CHAPTER 231

BOTTOM STRESS MODIFICATION BY BREAKING WAVES WITHIN A LONGSHORE CURRENT MODEL

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Introduction

Early radiation stress models of longshore current generation (Bowen 1969, Longuet-Higgins 1970a, 1970b, Thornton 1970) employing monochromatic wave models produced reasonable cross-shore current distributions over planar beaches, but relied heavily on horizontal mixing for smoothing of the velocity profile. Such mixing is required because the radiation stress associated with the alongshore component of the wave-induced momentum flux, S_{yx}, is, in theory, conserved outside the surf zone, but at the singular location of breaking predicted for monochromatic waves experiences instantaneous decay, and so an infinite gradient in radiation stress.

Waves observed in nature are seldom monochromatic and so more recent models of wave height transformation employ random wave height descriptions. This randomness is normally invoked through use of a representative statistic, such as H_{rms}, via either a probabilistic (eg. Thornton and Guza (1983)) or a deterministic/Monte Carlo (eg. Dally *et al.* (1985)) approach. Thornton and Guza (1986) found that for the near-planar beach at Santa Barbara, the distribution of breaker locations produced through such randomness, and the resulting smoothing of the rms-wave height decay, yielded a satisfactory velocity profile without the inclusion of a horizontal mixing term.

The random wave height model is not, however, able to explain longshore currents on barred beaches. The same radiation stress approach which performs well on a planar beach now predicts two maxima in forcing (over the bar and at the shore) and, if mixing is omitted, two maxima in longshore current velocity. This is in direct conflict with observations from the DELILAH experiment, (an acronym for *Duck Experiment on Low-frequency and Incident-band Longshore and Across-shore Hydrodynamics*), which generally show a single maximum in longshore current over the trough, where the radiation stress gradient is near zero.

Most longshore current models assume a spatially constant bottom friction coefficient (c_f), which is solved for empirically. Considerable range in the values is found in the literature. Given the extreme variation of fluid flow characteristics across the surf-zone, this assumption of constancy seems perhaps unrealistic. In general, while theories exist relating c_f to physical parameters such as bottom

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roughness and wave steepness (e.g. Jonnson 1967), data suitable to test these theories are extremely sparse.

The present study utilizes non-linear bottom stress in which c_f is likewise used to relate the free stream velocity to the bottom stress (i.e. the law of the wall), but incorporates the effects of breaking-wave produced turbulence, in that $c_f = c_{fl}+c_{fr}$, where l and r denote local (bottom boundary layer) and remote (breaking-wave) turbulence effects. In regions of breaking waves, the remotely generated turbulence is hypothesized to be a significant source of vertical mixing of the mean longshore flow and is thus essential to relating the free stream velocity to the bottom stress. Whereas the locally generated (boundary layer) turbulence is limited in magnitude by the restriction of equilibrium between the mixing it induces and the shear which produces it, the mixing potential of the remotely generated turbulence is essentially limitless. The result of this breaking-wave induced mixing is that for a given bottom stress, free stream velocity is decreased (Fig. 1). This modification of the longshore current's vertical profile by the breaking-wave induced turbulence is the essence of the present study. The cross-shore distribution of turbulent kinetic energy (modeled by a simple vertically integrated balance between breaking-wave production and dissipation), combined with a penetration parameter, is proposed to describe the intensity of near bottom breaking-wave induced turbulence, and thereby the modification of the relationship between bottom stress and free stream velocity. Model comparison with data from a barred beach (the DELILAH experiment at Duck, North Carolina) and data from a planar beach (NSTS data from Leadbetter Beach, Santa Barbara, California) yields improved agreement.

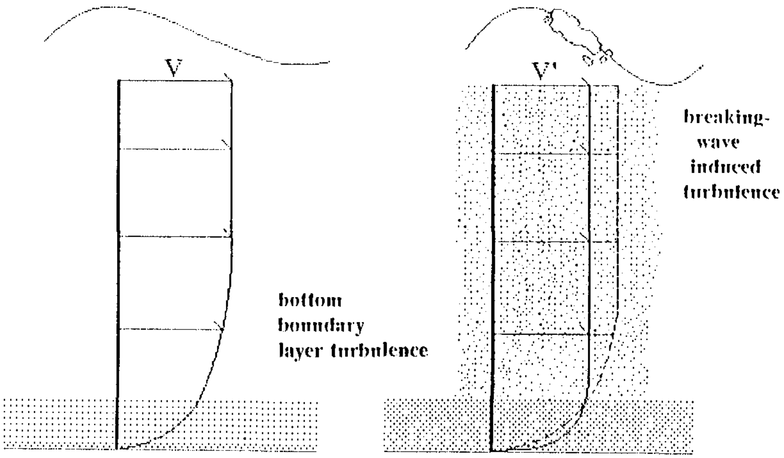


Fig. 1 Schematic of breaking-wave induced turbulence effects on vertical profile of longshore current.

Longshore Current Formulation

Assumptions

Linear wave theory is utilized, with x-axis perpendicular to the assumed straight and parallel, but arbitrary, bathymetry. Mean currents are assumed steady state, and are vertically integrated. All quantities are assumed uniform in the alongshore direction. Current shear is assumed sufficiently small that refractive interaction may be neglected. Narrowbandedness is assumed for both direction and frequency of the incident wave field.

Equations

The time averaged, depth integrated momentum equation in the alongshore direction produces a simple balance between the gradient of the radiation stress and the bottom stress, e.g. Phillips (1966):

1) $$\frac{\partial S_{yx}}{\partial x} = \frac{\partial \tilde{S}_{yx}}{\partial x} + \frac{\partial S'_{yx}}{\partial x} = \overline{R_y}$$

The radiation stress has been separated into two terms, one associated with the wave motion ($\tilde{}$) and the other due to turbulence ($'$). It is assumed the wave motion and turbulence are statistically independent of each other. The turbulent radiation stress, typically parameterized as a horizontal mixing term, is neglected. Although mixing is likely to be occurring on numerous scales, driven by numerous mechanisms, the purpose of this study is simply to examine the proposed modification of the bottom stress term. All subsequent references to "radiation stress" will pertain to wave associated radiation stress and the tilde will be omitted.

Radiation Stress Forcing

After applying Snells law for wave refraction based on the assumption of straight and parallel contours, the gradient of the alongshore momentum flux given by linear wave theory may be written as:

2) $$\frac{\partial S_{yx}}{\partial x} = \frac{\sin\alpha_0}{c_0}\frac{\partial}{\partial x}(EC_g\cos\alpha)$$

in which energy is given by $E=\frac{1}{8}\rho g H_{rms}^2$, with H_{rms} denoting rms-wave height. C_g is group velocity and α is incident wave angle. The subscript o indicates values at some initial point well seaward of breaking. The wave height transformation model of Thornton and Guza (1983) is applied. Two parameters are included in the model; γ, which describes the saturation conditions given by $\gamma=H_{rms}/h$, at which all waves are consider to be breaking, and B, a measure of the intensity of breaking as indicated by the portion of the foam region on the breaker face.

Bottom Stress

Neglecting molecular viscosity and surface wind stress, equation (1) becomes a simple balance between the radiation stress gradient and the bottom friction stress.

3) $$\frac{\partial S_{yx}}{\partial x} = \overline{\tau^b_y}$$

The general form of the component of the bottom stress in the alongshore direction is given by:

4) $$\overline{\tau^b_y} = c_f \rho \overline{|\vec{U}|(V+\tilde{v})}$$

where the overbar denotes time averaging and V is the mean longshore current velocity. The magnitude of $\bar{U}$, which represents the vector sum of the steady and wave-induced flow is obtained from

5) $$|\vec{U}| = (\tilde{u}^2 + 2\tilde{u}V\sin\alpha + V^2)^{\frac{1}{2}}$$

where the depth integrated cross-shore mean velocity, $\bar{U}$, is assumed equal to zero through conservation of mass. The result is then:

6) $$\overline{\tau^b_y} = \frac{1}{T}\int_T \rho c_f (\tilde{u}^2 + 2\tilde{u}V\sin\alpha + V^2)^{\frac{1}{2}} (V + \tilde{u}\sin\alpha)\,dt$$

In considering the non-linear form of the bottom stress for the case of random wave heights, specific treatment is required to maintain the ensemble-averaged nature of the radiation stress approach. Specifically, $\tilde{u}$ cannot be solved for based directly on H_{rms}, but instead the bottom stress for each wave height will be calculated and then ensemble averaged as given by:

7) $$\langle\overline{\tau^b_y}\rangle = \int_0^\infty \overline{\tau^b_y}(H)p(H)\,dH$$

An iterative method described in Thornton and Guza (1986) is used to calculate the longshore current velocity.

Bottom Stress and Free Stream Velocity:

The exact vertical profile of the longshore current within the boundary layer is not required, but it is inherently assumed, in accordance with Prandtl's mixing length hypothesis, that a state of equilibrium exists between the vertical mixing effect of the mechanically generated turbulence and the shear generated through the no-slip condition. In this manner, the friction coefficient, c_f, not only relates the free stream velocity, V, to the bottom stress, τ^b_y, but also to the characteristic turbulence/friction velocity, u_*, through:

8) $$\overline{\tau^b_y} = \rho u_*^2 = c_f \rho \overline{|\vec{U}|(V+\tilde{v})}$$

In most instances, for homogeneous fluids, the only source of turbulence (as represented by u_*) is local mechanical generation linked to the near-bottom current shear. Within the surf-zone, where breaking-wave generated turbulence is present, there are clearly two distinct sources. The intensity of the remotely generated turbulence is not limited by the equilibrium condition and so must be solved for separately. In regions where sufficient remotely produced turbulence is present, its vertical mixing effect may significantly alter the vertical profile of the longshore current. This modification to the relationship between bottom stress and free stream velocity is included in the proposed model through recognition of distinct components, c_{fl} and c_{fr}, producing:

$$\overline{\tau^b_y} = (c_{fl}+c_{fr})\rho\,\overline{|\vec{U}|(V+\tilde{v})} \tag{9}$$

Conceptually, for the same free stream longshore velocity, enhanced vertical mixing would increase the velocity near the bed and increase the bottom stress, or conversely, the same bottom stress would be associated with a reduced free stream velocity (see again Fig. 1).

In regions of high breaking-wave induced turbulence penetration, c_{fr} is expected to dominate, while away from breaking-wave induced turbulence, bottom stress is again governed by the local generation through c_{fl}. In the present work c_{fl} will be arbitrarily set to 0.0005. c_{fr} will be formulated based upon a suggested model of horizontal and vertical distributions of turbulent kinetic energy (tke), a turbulence penetration parameter, χ, and a fitting coefficient, Λ.

Horizontal Distribution of Turbulent Kinetic Energy:

A one-dimensional turbulent kinetic energy equation (tke-ε) (see for example Launder and Spalding (1972)) is used to solve for the temporally and vertically averaged breaking-wave induced tke, based on local balance of dissipation and production. Horizontal (cross-shore) advection of the turbulence has been neglected, as in Deigaard *et al.* (1986), based on the small cross-shore net particle velocities considered over the column. Vertical distribution is assumed to be through turbulent vortices injected from the surface, as concluded by Svendsen (1987) in his analysis of experimental data.

The resulting equation is then:

$$\frac{\partial(Ec_{gx})}{\partial x} = \rho \int_{-h}^{0} c_d\left(\frac{tke^{\frac{3}{2}}}{l_v}\right)dz \tag{10}$$

where the left hand side represents production of tke, and the right hand side, dissipation. Here c_d is a coefficient taken as 0.08 following Launder and Spalding (1972), and l_v is the length scale of the vortices estimated as 0.07h, with h representing depth, following Deigaard *et al.* (1986). Assuming vertically uniform tke and the combining of $c_d/l_v \approx 1.0/h$, as done by Roelvink and Stive (1989), integration of (13) yields:

$$tke = \left[\frac{1}{\rho}\frac{\partial Ec_{gx}}{\partial x}\right]^{2/3} \tag{11}$$

Vertical Distribution of Turbulent Kinetic Energy:

The vertical distribution of breaking-wave induced turbulence is quite likely non-uniform, but at present is unresolved. Deigaard *et al.* (1991) presents a theoretical model with significant vertical variation, while Svendsen (1987), summarizing a number of field and laboratory studies, found the turbulence to be surprisingly uniform. Different assumptions regarding advection and diffusion are made and the subject appears far from resolved.

The present work reflects this uncertainty in the magnitude of near-bottom breaking-wave induced turbulence through the use of a fitting coefficient, Λ, and a penetration parameter, χ, which scales the vertically averaged, wave induced turbulent kinetic energy contribution. This rather crude approach seeks the computational advantage of the assumption of vertically uniform tke, while recognizing that physically there must be some decay in tke close to the bed, ultimately going to zero. Intuitively, one might expect the turbulent penetration to be related to breaker type, with increasing penetration going from spilling, to plunging, to collapsing breakers. A new parameter,

12) $$\chi = \frac{\tan\beta(x)}{gH_{rmso}}$$

is employed. When the product is taken of χ and the vertically averaged tke the result is non-dimensional and is proposed to be parameterization of the near bottom mixing effect of the breaking-wave induced turbulence.

The wave height transformation model (Thornton and Guza 1983), which stresses an ensemble view of wave breaking, assumes the wave heights are described by the Rayleigh distribution. Waves may break at any location throughout the surf zone with the likelihood at any given point being some weighted portion of the Rayleigh distribution. Thus, some small portion of the waves might break on the shore side of the bar, a region in which the bottom slope, $\tan\beta$, is negative, producing a negative value of χ; use of this parameter in predicting the turbulence penetration would yield non-physical negative values over the shoreward side of the bar. To eliminate this problem with the least disturbance to the relative nature of the parameter χ, ($\tan\beta$+.03) was universally inserted in place of $\tan\beta$. In all of the cases studied, this was sufficient to ensure positive values throughout the surf zone, (i.e. the shoreward sloping faces do not exceed -0.03). In the planar beach case presented, this is of course unnecessary, but for comparison of the fitting parameter, results with and without this adjustment have been included.

An important point regarding the calculation of either χ over barred topography is that near the beach face, where the newly reformed wave rises up to break again, no history of the original wave height is maintained and so the utility of the original H_{rmso} in such a parameter as χ is limited. Instead, it is suggested that a new value of H'_{rmso}, i.e. that found over the trough, is used when considering breaking at the beach face. This implies (assumes) that surf characteristics are locally determined and cannot be represented by characteristics measured seaward of some previous breaking region. Values of $\chi(x)$ at the beach face are significantly increased through inclusion of the trough defined H'_{rmso} (175%). Recognition of the relevance of the trough region's H'_{rmso} to beach-face breaking is not only significant in the present penetration parameterization, but likewise in any application of a surf parameter on a barred beach. The proposed modification of bottom stress due to the near bottom mixing effect of the breaking-wave produced turbulence is completed by defining:

13) $$c_{fr} = \Lambda\ \chi(x)\ tke(x)$$

Solution Method

A Thornton and Guza (1983) wave height transformation model using bore dissipation theory is used to predict the gradient of cross-shore wave energy flux, used in eq(2) to calculate the radiation stress gradient (which serves as the forcing term in equation eq(3)). Additionally, the penetration parameter (used in eq(13)) is similarly given by the model. The cross-shore distribution of tke is solved for through eq(11) and is then used in eq(13) to estimate the vertical mixing effect of the breaking-wave induced turbulence. The modification of the bottom stress due to the breaking-wave induced turbulence is then modeled through eq(9). Balancing the radiation stress gradient with the bottom stress in eq(3) allows solution for the longshore current velocity.

DELILAH Experiment Description

The DELILAH experiment was conducted between October 1 and 21, 1990 at the US Army Corps of Engineers' Field Research Facility (FRF) at Duck, North Carolina, a barred beach which was the site of the previous experiments DUCK 85 and SUPERDUCK. Site selection was based upon the presence of the FRF and its infrastructure, including the permanent directional wave array which the FRF maintains in 8 meters of water, and the relative isolation of the beach. The cross-shore array consisted of 9 current meter/pressure gage stations deployed from the beach face to just beyond the 4 meter contour (Fig. 2). An autonomous Coastal Research Amphibious Buggy (CRAB), was used for daily bathymetric measurements.

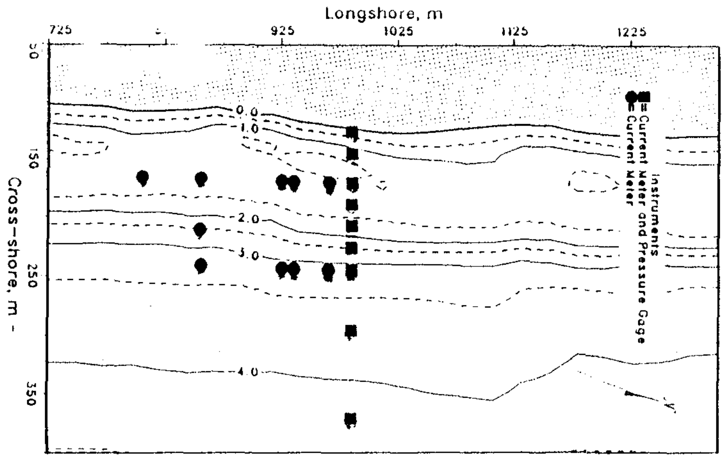
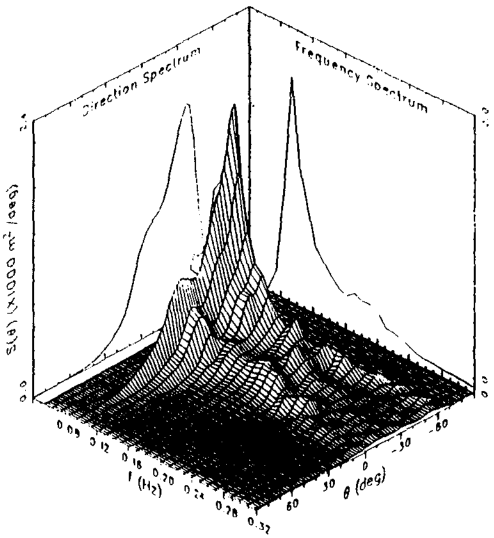


Fig. 2 Meter/gage locations.

Wave conditions on Oct. 10, chosen for model comparisons in the present consisted of an rms wave height of .77m arriving at 16.7 deg. and peak wave frequency of .094. Reasonable narrowbandedness in both frequency and direction can be seen in the two-dimensional energy density spectra (Fig. 3). Measured bathymetry is shown in Fig. 4. Bathymetry for Santa Barbara on 4 Feb is described in Thornton and Guza (1983) and is near planar with a slope of .038.



DELILAH 10 October, 1990

Fig. 3 Frequency/directional energy density spectrum.

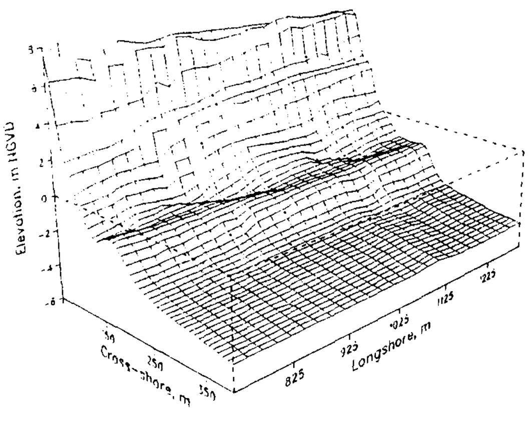


Fig. 4 Measured bathymetry for 10 Oct DELILAH.

Comparison with Data

Model results are presented in Fig. 5 for Feb. 4 Santa Barbara NSTS data and Fig. 6 for Oct. 10 DELILAH. Each wave height plot contains measured rms-wave height and bathymetry together with predicted rms-wave height and tke distributions. Agreement between observed and predicted H_{rms} is for both days generally good. Values for the two coefficients contained in the wave height transformation model, obtained by fitting the model to the data in a least square sense, are γ=.41 for both days, and B=1.28 for Santa Barbara and 1.30 for DELILAH. Each longshore current plot contains bathymetry and 3 longshore current profiles (one for a linear bottom stress term, one for a non-linear term, and one for the proposed term which is also non-linear, but includes the effects of breaking-wave induced turbulence).

Longshore Current Modeling

Fitting of the Santa Barbara velocity profiles (Fig. 5), for the two spatially constant c_f cases, linear and non-linear (without breaking-wave induced turbulence), produces c_f values of 0.008 and 0.006. The proposed model with Λ=4.0 and c_{fl}=.0005 produces a broader profile with increased velocity on the seaward extreme (where c_{fl} dominates). The overall result is slightly better agreement with observations. It should be remembered that the adjustment of tanβ necessitated by the bar has been included strictly for comparison and is not physically necessary. Without this adjustment a value of Λ=7.5 is found (the resulting velocity profile has been omitted as it is essentially identical to that shown).

In the case of the barred beach (Fig. 6), longshore current profiles for the linear and non-linear (without breaking-wave induced turbulence) cases show maxima at the seaward face of the bar and at the beach face. These are offered for comparison, without fitting, using the values of 0.008 and 0.006 (those found for the Santa Barbara data). Again, none of the velocity profiles include horizontal mixing. For the case of the proposed bottom stress form, fitting of the predicted profiles in the high turbulence regions (the vicinity of the bar and beach face), where c_{fr} dominates, yields Λ values of 3.0. c_{fl}, which is important away from the breaking-wave induced turbulence, has been set at 0.0005 arbitrarily in order to demonstrate that significantly lower values may in fact be plausible.

c_{fr} and its two spatially variable components, χ and tke, are shown in Fig. 7. Sensitivity tests on Λ and c_{fl} are shown for in Fig. 8. The three profiles of V shown represent three values of Λ (2.5, 3.0, and 3.5) with c_{fl} held constant at .0005. It can be seen that the profiles are not overly sensitive to Λ with approximately a 40% change in Λ producing only a 15% change in V_{max}. The converse situation is also shown where Λ is held constant (3.0) and c_{fl} is varied. As expected, the changes are found in the trough and seaward of the bar, regions away from the domination of breaking-wave induced turbulence.

Discussion and Conclusions

The assumption that the relationship between the free stream longshore current velocity and the bottom stress is constant across the surf zone has been brought into question. Field data from DELILAH have been used to demonstrate that inclusion of breaking-wave induced turbulent effects reduces reliance on horizontal mixing for all regions except the trough of a barred beach. In the cases of the two spatially constant c_f's (linear and non-linear without wave induced turbulence), current velocities outside the breaking region are greatly under-predicted. Improved agreement with observations is obtained using the proposed form with c_{fl}=.0005. Data

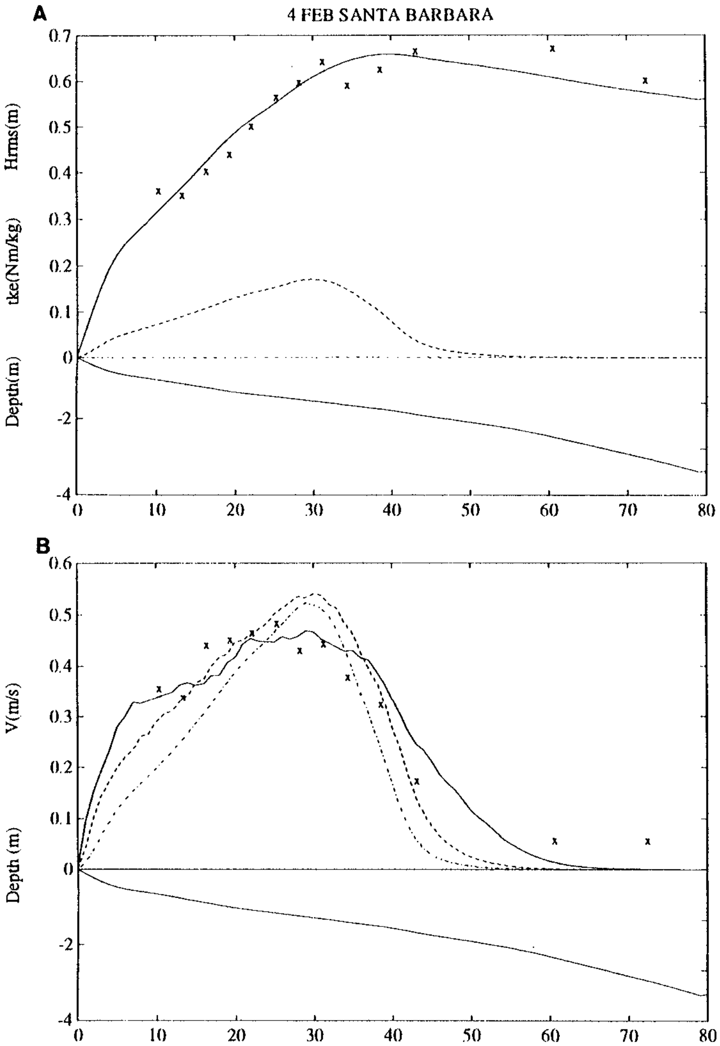


Fig. 5 a) Hrms model prediction (solid), tke (dashed), and measured bathymetry. b) Longshore current model predictions with linear bottom stress (dot-dash, Cf=0.008), non-linear (dashed, Cf=0.006), and non-linear with wave induced turb. effects (solid, Λ=4.0, Cfl=0.0005).

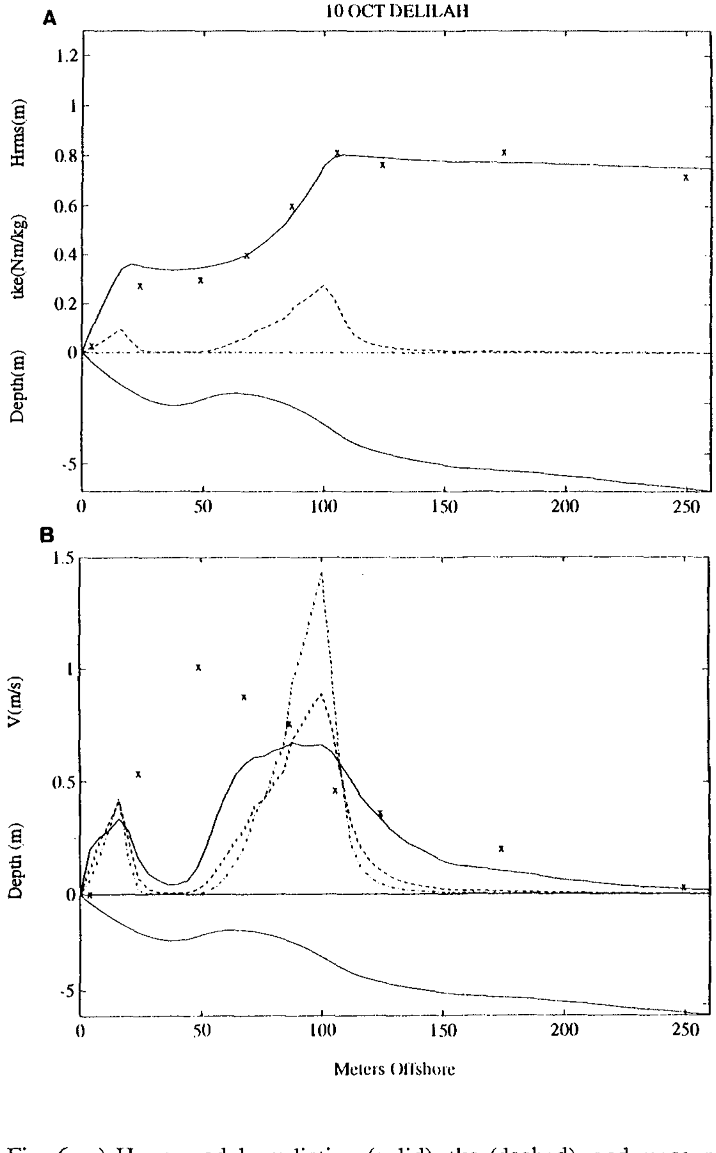


Fig. 6 a) Hrms model prediction (solid), tke (dashed), and measured bathymetry. b) Longshore current model predictions with linear bottom stress (dot-dash, Cf=0.008), non-linear (dashed, Cf=0.006), and non-linear with wave induced turb. effects (solid, Λ=3.0, Cfl=0.0005).

from NSTS Santa Barbara have shown that the proposed model is similarly applicable to planar beaches. Although the calculated value of c_{fl}=.0005, used throughout this study, is significantly lower than c_f values used in previous studies, it should be noted that cross-shore mean of c_f=c_{fr}+c_{fl}, calculated from the shore out to 200m, is .0023 which is more comparable to the spatially constant values found in the literature.

As was noted earlier, horizontal momentum mixing, which has been omitted from the proposed model, does occur to some extent in nature and the contributing roles of mean cross-shore flow and shear instabilities are being explored. Certainly, such mixing would be likely to transfer some longshore momentum into the trough region. It is worthy of note that generation of a velocity maximum between two predicted maxima, via horizontal mixing length theory, requires up-gradient momentum transfer, and therefore does not appear appropriate.

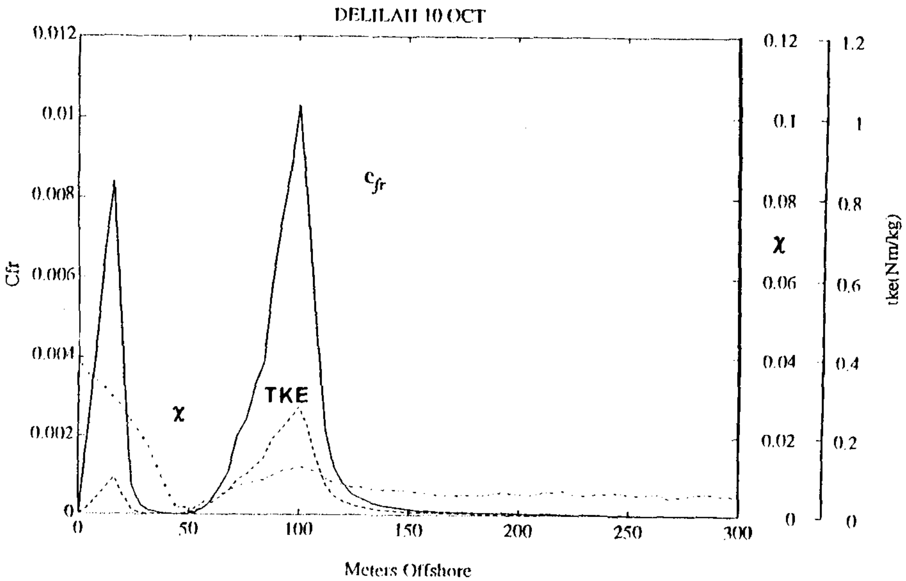


Fig. 7 Bottom friction coefficient associated with breaking-wave turbulence modification (Cfr) with its principle components tke and the penetration parameter χ.

Summary: A spatially variable bottom stress is proposed through the inclusion of the effects of breaking-wave induced turbulence. Employing reasonable assumptions regarding the horizontal and vertical distributions of this turbulence, it is shown that inclusion of breaking-wave induced turbulence effects improves agreement between predicted and observed longshore current velocities for both a planar and barred beach. It is suggested that disparities between predicted and observed velocities over the trough are most likely due to a failure to identify a significant alteration of the forcing mechanism in this particular region.

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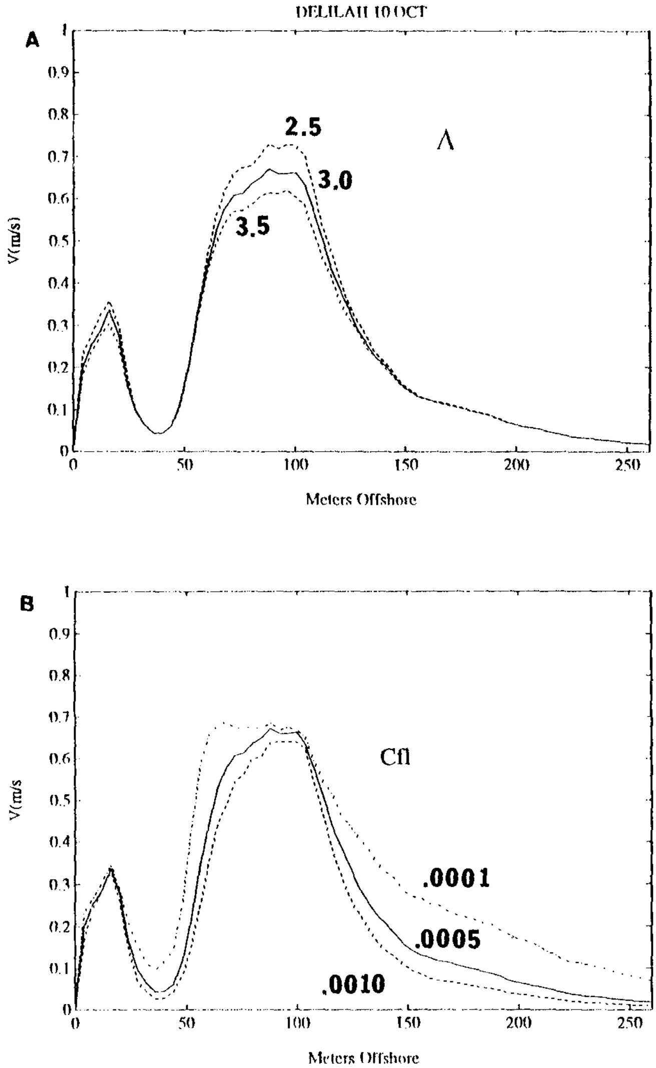

Fig. 8 Sensitivity tests a) Λ, Cfl held at 0.0005; b)Cfl, Λ held at 3.0

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