CHAPTER 3

LABORATORY COMPARISON OF DIRECTIONAL WAVE MEASUREMENT SYSTEMS AND ANALYSIS TECHNIQUES

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Abstract

In order to define a directional wave sensor for laboratory experiments, three measuring systems as well as seven directional analysis methods are combined, applied and compared on three different tests performed in a directional wave basin. "Single-point" gauges are found to accurately analyse unimodal spectra when associated to advanced methods (Fit to bimodal model, Iterative Maximum Likelihood Method, Maximum Entropy Method, Bayesian Method). The heave-pitch-roll gauge used in this study is in particular very simple and shows promising capabilities. For bimodal spectra (two directional peaks at same frequency) however, only the wave probe array combined with the Maximum Entropy Method or the Bayesian Method appears to be able to produce reliable estimates.

1. INTRODUCTION — SCOPE OF THE STUDY

The measurement of directional wave spectrum may be performed through various systems, including co-located gauges (directional buoys, pressure sensor combined with a 2D currentmeter,...), arrays of gauges (wave probe arrays or mixed instruments arrays) or remote-sensing systems (satellite synthetic aperture radar, aerial stereo-photography techniques,...). Each of these measuring devices delivers a rather limited amount of information and the estimation of directional wave spectrum is then an awkward inverse problem, mathematically speaking. In order to get an estimate from the data anyway, various directional analysis methods have been proposed : Fourier Series Decomposition, Fit to parametric models, Maximum Likelihood Methods, Maximum Entropy Methods, Bayesian Methods,...

From practical point of view these methods exhibit different behaviours and characteristics for instance in mathematical complexity, directional accuracy, directional spectrum shape dependency, computing time, numerical convergence,... A good number of these methods have recently been implemented at Laboratoire National d'Hydraulique (LNH) and tested quite extensively on numerical simulations using heave-pitch-roll data (Benoit, 1992) as well as gauge array data (Benoit, 1993). The present study aims to proceed a step further in this comparative analysis by evaluating the capabilities of the methods on laboratory data.

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We briefly recall that the main unknown of the problem is the directional wave spectrum $S(f,\theta)$, a function of wave frequency f and direction of propagation θ. The following conventional decomposition is used : $S(f,\theta) = E(f).D(f,\theta)$

E(f) is the classical variance or 1D-spectrum that may be estimated by a single record of free-surface elevation. and $D(f,\theta)$ is the Directional Spreading Function (DSF) satisfying two important properties :

$$D(f,\theta) \geq 0 \text{ over } [\,0\,,\,2\pi\,] \quad \text{and} \quad \int_0^{2\pi} D(f,\theta)\, d\theta = 1$$

The directional analysis procedure may be roughly decomposed into three steps :

a. record simultaneously one or several wave properties (elevation, velocities, pressure, slopes,...) at one or more locations : $X_i(t)$, ... , $X_N(t)$ $(N \geq 3)$

b. compute the cross-spectra between each pair of recorded signals :

$$G_{ij}(f) = \int_{-\infty}^{+\infty} R_{ij}(\tau)\, e^{-i2\pi f\tau}\, d\tau \quad \text{with} \quad R_{ij}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_0^T Xi(t).Xj(t+\tau)\, dt$$

c. estimate the directional spectrum by inverting the following set of equations :

$$G_{ij}(f) = \int_0^{2\pi} H_i(f,\theta).\,\overline{H_j}(f,\theta).S(f,\theta\,).\exp(-\vec{k}.\vec{x_{ij}})\, d\theta$$

In this study, three measuring systems are set up in LNH directional wave basin (see section 2) and seven directional analysis methods are selected (see section 3). The experimental lay-out is described in section 4 and the three test-cases are presented in section 5. The comparative analysis of results is reported in section 6.

2. DIRECTIONAL MEASURING DEVICES

Three measuring systems are considered for laboratory measurements (figure 1) :

— a wave probe array : the array is composed of five probes (numbered from 1 to 5) laid out on the same configuration as the one used by Nwogu (1989). The wave probes are resistive-type wires mounted on a frame that allows a precise positionning. The radius R of the array is 0.40 m. As it will be presented in section 5, the wavelength corresponding to peak frequency is L_P = 2.42 m, and thus the ratio R/L_P is about 16 %.

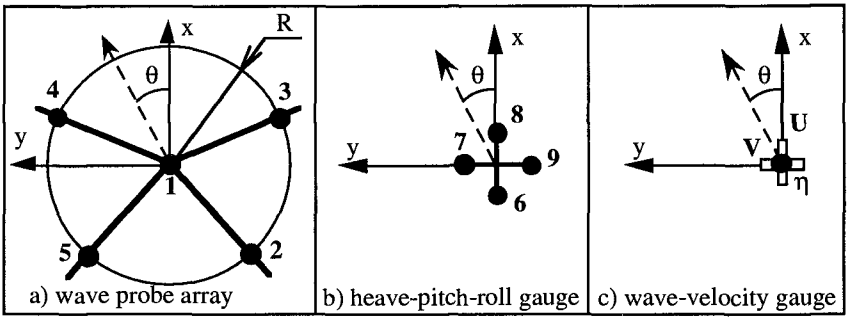

Figure 1 : the three directional measuring devices used for experiments.

— a "heave-pitch-roll" gauge : this gauge aims to deliver the same type of signals as the heave-pitch-roll buoy used in the field. To that extent, four wave probes are set up close to each other in a very simple way (see figure 1-b). From the four recorded free-surface elevation time series, the elevation and two orthogonal slopes of free-surface at the center of the gauge are computed :

$$\eta(t) = (\eta_6 + \eta_7 + \eta_8 + \eta_9)/4.$$

$$\frac{\partial \eta}{\partial x}(t) = \frac{\eta_8(t) - \eta_6(t)}{d_{6\text{-}8}}$$

$$\frac{\partial \eta}{\partial y}(t) = \frac{\eta_7(t) - \eta_9(t)}{d_{7\text{-}9}} \qquad \text{with : } d_{6\text{-}8} = 11.4 \text{ cm} \quad \text{and} \quad d_{7\text{-}9} = 13.2 \text{ cm}$$

— a wave-velocity gauge : as the previous one, this gauge is also a "single-point" gauge, recording at the same location the free-surface elevation (through a wave probe) and the two horizontal components of velocity (through a 3D acoustic velocimeter, from which only the two velocity signals U and V are kept).

3. DIRECTIONAL ANALYSIS METHODS

Among the methods available at LNH, the seven following ones are considered, because each of them may be used both for "single-point" systems and arrays :

— **Weighted Fourier Series (WFS) :** the directional spreading function is expressed as a truncated Fourier series whose first coefficients are computed from the cross-spectra (Borgman, 1969). A weighting function is used to avoid possible negative values taken by this estimate (Longuet-Higgins *et al.*, 1963).

— **Fit to bimodal Gaussian model (2MF2) :** A bimodal parametric model obtained from linear combination of two unimodal Gaussian-type models is used. Its five unknown parameters are determined from the cross-spectra. In the case of "single-point" systems, the problem becomes awkward because there are only four information avalaible and additional constraints are thus needed (Benoit, 1992).

— **Maximum Likelihood Method (MLM) :** By this method the directional spectrum is regarded as a linear combination of the cross-spectra. The weighting coefficients are calculated with the condition of unity gain of the estimator in the absence of noise (Oltman-Shay and Guza, 1984 ; Krogstad, 1988). Recently, Haug and Krogstad (1993) proposed a modified version of MLM (the constrained MLM) for gauge arrays which is not taken into account here.

— **Iterative Maximum Likelihood Method (IMLM2) :** The estimate obtained from the former method is not consistent with the measured cross-spectra. It may be iteratively modified to let its cross-spectra become closer to the ones obtained from the data (Oltman-Shay and Guza, 1984).

— **Maximum Entropy Method (MEM2) :** The method is based on the definition of Shannon for entropy : $\chi = -\int_0^{2\pi} D(f,\theta).\ln(D(f,\theta))\, d\theta$.

This entropy is maximized under the constraints given by the cross-spectra. The application to single-point systems or gauge arrays is described by Kobune and Hashimoto (1986), Nwogu *et al.* (1987) and Nwogu (1989).

This entropy definition is different from the one used by other authors (e.g. Lygre and Krogstadt, 1986) which generally appears to be less powerfull.

— **Bayesian Directional Method (BDM) :** No *a priori* assumption is made about the spreading function which is considered as a piecewise-constant function over [0 , 2π]. The unknown values of $D(f,\theta)$ on each of the K segments dividing [0 , 2π] are obtained by considering the constraints of the cross-spectra and an additional condition on the smoothness of $D(f,\theta)$ (Hashimoto *et al.*, 1987).

— **Variational Fitting Technique - Long-Hasselmann Method (LHM) :** Long and Hasselmann (1979) developed this method by which an initial simple estimate is iteratively modified to minimize a "nastiness" function that takes into account the various conditions on the spreading function. The application to buoy data is described in detail by Long (1980).

When referring to previous LNH numerical comparative surveys (Benoit, 1992), the Fit to unimodal model method (e.g. Borgman, 1969) and the Eigenvector Method (Mardsen and Juszko, 1987) have been dropped. The former is definitely unable to analyse bimodal cases and the latter has not been applied to gauge arrays.

4. EXPERIMENTAL LAY-OUT

The LNH multirectional wave facility is a rectangular wave basin of 50 m by 30 m used for coastal studies. The segmented wavemaker is composed of 56 piston-type paddles. The width of the paddles is 0.40 m. The total wavemaker length is thus 22.4 m. It is movable along the main side of the basin. The maximum water depth in the basin is 0.80 m. The facility is equipped with numerous mobile upright progressive wave absorbers. Each absorber unit measures 2.8 m by 2 m, allowing variable and adpatable absorber configurations in the basin. Tidal currents may also be simulted in addition to waves.

For the present experiments only the eastern part of the basin was used as there was a breakwater model set up in the remaining part of the basin. Only the first 30 paddles were activated, giving an effective wavemaker length of 12 m. The test area was then a rectangle of 12 m by 25 m (figure 2) limited by wave absorbers. Fully reflective sheets over a length of 6 m were set-up at each side of the wavemaker in order to increase the work area through the corner reflection method. The bottom was flat over the whole test area. The water depth was kept constant at 0.60 m.

The three measuring devices were located 6 m apart from the wavemaker. They were set up every 0.40 m on a line parallel to the wavemaker (see figure 2).

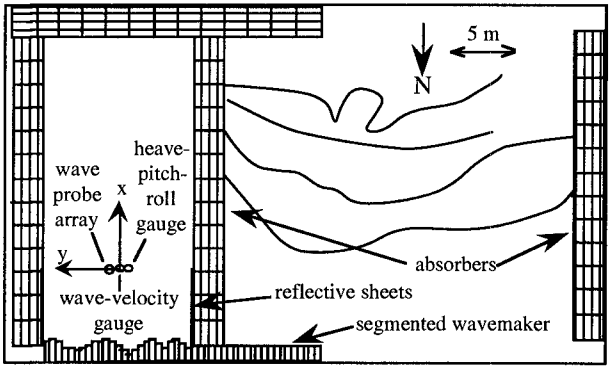

Figure 2 : definition sketch of experimental setup.

5. CHARACTERISTICS OF LABORATORY TEST-CASES

5.1 Wave simulation characteristics

The directional wave simulation is achieved by using a "single summation " method (also called "single direction per frequency method") (Miles, 1989) :

$$\eta(x,y,t) = \sum A_n \cos(2\pi.f_n.t - k_n(x.\cos\theta_n + y.\sin\theta_n) + \varphi_n)$$

The sea surface elevation is obtained through a linear superposition of numerous elementary components. The amplitude of each component is related to the target spectrum through : $A_n = \sqrt{2\, S(f_n, \theta_n)\, \Delta f_n\, \Delta\theta_n}$

The phases are uniformly and randomly distributed over [0 , 2π]. The directions θ_n are of the form $k.\Delta\theta$, but randomly distributed over [0 , 2π].

The frequency spectrum E(f) is a classical JONSWAP spectrum with a significant wave height of 0.10 m, a peak period of 1.3 s and a peak-factor $\gamma = 5$. The simulated directional spreading function (DSF) is frequency independent :

$$\Pi_{s1,\alpha1,s2,\alpha2,\lambda}(\theta) = \lambda.\Pi_{s1,\alpha1}(\theta) + (1-\lambda).\Pi_{s2,\alpha2}(\theta) \qquad \text{with } 0 \le \lambda \le 1$$

with : $\Pi_{s,\alpha}(\theta) = \Delta(s)\ \cos^{2.s}[(\theta-\alpha)/2]$ if $\theta-\alpha \in [-\theta_m ; \theta_m]$ $(\theta_m = 60°)$

Three DSF are simulated with the following characteristics (see figure 3) :

Test	Description	s1	α1	s2	α2	λ
A1	Unimodal Broad DSF	1	0.			1.
A3	Unimodal sharp DSF	15	0.			1.
A5	Bimodal DSF	25	-40.°	5	30.°	0.5

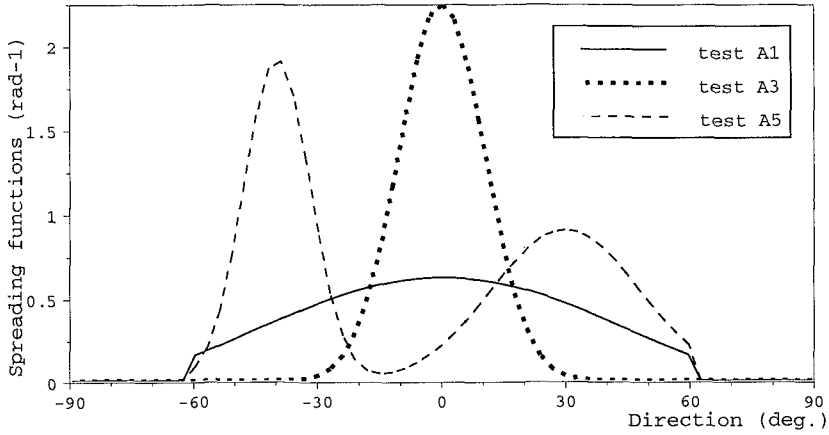

Figure 3 : The three simulated directional spreading functions.

5.2 Signal recording characteristics

The signals are recorded with a time step of 0.05 s over a duration of 819.2 s.

5.3 Cross-spectral analysis characteristics

The spectral analysis procedure is based on the technique of the averaged periodogram on the whole recorded signals partitioned in segments of 512 points. An overlapping of 25% between adjacent segments is used. The resulting frequency resolution is 0.039 Hz. Directional analysis is carried out between 0.5 and 1.25 Hz.

6. PRESENTATION AND DISCUSSION OF LABORATORY TESTS RESULTS

The directional spectra analysed on the three laboratory test-cases are presented using 2D-plots on figures 4-a and 4-b (test A1), 5-a and 5-b (test A3), 6-a and 6-b (test A5). Figure 7 gives a 3D-view of directional spectra analysed by MEM2 and BDM for the three measuring devices on test A5.

6.1 Analysis of test A1 — Unimodal broad spectrum — Figures 4-a and 4-b :

— Wave probe array : Reliable estimates are obtained by the BDM and MEM2 methods only. The WFS, MLM and LHM estimates are broader than the target spectrum. The 2MF2 and IMLM2 estimates are either too sharp or quite bimodal and reveal some instable behaviour of the methods.

— Heave-pitch-roll gauge : Very accurate and similar estimates are obtained from the 2MF2, IMLM2, MEM2 and BDM methods. This similarity allows a certain confidence in the results of this gauge. The spectra analysed by the WFS, MLM and LHM methods are too broad.

— Wave-velocity gauge : the behaviour of analysis methods is very similar to the heave-pitch-roll gauge, but the estimated spectra are a little bit sharper than the former ones. Correct estimates are again obtained from the 2MF2, IMLM2, MEM2 and BDM methods.

6.2 Analysis of test A3 — Unimodal sharp spectrum — Figures 5-a and 5-b :

— Wave probe array : Best estimates are obtained from MEM2 method. The BDM and IMLM2 methods produce acceptable results, but the latter one shows some numerical instabilities out of peak region. The 2MF2 method also exhibits some numerical instabilties, resulting in spurious peaks of the spectrum. The WFS, MLM and LHM methods are unable to model the sharpness of the spectrum and appear to be unefficient for sea-states with narrow angular spreading of energy.

— Heave-pitch-roll gauge : the best estimates are given by IMLM2, BDM and MEM2 methods. The spectrum analysed by the 2MF2 method is clearly too sharp, while the spectra analysed by the WFS method especially, but also by the MLM and LHM methods, are far too broad.

— Wave-velocity gauge : As for test A1, the observations for this measuring device are very close to the ones of heave-pitch-roll gauge. Again the most accurate estimates are achieved by the IMLM2, BDM and MEM2 methods. On this second case however the directional widths of the estimates are very close to those obtained from heave-pitch-roll gauge.

6.3 Analysis of test A5 — Bimodal spectrum — Figures 6-a, 6-b and 7 :

— Wave probe array : Reliable estimates are obtained from the 2MF2, MEM2 and BDM methods. For these three methods the bimodal nature of the sea-state (with a difference in the shapes of the two peaks) is clearly reproduced. On figure 7 it may be seen that the spectra given by MEM2 and BDM agree quite well with the theoretical spectrum. One must emphasis that this bimodal ease with two peaks at the same frequency only separeted by 70 degrees is very severe. Bimodality of spectrum is harly detected by the LHM and the IMLM2. The WFS and MLM methods only produce an unimodal and very broad spectrum.

— Heave-pitch-roll gauge : The results given by the various methods are definitely worse than for the wave probe array. The quite low number of information recorded by the single point-system undoubtedly limits here the resolution capabality of the analysis methods. The bimodal nature of the spectrum is

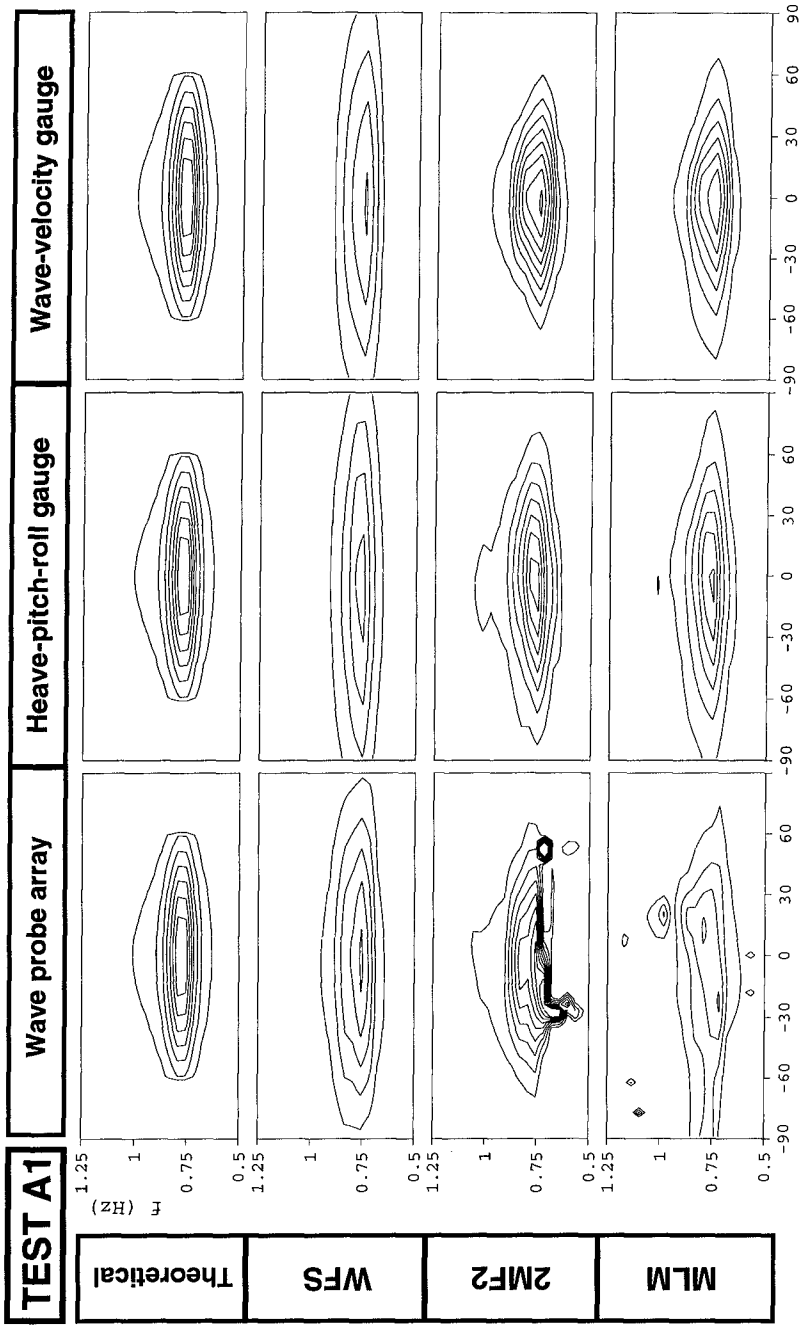

Figure 4-a : Directional wave spectra analysed on laboratory test A1 (broad unimodal case) Comparison of measuring systems and analysis methods (WFS, 2MF2, MLM)

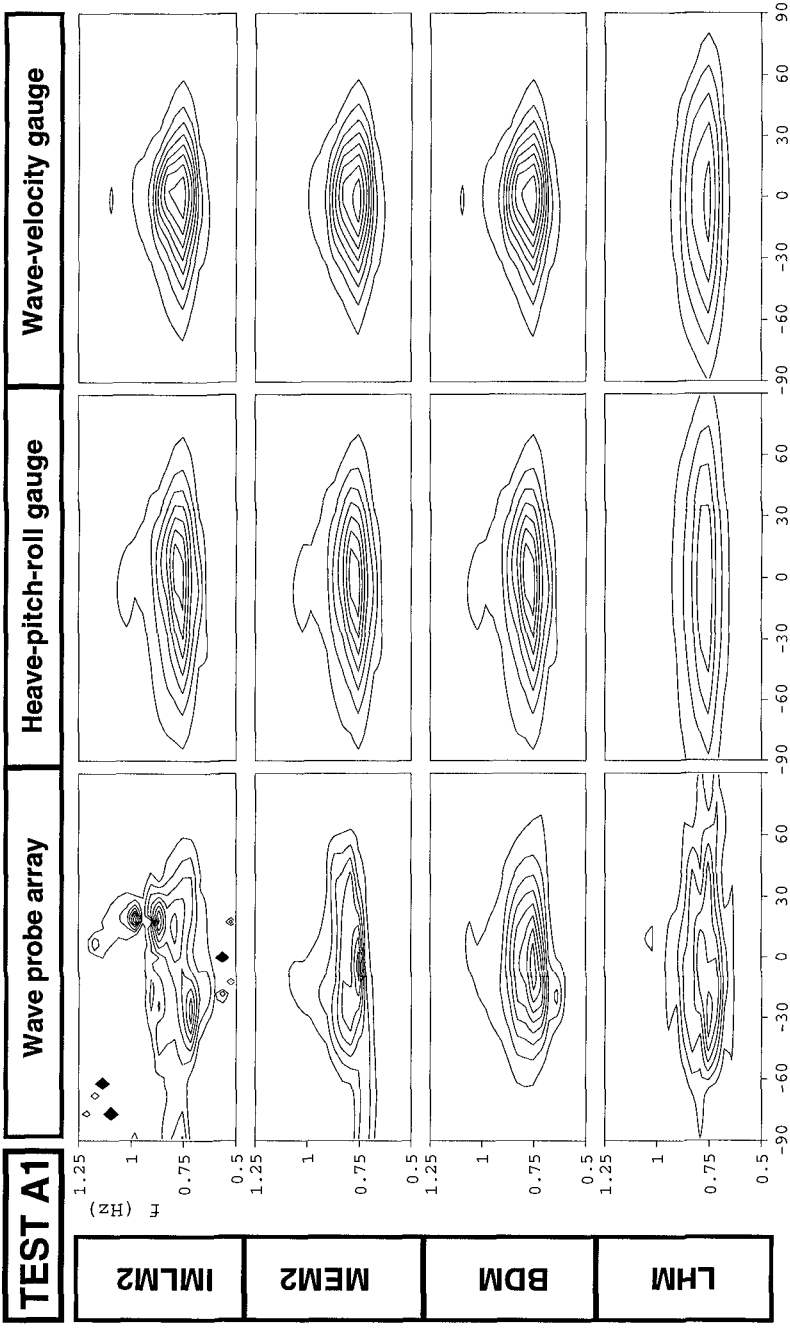

Figure 4-b : **Directional wave spectra analysed on laboratory test A1 (broad unimodal case) Comparison of measuring systems and analysis methods (IMLM2, MEM2, BDM, LHM)**

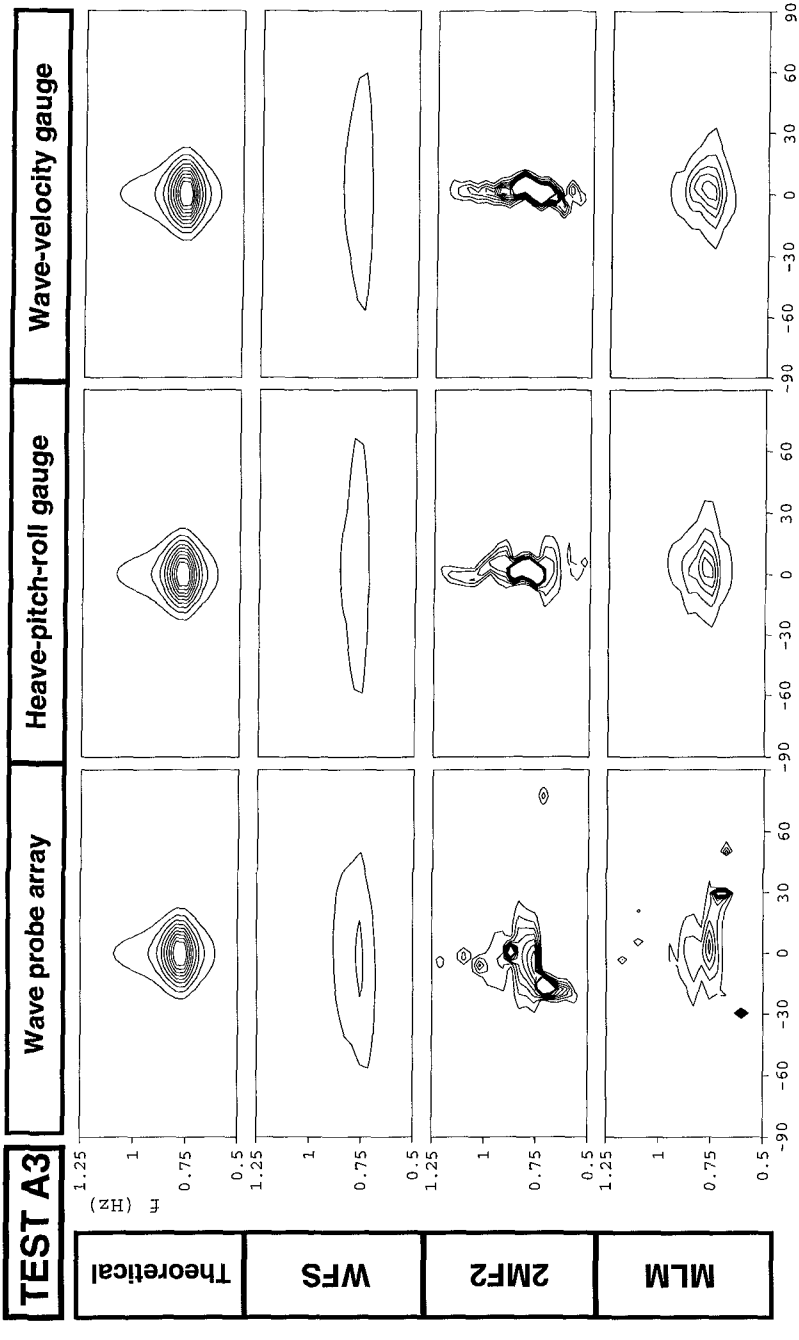

Figure 5-a : Directional wave spectra analysed on laboratory test A3 (Sharp unimodal case) Comparison of measuring systems and analysis methods (WFS, 2MF2, MLM)

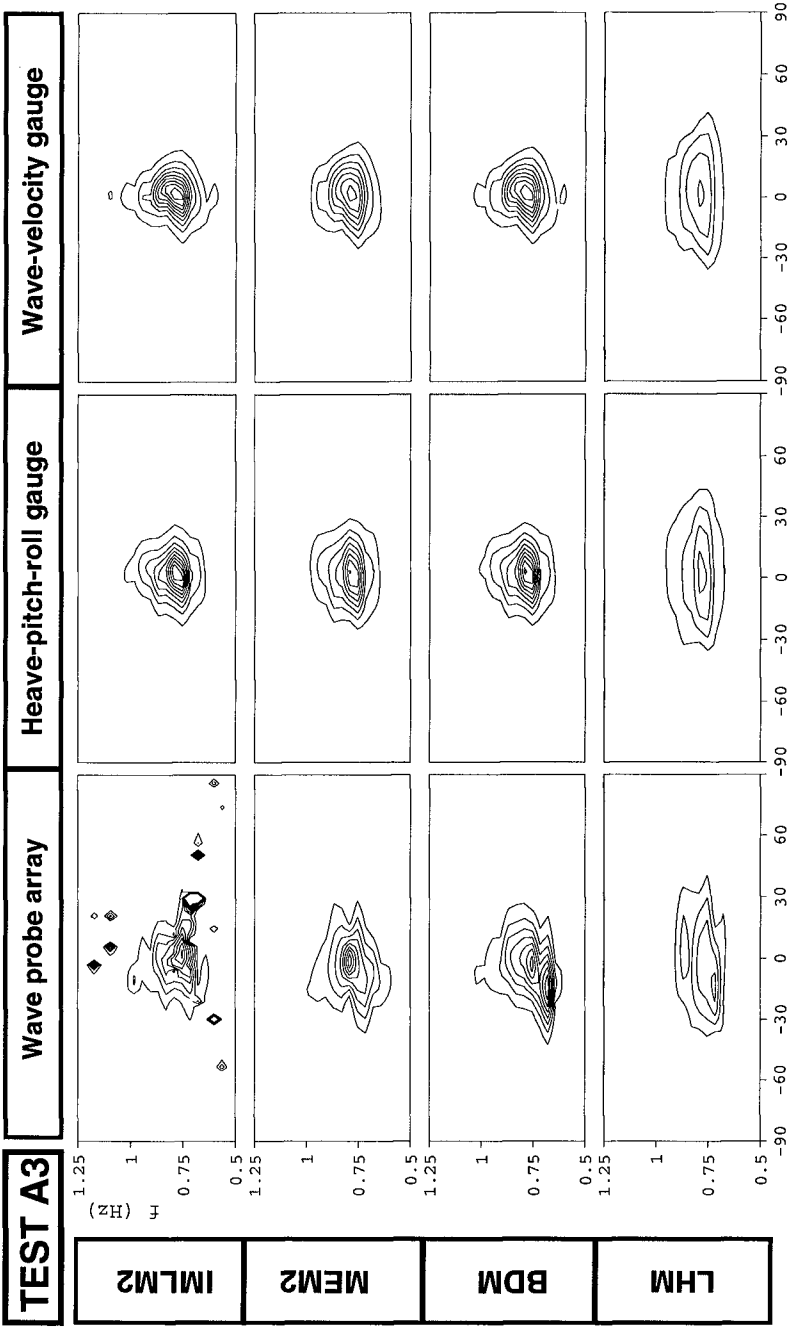

Figure 5-b : Directional wave spectra analysed on laboratory test A3 (Sharp unimodal case) Comparison of measuring systems and analysis methods (IMLM2, MEM2, BDM, LHM)

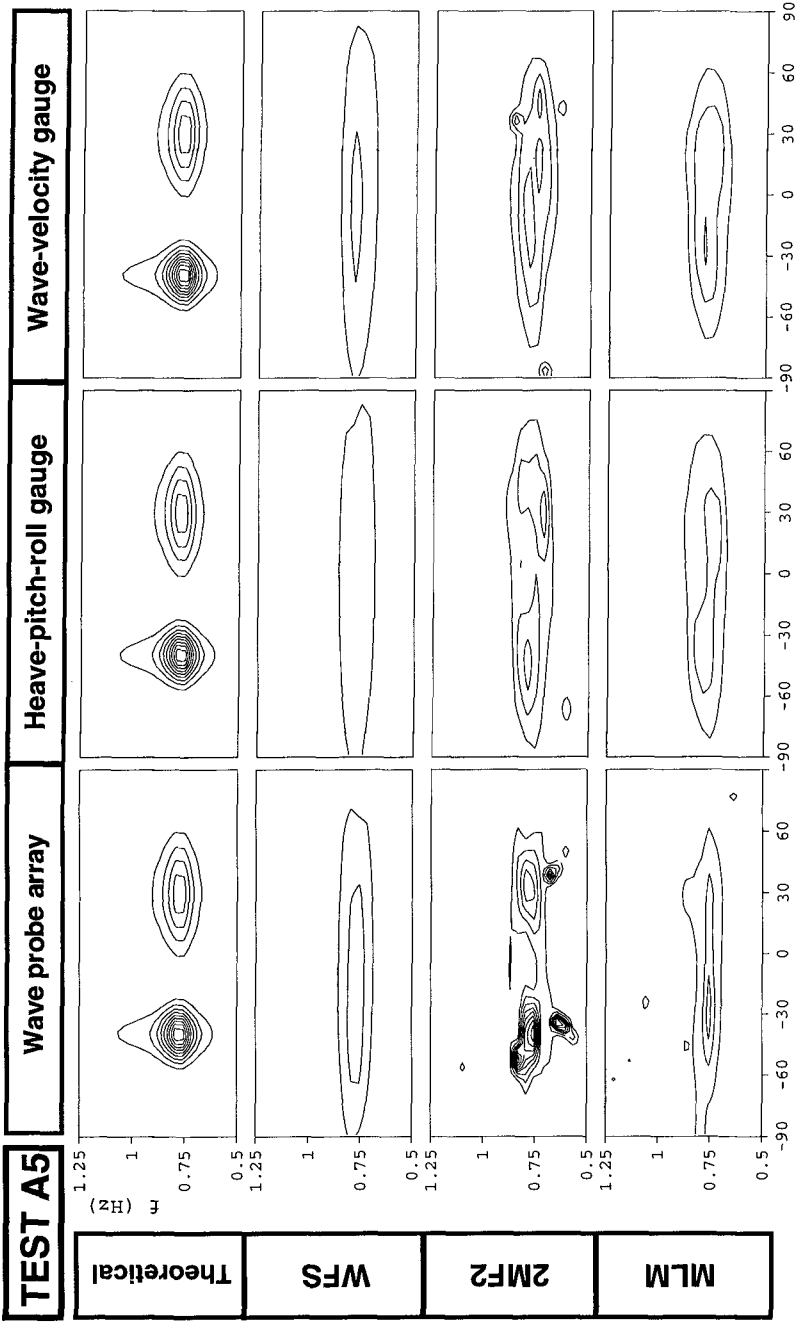

Figure 6-a : Directional wave spectra analysed on laboratory test A5 (Bimodal case) Comparison of measuring systems and analysis methods (WFS, 2MF2, MLM)

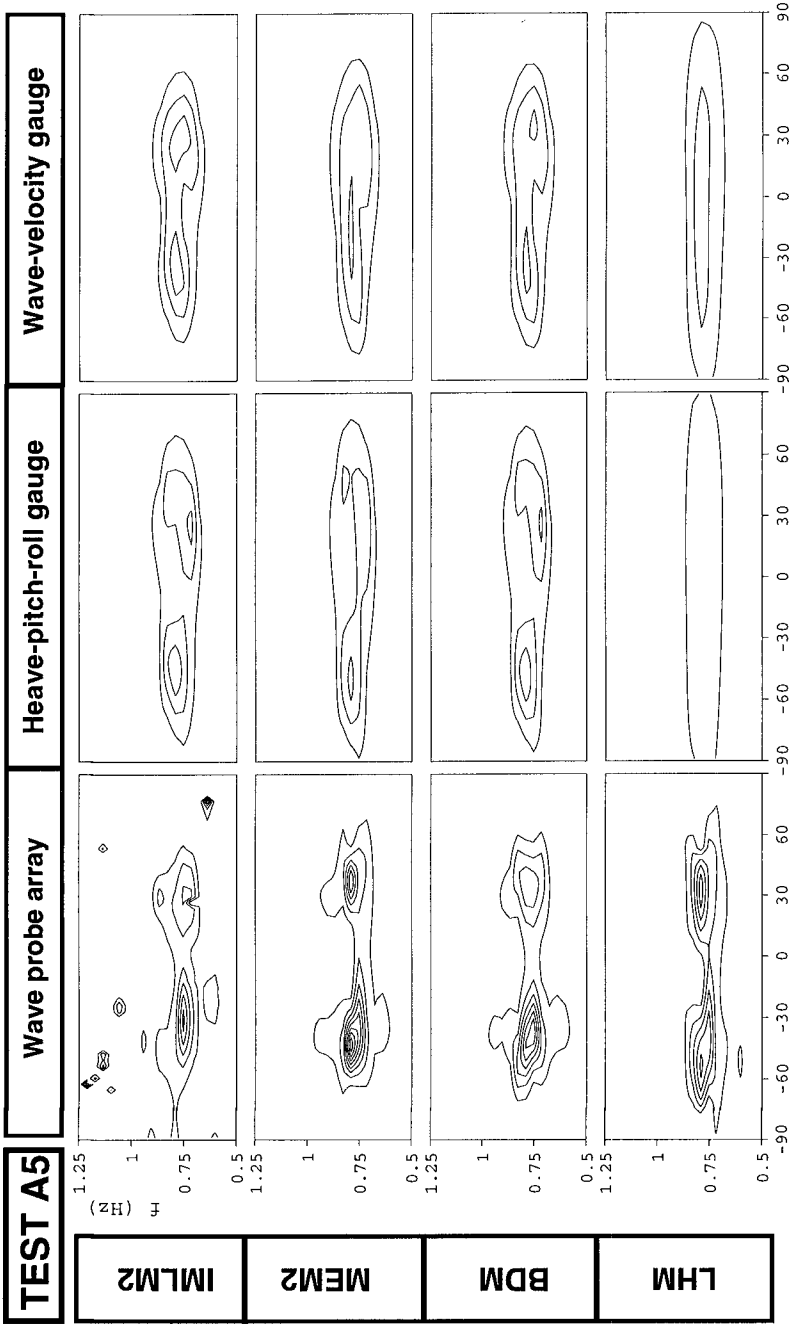

Figure 6-b : **Directional wave spectra analysed on laboratory test A5 (Bimodal case)**
Comparison of measuring systems and analysis methods (IMLM2, MEM2, BDM, LHM)

slightly noticeable for the 2MF2, IMLM2, MEM2 and BDM methods, but it is almost impossible to get detailed information about the relative shapes of the two peaks. Again these four methods give very similar and concordant results.

— Wave-velocity gauge : Most of the estimates are of same quality —and generally even a little bit worse— than the ones obtained from the heave-pitch-roll gauge. An exception is maybe the IMLM2 method which shows some better results than for the heave-pitch-roll gauge. Figure 7 however indicates that on this test-case the heave-pitch-roll gauge is superior to the the wave-velocity gauge when associated with sophisticated methods (MEM2 and BDM).

7. CONCLUSIONS — FUTURE WORK

Based on the comparative analysis of the various laboratory experiments carried out during this study, the following conclusions may be expressed :

- Great care should be given to the preliminary steps of signal recording and spectral analysis. There is a strong need to record rather long time series in order to get minimum variance spectral estimates. It seems worthwhile to increase the number of degrees of freedom of the cross-spectra as much as possible.
- The analysis of unimodal directional sea-states may be quite efficiently achieved by "single-point" (or "co-located") measuring systems, recording only three wave signals : heave-pitch-roll gauge or wave-velocity gauge.
- Each of these gauges usually produces concordant estimates from the 2MF2, IMLM2, MEM2 and BDM methods. When associated to one of these analysis methods, the "single-point" gauges exhibit good resolution capabilities for unimodal spectra, whatever the directional spreading of energy is.
- In addition, the comparison of tests results seems to indicate that the heave-pitch-roll gauge is somewhat superior to the wave-velocity gauge. This point needs however to be confirmed on more extensive experiments with velocity measurement at various depths. Furthermore, the heave-pitch-roll gauge used for these laboratory experiments is very simple and may be set up at moderate cost. Although these performances have to be confirmed on additional test-cases, the present experimental results indicate promising capabilities for operational laboratory measurements.
- For complex bimodal sea-states, the "single-point" systems usually fail to reproduce the shape of the target spectra. The bimodal nature of sea-state is hardly detected by these systems, even by using sophisticated analysis methods (MEM2, IMLM2, BDM). The output resolution seems too low to allow physical interpretation of results. For such bimodal cases, the amount of input information must be extended (up to five or more). The gauge array composed of five wave probes used in these experiments has proven to be able to produce reliable estimates on the three tests.
- The gauge array however usually requires more refined and complex numerical treatment because of the additional assumption it needs about spatial homogeneity of sea-state. For this type of measuring system, one must emphasis the need of advanced methods (MEM2 or BDM) in order to get reliable results .

Additional laboratory experiments with differents conditions will be performed in order to confirm or modify the present conclusions. The extension of these methods for the measurement of directional wave spectra close to a reflective structure is also a major research field for the next future.

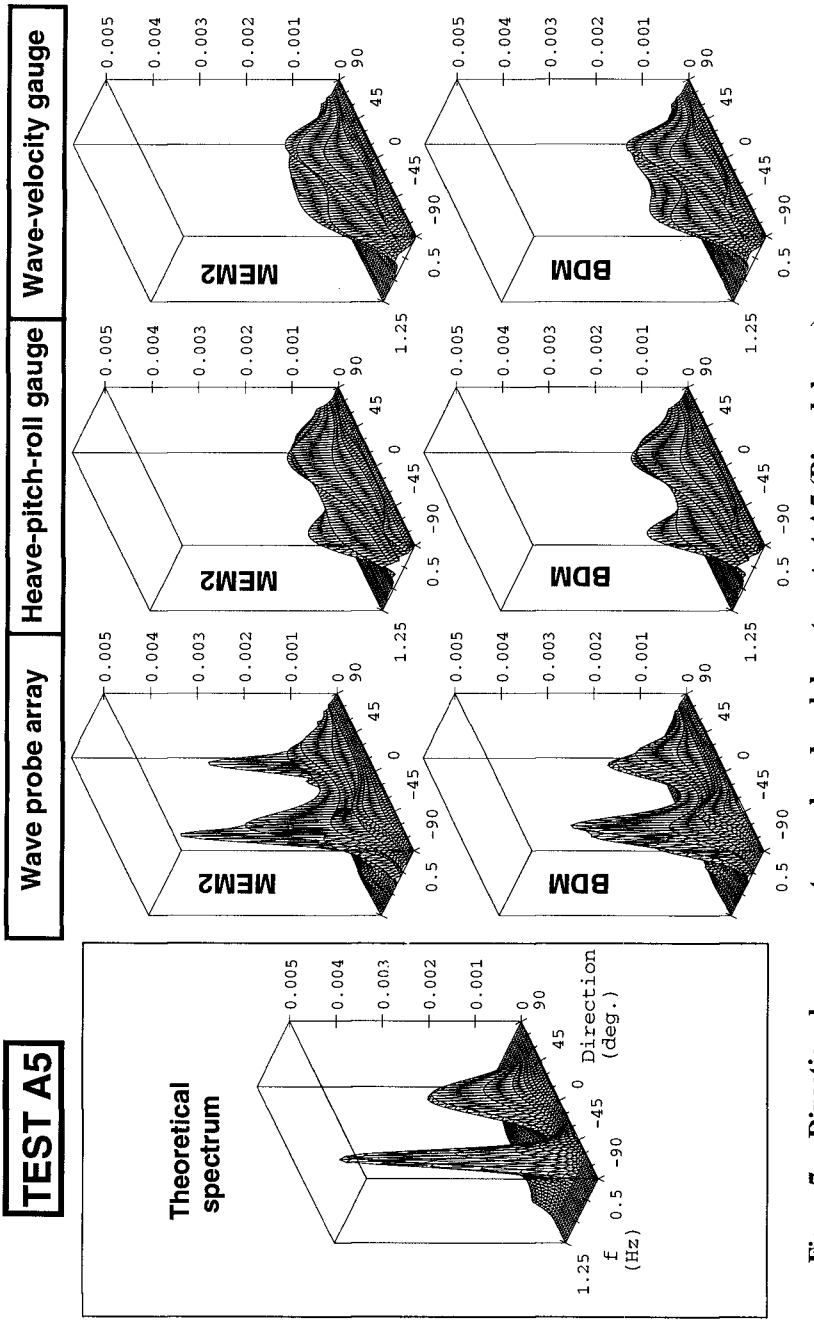

Figure 7 : Directional wave spectra analysed on laboratory test A5 (Bimodal case) Comparison of measuring systems and analysis methods (MEM2, BDM)

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