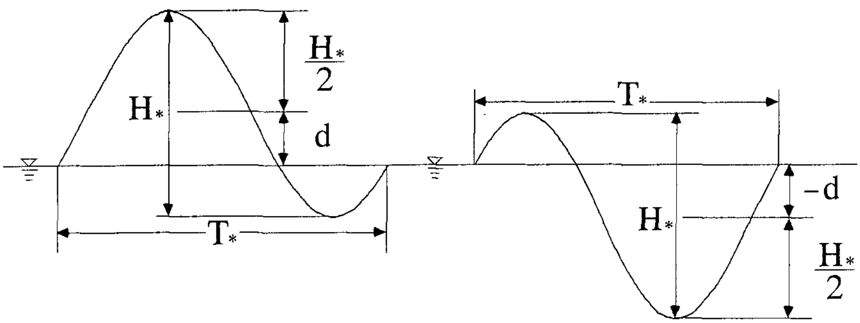

Fig.2 Zero-crossing waves with the same wave height and period but different crest heights.

Probability of eq.(8) is theoretically given as follows. Putting $\varepsilon = d/H_*$, combined distribution of ε and H is given as (Kimura and Ohta, 1992b),

$$p(\varepsilon,H) = \frac{\pi^2 H^3(1-4\varepsilon^2)}{1-\kappa_2^2}\exp\left(-\frac{\pi H^2(1-4\varepsilon^2)}{2(1-\kappa_2^2)}\right) I_0\left(-\frac{\pi\kappa_2 H^2(1-4\varepsilon^2)}{2(1-\kappa_2^2)}\right) \tag{9}$$

in which H is the normalized wave height with its mean ($H=H_*/H_m$). The condition 3 (eq.8) is also expressed as

$$p_{f2}(H) = \int_{0.15}^{0.5} p(\varepsilon | H)\, d\varepsilon \ , \tag{10}$$

in which $p(\varepsilon | H)$ is determined as

$$p\,(\varepsilon | H) = p\,(\varepsilon, H) \,/\, p^*(H) \ , \tag{11}$$

where

$$p^*(H) = \int_0^{2H} p\,(A_1, H)\, dA_1 \ , \tag{12}$$

and

$$p(A_1, H) = \frac{\pi^2 A_1(2H - A_1)}{2(1-\kappa_2^2)} \exp\left(- \frac{\pi \left\{ A_1^2 + (2H - A_1)^2 \right\}}{4(1-\kappa_2^2)} \right) I_0\left(\frac{\pi \kappa_2 A_1 (2H - A_1)}{2(1-\kappa_2^2)} \right) \ . \tag{13}$$

In eqs.(9) and (13), κ_2 is the correlation parameter. This value is calculated using the values 0, ∞ and $T_m/2$ instead of f_d, f_u and T_m respectively in eqs.(4), (4)'.

Solid line in Fig.3 shows the calculated p_{f2} (for a fully saturated sea condition, for example, r=5).

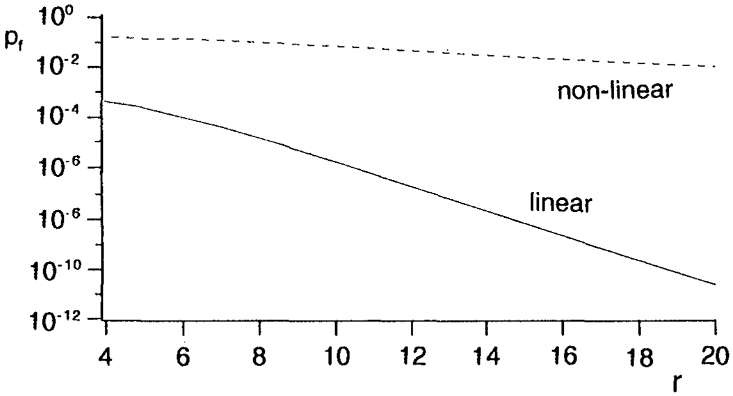

Fig.3 p_{f2} with non-linearity (dotted line) without non-linearity (solid line)

If we take the non-linearity of wave profile into account, p_{f2} increases considerably as follows.

In a deep sea condition, the 3rd order wave profile is given as,

$$\eta = a\cos\left(2\pi\theta\right) + \frac{\pi a^2}{L}\cos\left(4\pi\theta\right) + \frac{3\pi^2 a^3}{2L^2}\cos\left(6\pi\theta\right) \ , \tag{14}$$

in which θ is a phase, L is a wave length and the relation between wave height H_* and a is given by

$$H_* = 2a + 3\frac{\pi^2}{L^2}a^3 \ . \tag{15}$$

Wave steepness of a significant wave for fully saturated wind waves is about 0.04 ~ 0.05 (Goda, 1975). Since $H_f > 2H_{1/3}$ (H_f : freak wave height), wave steepness of the freak wave may be larger than 0.1, and d/H_* may be approximately,

$$\pi a^2 / LH_* = \pi / 4 \cdot H / L \ . \tag{16}$$

Putting H/L in eq.(16) equals to 0.1, we obtain ε to be about 0.08. Therefor taking the non-linearity into account, the condition 3 may be able to change as,

$$p_{f2}(H) = \int_{0.15-0.08}^{0.5} p(\varepsilon \mid H)\, d\varepsilon \ . \tag{17}$$

6. Formulation of the condition 1

Assuming the conditions 2A and 2B and the condition 3 to be independent, the condition 1 together with 2A, 2B and 3 is formulated as,

$$p_f = \int_{2H_{1/3}}^{\infty} p_{f1}(H)\, p_{f2}(H)\, dH \tag{18}$$

7. Result of the calculation

Result of the calculations are listed in Table-1. If we apply eq.(10) for the condition 3, p_f is about $0.155\text{x}10^{-4}$ when r=5 in eq.(5). Narrower spectrum brings far smaller value for p_f. The effect of non-linearity on the condition 3 is compared in Fig.3. Broken line show p_f from eqs.(17) instead of eq.(10). p_f with non-linearity gives a far larger value. ε does not distribute widely when wave height is very large (Kimura and Ohta, 1992b) and large waves have non-linearity on their profiles, the condition 3 may not be important.

Furthermore, the second jump (H_j > H_{j+1}) in the condition 2 may not be an important also. To realize only a hazardous property of the freak wave, consecutive wave height after the freak wave may not be important.

Table-1 Appearance probability of the freak waves

Conditions considered	uni-directional	directional
1	0.321×10^{-3} (1/3,100)	0.477×10^{-3} (1/2,100)
1, 2A, 2B, 3	0.155×10^{-4} (1/65,000)	-
1, 2A, 2B	0.106×10^{-3} (1/9,400)	-
1, 2A	0.198×10^{-3} (1/5,000)	0.278×10^{-3} (1/3,600)

The calculated result is listed in Table-1 when the conditions 2B and 3 are neglected.

8. 3-dimensional sea condition

If we use a single wave gauge in the measurements, the wave gauge can not always record the local maximum in a directional sea condition as shown in Fig.4. If wave gauges can selectively record wave profiles at the local maxima of short crested waves, we may have larger appearance probability of large waves. In this section, the change in the appearance probability of the freak wave is introduced when the maximum

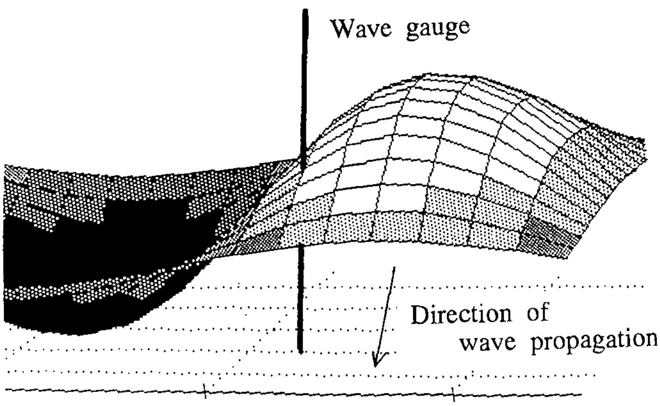

Fig.4 Short crested wave and wave gauge

wave height within a certain distance from a fixed point is applied.

We place a plane Q which is vertical to the horizontal still water plane (x'- y') and is perpendicular to the dominant wave direction. x' is taken in the dominant direction of waves and y' is taken on Q. Figure 5 shows schematically a wave envelope for the cross section of short crested wave profile on Q at a certain instance. A wave gauge is placed at point A and this point is taken as an origin (x'=0,y'=0). Using the Taylor series expansion, the envelope R(y') is expanded around A as,

$$R(y') = R_A + R_A'(y') + R_A''(y')^2/2 + \cdots . \tag{19}$$

R_A' and R_A'' are the first and second derivatives of R at point A. Since we only discuss the wave height in the vicinity around A, we apply three terms in eq.(19) : R is approximated with a quadratic function around A. The value of R at the local maximum (R_B) is given as

$$R_B = \left\{2R_A R_A'' - R_A'^2\right\} / 2R_A'' . \tag{20}$$

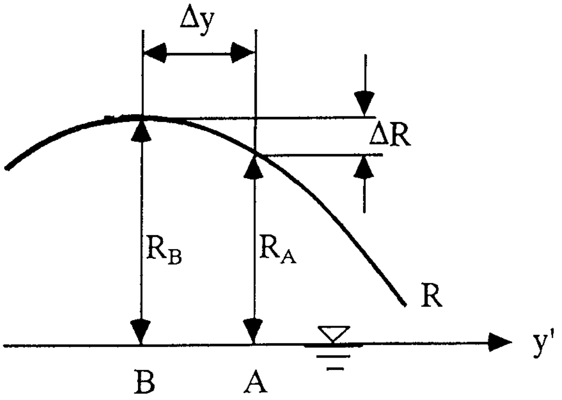

Fig.5 Wave envelope on Q

A distance from A to B is given as

$$\Delta y = \left| R_A' / R_A'' \right| . \tag{21}$$

The probability that the value R_B-R_A exist within ΔR ~ ΔR+dR is given by

$$P_F = \int_S P(R', R''; R_A)\, dR'\, dR'' \tag{22}$$

in which $P(R_A', R_A''; R_A)$ is the conditional probability distribution of R_A' and R_A'' for the given value of R_A. S is the region of integration. Figure 6 shows the region S schematically. Solid lines shows the relations,

$$\Delta R = -R_A'^2 / 2R_A'' \ , \tag{23}$$

and

$$\Delta R = -R_A'^2 / 2R_A'' + dR \ , \tag{24}$$

respectively where $\Delta R = R_B - R_A$. Dotted line shows the relation,

$$\Delta y = | RA' / RA'' | = const. \tag{25}$$

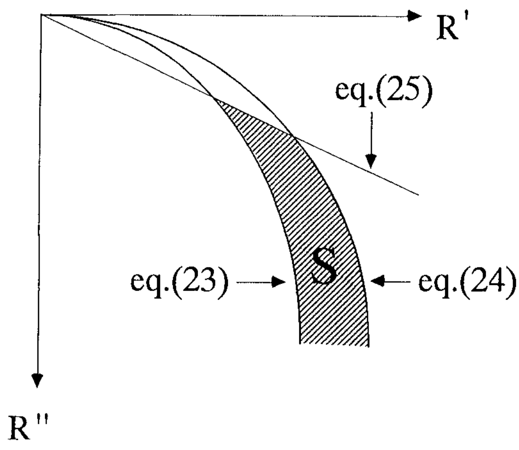

Fig.6 Region of integration S

If the shadowed part is taken as a region S, eq.(22) gives a probability of $R_B - R_A = \Delta R \sim \Delta R + dR$ within a distance Δy on Q from A.

$P(R_A', R_A''; R_A)$ is introduced as follows. The combined distribution of R_A, R_A', R_A'' : $P(R_A', R_A'', R_A)$, is derived theoretically by Rice(1945) as

$$p(R, R', R'') = 2\alpha \int_0^\infty \exp\left(-\beta\phi'^4 - \gamma\phi'^2\right) d\phi' \ , \tag{26}$$

in which

$$\alpha = \frac{R^2}{(2\pi)^{3/2}\sqrt{B_4}}\exp\left\{-\frac{1}{2B^2}(B_0R^2 - 2B_2RR'' + B_{22}R'^2 + B_4R''^2\right\}, \tag{27}$$

$$\beta = B_4R^2 / 2B^2,$$

$$\gamma = (B_{22}R^2 - 2B_4RR'' + 2B_2R^2) / (2B^2),$$

where

$$\begin{aligned} &B = b_0b_2b_4 + 2b_1b_2b_3 - b_2^3 - b_0b_3^2 - b_4b_1^2 \\ &B_0 = (b_2b_4 - b_3^2)\,B, \qquad B_{22} = (b_0b_4 - b_2^2)\,B\,, \\ &B_1 = -(b_1b_4 - b_2b_3)\,B, \qquad B_2 = (b_1b_3 - b_2^2)\,B\,, \\ &B_3 = -(b_0b_3 - b_1b_2)\,B, \qquad B_4 = (b_0b_2 - b_1^2)\,B\,. \end{aligned} \tag{28}$$

b_i (i=0,1,2,3,4) is given as follows,

$$\begin{aligned} &b_0 = \overline{(l_{c1}^2)} = \overline{(l_{s1}^2)}\,, \qquad b_1 = \overline{(l_{c1}l_{s2})} = \overline{(l_{c2}l_{s1})}\,, \\ &b_2 = \overline{(l_{c2}^2)} = \overline{(l_{s2}^2)}\,, \qquad b_3 = \overline{(l_{s2}l_{c3})} = \overline{(l_{c2}l_{s3})}\,, \\ &b_4 = \overline{(l_{c3}^2)} = \overline{(l_{s3}^2)}\,, \end{aligned} \tag{29}$$

where

$$\begin{aligned} l_{c1} &= \sum_{n=1}^{\infty} C_n \cos\left(u_n'x' - u_m'x' + \varepsilon_n\right) \\ l_{s1} &= \sum_{n=1}^{\infty} C_n \sin\left(u_n'x' - u_m'x' + \varepsilon_n\right) \end{aligned} \tag{30}$$

and

$$l_{c2} = (l_{c1})', \quad l_{s2} = (l_{s1})', \quad l_{c3} = (l_{c1})'', \quad l_{s3} = (l_{s1})''. \tag{31}$$

( )' and ( )" are the first and the second derivatives, C_n is calculated by the relation (Longuet-Higgins,1957),

$$\sum_{u,v}^{u+du,\,v+dv} \frac{C_n^2}{2} = E(u,v)\,du\,dv\,, \tag{32}$$

where E(u, v) is the directional wave spectrum in which u and v are the wave number of component wave in x' and y' directions respectively. Σ means to take the total of $C_n^2/2$ in the region u ~ u+du and v ~ v+dv. The conditional distribution $P(R_A', R_A''; R_A)$ is given as,

$$P(R', R''; R_A) = P(R', R'', R) / p(R)\big|_{R=R_A}\,, \tag{33}$$

in which p(R) is the Rayleigh distribution.

Rice (1945) gave a theoretical expression for eq.(26), the integration was made numerically, for the simplicity.

In the calculation of E(u, v), Bretschneider-Mitsuyasu spectrum with the significant wave of $H_{1/3}$ =5.5m, $T_{1/3}$ =10s is used. For the directional function, Mitsuyasu type directional function (Goda,1985) with S_{max} =10 is used. When the sea is in a fully saturated condition, above value for S_{max} is recommended (Goda,1985). The power spectrum and the directional function is multiplied and transformed into E(u, v), applying the dispersion relation of component waves. Above power spectrum brings same statistical properties of freak waves when r=5 in eq.5 and directionarity is ignored.

Figure 7 shows the probability that R_B exceed 2 times of $R_{1/3}$ (1/3 highest amplitude) in terms of $R_A/(2R_{1/3})$. Considerable probability exists in the region $R_A/(2R_{1/3}) >$ 0.85.

If we take the waves of $R_A < 2R_{1/3}$ but $R_B > 2R_{1/3}$ into account as freak waves, appearance probability of the freak wave can be calculated as follows.

When only the condition 1 is used for the freak wave definition,

$$p = \int_0^\infty p_F \, p(H) \, dH \quad , \tag{34}$$

in which

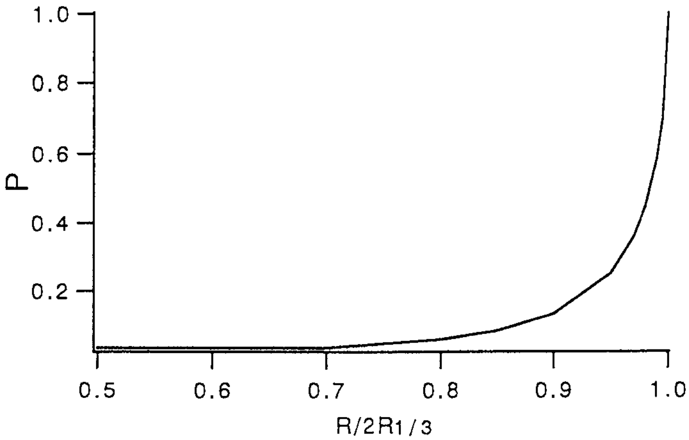

Fig.7 Appearance probability of the freak wave within the vicinity 0.1 $L_{1/3}$.

$$p_F = \begin{cases} p_F & : 0 < R < 2R_{1/3} \\ 1 & : R > 2R_{1/3}\ , \end{cases} \qquad (35)$$

p_F is given by eq.(22).

If the conditions 2B and the 3 are neglected, the appearance probability is given by,

$$p = \int_0^\infty p_F p_{12}(H)\, dH \qquad (36)$$

Equation (35) is also applied in eq.(36) for p_F.

The results from eqs.(34) and (36) are also listed in Table-1. If we take the directional property of waves into account, the appearance probability increases about 45% when the sea condition is fully saturated and the condition 1 and 2A are applied in the freak wave definition.

9. Conclusion

If only the conditions 1 and 2A given by Klinting and Sand are applied, the appearance probability of the freak wave is about 2.78×10^{-4}. This means a freak wave appears once every 3,600 waves on the average. Considering the extremely disastrous properties of this wave, this is slightly too frequent. We may have to use a higher wave height for the freak wave. We also have to continue looking for the possibility that freak wave may not follow the ordinary statistical law but comes from other physical mechanism.

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