CHAPTER 16

MEASURING WAVES WITH MANOMETER TUBES

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Abstract

A new type of wave gauge has been developed for the measurement of waves near a beach or near an existing coastal structure. It consists of a nylon tube with diameter between 0.5 and 1cm and length up to 500m. The seaward end of the tube is open so that the wave induced pressure fluctuations can be transmitted through the water in the tube. The landward end, which is conveniently above the water, is fitted with a pressure transducer. This is simple and reliable technology well suited for use in developing countries. Maintenance and running coasts are also very low. The frequency response function of the system is somewhat complicated but a workable formula is presented and "once and for all" calibration of the system can be done very easily.

Background

The authors were prompted to look for a new type of nearshore wave gauge by the difficulty encountered with getting representative wave data for the Brunswick Heads field site during storm conditions. The problem with that particular site is that the nearest offshore waverider, which is off Cape Byron, tends to go a drift during "interesting" weather conditions. This leads to increased difficulty with interpreting the most interesting data.

The above mentioned example is not isolated. There is a considerable general need for a simple, inexpensive method of measuring nearshore wave heights, i e within 50 to 500 metres of a beach or an existing structure.

The existing devices for nearshore wave measurements include surface piercing gauges, "Schwartz poles", and bottom mounted current meters and pressure transducers. All of these are well proven but not without problems.

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Not all locations allow the installation of a "Schwartz pole" because the supporting structure becomes very expensive with increasing depth and it may present a navigation hazard.

One problem with bottom mounted pressure transducers/current meters is that they tend to get lost to trawlers etc. Secondly, the fact that it is impossible to check their performance during the deployment has lead to many diappointments when the instruments, upon recovery, have been found to contain no useful data. The tubes of the present system are usually buried in the sand and thus protected from "trawler attack", see Figure 1.

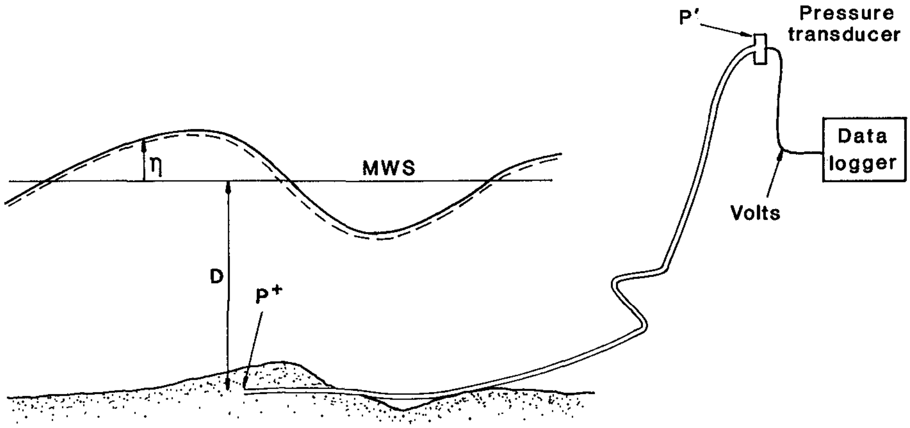

Figure 1: The idea is to estimate the water surface elevation time series $\eta(t)$ on the basis of measured pressure fluctuations $p'(t)$ at the landward end of a water filled nylon tube.

A new type of cheap and reliable nearshore wavegauge

The present study has investigated the possibility of measuring waves by monitoring the pressure fluctuations $p'(t)$ at the landward end of nylon tubes of *10mm* OD and lengths between 50 and 500 metres. The seaward ends of the tubes are exposed to the wave induced pressure fluctuations $p^+(t)$, See Figure 1.

The main problem addressed here is that of estimating of p^+ on the basis of p'. However, the second step in the process of getting wave data from p', namely determining the surface elevation η from p^+ is also discussed briefly.

Initial field testing

Initial field testing has been performed with the new tube-transducer system in order to assure that the transmitted pressure signal $p'(t)$ is of adequate strength and that the system is easy to operate in the field. The system performed very satisfactorily in these tests. The transducer connected to the tubes which are permanently installed at Brunswick Heads. Connection and evacuation of the sytem takes only a few minutes.

The pressure signal was initially recorded by a chart recorder in the field. The $p'(t)$-signal has a low noise level and is of adequate strength to be recorded with the chart recorders *0-50mV* range. The transducers applied in the present study are Model AB Pressure Transducers from Data Instruments Inc, Ma, USA. In later field tests the data was recorded in digital form on a portable PC. Various combinations of pressure transducers and recording equipment can be used but it should be noted that the working range for the pressure transducer should be about 0.5 - 1 atmosphere absolute, i e, the working pressure is below atmospheric pressure.

Pressure transducer on the bed

Relationships between the dynamic bottom pressure $\overset{+}{p}$ $(p(t) = \bar{p} + \overset{+}{p}(t))$ and the surface elevation η may be taken from linear wave theory for monocromatic small amplitude waves

$$\eta = \frac{\overset{+}{p}}{\rho g} \cosh kh \tag{1}$$

where k is the wave number $2\pi/\lambda$, λ is the wave length and h is the water depth.

For irregular, non-linear waves, local approximations may be used. Nielsen (1989) recommended the formula

$$\eta_n = \frac{\overset{+}{p}_n}{\rho g} \exp\left\{\frac{2}{3} \frac{-\overset{+}{p}_{n-1} + 2\overset{+}{p}_n - \overset{+}{p}_{n+1}}{\overset{+}{p}_n\, g\, \delta_t^2} (h + \frac{\overset{+}{p}_n}{\rho g})\right\} \tag{2}$$

based on a measured time series $\overset{+}{p}_1, \overset{+}{p}_2, \overset{+}{p}_3, \cdots$.

As an alternative to the local approximations approach, the classical spectrum transformation approach may be applied. With this method a Fourier transform is applied to the pressure record. Then each spectral estimate is transformed in accordance with Equation (1). Finally, the inverse Fourier transform is applied to the transformed spectral estimates to obtain an estimate of the surface elevation time series. The relative merits of the two methods has been discussed by Nielsen (1989).

Transducer burried in the bed

It is possible that a layer of sand on top of the "open" end of the tube can cause extra damping. Most likely however, the effect is negligible for typical beach sand and typical wave frequencies.

Sleath (1970) and Maeno & Hasagawa (1987) measured pore pressures inside the bed simultaneously with pressures at the bed surface. The pressure amplitude ratios agreed reasonably with the formula

$$|p^+| = \rho g \frac{H}{2} \frac{\cosh k(z+h_1)}{\cosh kh \ \cosh kh_1} \tag{3}$$

where z is the transducer elevation measured from the sand surface h is the water depth and h_1 is the thickness of the sand bed, see e g Sleath (1984).

According to this formula it makes very little difference whether a transducer, at a fixed depth below the water surface, is covered with sand or not. Consider for example, the situation in Figure 2.

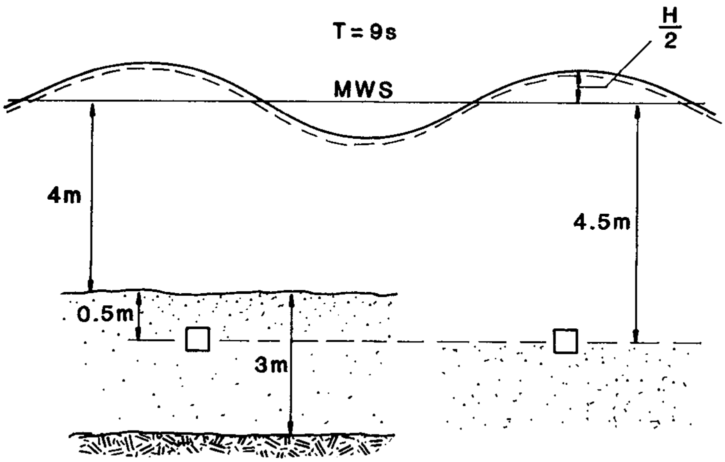

Figure 2: The pressure felt by a transducer *4.5m* below the MWS under a *9s* wave changes very little due to a sand cover of *0.5m*.

A pressure transducer is placed *4.5m* below the MWS. The wave period is 9s, and we consider the situation where the sand level is at the transducer level as well as the situation where the sand level is 0.5m above the transducer. The bed is assumed impermeable below -7m.

Linear wave theory plus Equation (3) gives $|p^+| = 0.889\rho gH/2$ for the "uncovered case" and $|p^+| = 0.885\rho gH/2$ for the "covered" case. This difference is negligible compared to the accuracy of linear wave theory. It should be noted however that finer material like silt or mud will provide a stronger damping of the pressure signal than 0.2mm sand.

The speed of pressure waves in a flexible tube

The frequency response of the tube-transducer system in Figure 1 depends crically on the speed of pressure waves in the tube. This speed, in turn, is a function of the compressibility of the water, the rigidity of the tube walls and, if air bubbles are present, of the bubble concentration.

In an infinite fluid, the speed c of a plane sound wave is determined by the density ρ and the compressibility K

$$c = \frac{1}{\sqrt{\rho K}} \tag{4}$$

The speed of sound in sea water is approximately 1500m/s, corresponding to a compressibility K of $4.4 \cdot 10^{-10} Pa^{-1}$.

If the fluid is contained in a flexible tube, the speed of sound will be reduced in accordance with the formula

$$c = \frac{1}{\sqrt{\rho\,(K+D)}} \tag{5}$$

where the distensibility D of the tube is defined in terms of the normal cross sectional area A_o and the excess pressure p_e by

$$D = \frac{1}{A_o}\frac{dA}{dp_e}\Big|_{p_e=o} \tag{6}$$

If the tube cross section is circular, the distensibility can be due to stretching of the wall only. However, if the normal tube cross section is not circular a greater distensibility may be partly due to this non-circularity. In this case, an area increase may be obtained by bending the wall towards the circular shape. Thus, the speed of sound in an oval shaped tube will be lower than in a perfectly circular tube for the same wall thickness.

Experimental determination of the distensibility

Experiments were performed to determine the distensibility of *10mmOD* Nylex tubing by monitoring the volume increase in *60m* of tube as function of excess pressure.

The results are shown in Figure 3 and the best-fit distensibility $(\frac{1}{V_o}\frac{dV}{dp})$ was found to be $2.0 \cdot 10^{-8} Pa^{-1}$. We note that the behaviour of the tube material is linear up to excess pressures of at least *120 kPa*, corresponding to *12m* excess head of water.

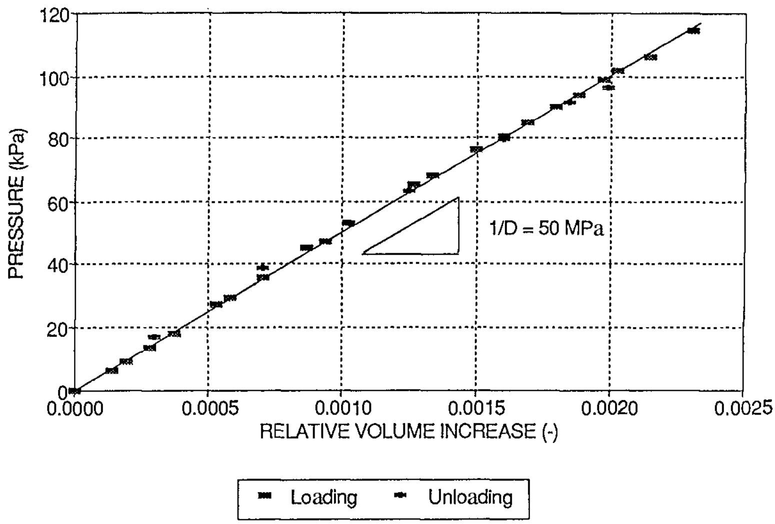

Figure 3: Expansion test data for *10mm OD* Nylex pressure tubing (standard).

According to Equation (5) this corresponds to the speed of sound c = 224m/s for a *10mm OD* Nylex tube with no air bubbles.

A complementary streching test was performed on a short (*150mm*) length of tube to determine Young's modulus E for the tube material. Based on a measured ID of *6.7mm* and wall thickness δ of *1.67mm* the result was $E = 2.03 \cdot 10^{8} Pa$.

Through the simple relationship

$$D \approx \frac{d}{\delta E} \qquad (7)$$

this gives a distensibility of $1.95 \cdot 10^{-8} Pa^{-1}$ in close agreement with the directly measured value above. The manufacturer's value for E is $35 kg/mm^2$ corresponding to 3.4·108Pa for standard tubing (all colours) and $100 kg/mm^2$ (They must be thinking in terms of "kg force") corresponding to 9.8·108Pa for "semi rigid tubing (only black). The discrepancy (3.4 versus 2.0) being due to uncertainty of tube dimensions in test and to variable humidity. The laboratory tests were perfomed with fully wet tubes.

The effect of air bubbles on the speed of sound

Small, isolated air bubbles can also slow down the pressure waves in the tube. They do this by effectively increasing the distensibility. To quantify this effect, consider for simplicity an air bubble of volume V_o at the ambient pressure p_o, which is compressed isothermally. Its volume is then given by $V(p) = V_o\, p_o/p$ and hence,

$$\frac{dV}{dp} = -V_o \frac{p_o}{p^2} \approx -\frac{V_o}{p_o} \tag{8}$$

The presence of air bubbles with concentration C_{air} (vol/vol) will therefore increase the distensibility by the amount

$$D_{air} \approx \frac{C_{air}}{p_o} \tag{9}$$

leading to the reduced speed of sound

$$c = \frac{1}{\sqrt{\rho\,(K + D + C_{air}/p_o)}} \tag{10}$$

Test for linearity with regular waves

A series of measurements were conducted at the University of Queensland in the period August to October 1993 to establish the possible existence and importance of nonlinearity of the systems response to regular waves.

Regular but not quite simple harmonic pressure waves with "heights" in the range $0.5m < H < 4.5m$ and periods in the range $2s < T < 7s$ were generated by moving a small reservoir with mercury up and down in a quasi simple-harmonic fashion. The test tube was approximately *100m* of *10mm* OD Nylex standard tubing.

The data indicate that the gain is only weakly dependent upon the amplitude and hence, the use of a linear frequency response model (Equation (15)) developed below for the system is reasonably well justified.

Frequency response for the tube-transducer system

As a working hypothesis, it was assumed that the tube-transducer system can be modelled in analogy with a dampened "quarter length resonator". That is, it has resonnant pressure wave modes of the form indicated in Figure 4. Such a system has the approximate frequency response function

$$F(f) = \frac{1}{\cos\left(\frac{\pi}{2}\frac{f}{f_o}\right) + i\,D_E\left(\frac{f}{f_o}\right)} \tag{11}$$

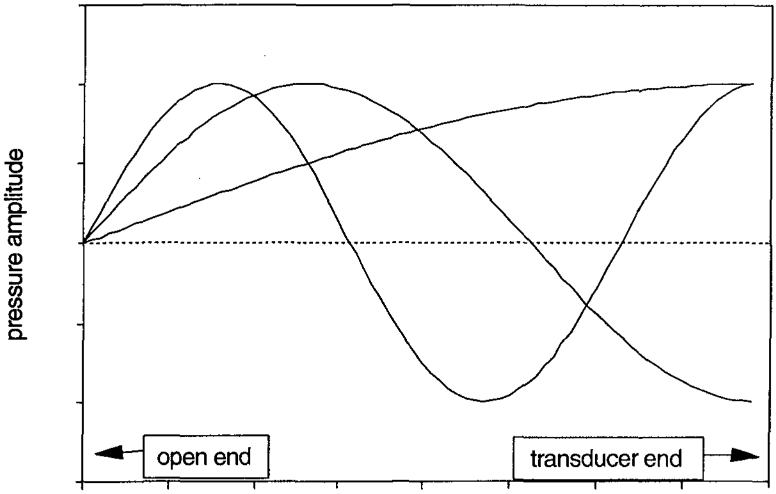

Figure 4: Assuming that the pressure transducer forms a hard, reflecting boundary, the tube/transducer system will behave as a dampened quarter length resonater and have infinitely many resonant wave modes of which the first three are shown here.

where $D_E\ (f/f_o)$ is an energy dissipation function. The corresponding gain function is

$$G(f) \;=\; |F(f)| \;=\; \frac{1}{\sqrt{\cos^2\left(\dfrac{2}{\pi}\dfrac{f}{f_o}\right) \;+\; D_E^2\left(\dfrac{f}{f_o}\right)}} \qquad (12)$$

In these expressions, f_o is the lowest resonance frequency corresponding to the resonance period T_o. It is seen from Equation (12) that the gain function has peaks for all odd multiples of the resonance frequency f_o. This model is in reasonable agreement with experiments see Figure 5

The experiments show the first two peaks of the gain function rather clearly.

The mode of resonnance has a pressure antinode at the transducer end and a node at the open end of the tube, see Figure 4. Hence, the resonant pressure wave in the tube resembles a seiche in a bay. The length of the tube must in that case be 1/4 of the wavelength of the first mode: $L \;=\; \lambda_o/4 \;=\; c\,T_o/4$, corresponding to the resonance frequency

$$f_o \;=\; \frac{c}{4\,L} \qquad (13)$$

the frequencies of the higher resonant modes are all the odd multiples of f_o.

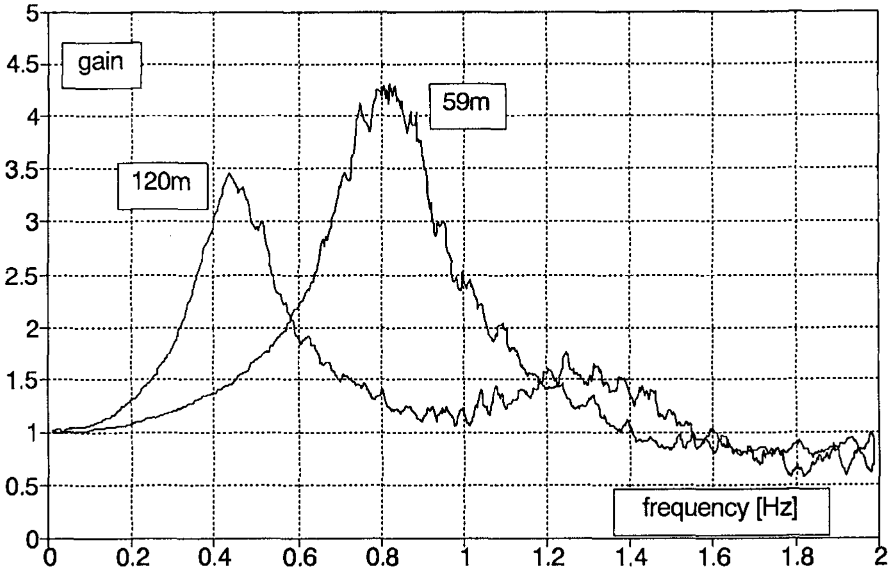

Figure 5: Gain functions measured in the laboratory for a 120m and a *59m OD10mm* tube (Nylex standard pressure tubing). The frequency response data was obtained by comparing the output of two transducers. One at the closed end of the tube and one at the "open" end where irregular pressure focing was provided by moving a water filled open ended tube up and down,

Energy dissipation and damping

The maximum gain values observed with *10mm* Nylex tubes of lengths *60m* to *120m* are of the order *4.5* and *3.5* respectively, see Figure 4.

These finite gain values indicate some damping in the system. The nature of this damping i e, the loss of energy is not completely understood. - The energy may be turned into heat in the fluid and in the tube walls. Alternatively, it may be radiated away. Radiation may occur along the full length of the tube or mainly from the open end.

Energy loss in the form of heating of the fluid is caused by the viscosity ν and may be estimated as follows. The rate of heat generation in a boundary layer is $\overline{u_\infty \tau}$ where u_∞ is the velocity at the edge of the boundary layer and τ is the wall shear stress. The wall shear stress may, under the assumption $\sqrt{\nu T} << d$, be estimated by the formula $|\tau| = \rho \sqrt{2\pi f \nu}\ |u_\infty|$ which holds for a plane, oscillatory boundary layer, see e g Nielsen 1992, p 21. The velocity amplitude $|u_\infty|$ is related to the pressure amplitude by $u = p/\rho c$, see Lighthill 1978, p 4. Hence the energy dissipation due to fluid viscosity in a tube of length L and diameter d over one period can be estimated by

$DE_{fluid} \approx \rho\, u^2\sqrt{2\pi f \nu}\; L\, d\, T.$

The loss of energy as heat in the tube walls may be estimated as follows. For a given pressure amplitude $|p|$, the deformation of the tube wall is of the magnitude $\frac{|p|\, d^2}{\delta E}$ where δ is the wall thickness and E is Young's modulus for the tube material. Hence, the work done on the tube wall per unit length through one cycle is of the magnitude $\frac{|p|^2\, d^3}{\delta E}$ and the work done on a tube of length L in one period is $\frac{|p|^2\, d^3}{\delta E} L$. Since the general magnitude of $|p|$ along the tube is $|p'|$, this may be written

$$DE_{wall} \sim \frac{|p'|^2\, d^3}{\delta E} L \approx \frac{|p'|^2 L\, d^2}{\rho\, c^2}, \text{ since } \rho\, c^2 \approx E\frac{\delta}{d} \text{ for relatively flexible tubes,}$$

cf Equations (5) and (7).

The energy flux through the tube cross section is of the order $|p|cd^2$ corresponding to an energy input of $|p^+|\; c\; d^2 T$ at the open end during one wave period.

Based on these considerations, and with $p' = G\, p^+$, we find that the relative energy loss D_E can be quantified approximately by

$$D_E = \frac{\textit{energy loss}}{\textit{energy input}} \sim \frac{\frac{|p'|^2\, d^2 L}{\rho\, c^2} + \frac{|p'|^2}{\rho\, c^2}\sqrt{2\pi f \nu}\; T\, d\, L}{|p^+|\, c\, d^2\, T}$$

$$D_E = \frac{\textit{energy loss}}{\textit{energy input}} = G\frac{|p'|}{\rho\, c^2}\frac{L}{c}\left[C_1 f + C_2\frac{\sqrt{\nu}}{d}\sqrt{f}\,\right] \qquad (14)$$

where C_1 and C_2 are dimensionless coefficients.

A semi enmpirical gain function

Based on the analysis above, which indicates the existence of some damping terms proportional to the square root of the frequency and some which are proporttional to the frequency it seems reasonable to suggest a semi empirical gain function of the form

$$G = \frac{1}{\sqrt{\cos^2\left(\frac{\pi}{2}\frac{f}{f_o}\right) + \left[B_1\frac{f L}{c}\left(1 + B_2\sqrt{\frac{\nu}{f d^2}}\right)\right]^2}} \qquad (15)$$

Based on the data shown in Figure 5 the values of the dimensionless constants were found to be $(B_1, B_2) = (0.58, 5.0)$. The fact that $B_2 >> B_1$ indicates that the loss due to fluid viscosity is dominant compared to the loss due to deformation of the tube walls for these tubes. The formula (15) with these values of B_1 and B_2 is compared with the data in Figure 6.

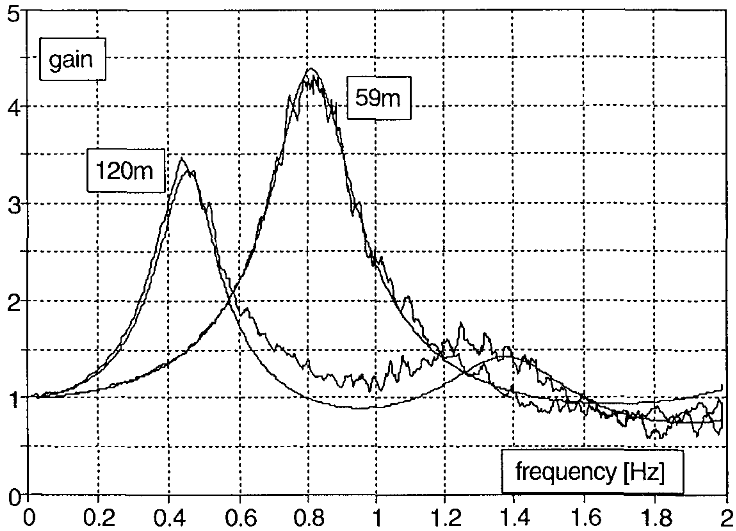

Figure 6: The semi empirical gain function (15) compared with laboratory measurements.

The matching of the shape of the gain function is not perfect, but the discrepancies seem to be due to the cosine function not having the right shape.

Calibration

The values of the constants B_1 and B_2 which were determined on the basis of the data in Figure 5 may not be universal. Hence, when working with systems with different lengths, diameters and tube materials it would be wise to calibrate the system before deployment. This calibration is best done in the way described in connection with Figure 5.

Evaluation of the system

The indication of the tests carried out so far is that the new wave gauge offers a cheap and reliable alternative to existing gauges for measuring the wave conditions in shallow (< *5-6m*) depths near (*0-500m*) beaches or existing structures. The main advantages of the new system are that it is cheap and easy to service and interrogate because all electronics are kept "high and dry".

Based on the field, it can be concluded that using the sytem in the field with tubes which are already in place is easy (installation time about *5* minutes in fair weather). Earlier tests also show that deployment of the tubes for a one or two day experiment on a beach is manageable, provided a 4 wheel drive vehicle is at hand to help pull the tubes back ashore.

The pressure signal is of adequate strength, and the noise level is very low provided the tubes are prevented from moving with the waves. This is normally achieved by tying the tubes to an 8mm steel chain.

Variable degrees of sand cover over the seaward end of the tube seem not to cause problems with the translation of dynamic bottom pressures into water surface elevations. However, thich layers of silt or mud may have a very strong dampening effect.

The frequency response function for the system is fairly complicated. It has several peaks and the dampening is non-linear. However, with the use of Nylex standard pressure tubing the second peak of the gain function will generally be well outside the frequency range of ocean waves. For example, a *600m 10mmOD* tube system will have its second peak at f = 0.45Hz (fo = 0.15Hz) and most of the energy in the pressure spectra from wind waves on a beach lies at freqencies below 0.25Hz.

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