CHAPTER 211

A MIXING MECHANISM IN THE NEARSHORE REGION

Uday Putrevu, Associate Member, ASCE and Ib A. Svendsen,[1] Member, ASCE.

ABSTRACT

A dispersive mixing caused by the interaction of the longshore currents and the undertow is found to exist in the nearshore region. It is found to depend critically on the depth variation of both the cross-shore and the longshore currents. The effect is calculated for the simplest possible current profiles which yield a non-zero contribution. It is demonstrated that the dispersive mixing totally dominates the mixing in the nearshore region, exceeding the effect of turbulence by an order of magnitude even inside the surf-zone. In consequence, accounting for this interaction of the nearshore currents makes it possible to model longshore currents using realistic turbulent mixing levels. The vertical variation of those currents become part of the results provided. The predicted depth variation of the longshore currents is shown to be consistent with the only set of such measurements presently available.

1. INTRODUCTION

Longshore currents on beaches have been analyzed using the concept of radiation stress since the pioneering works of Bowen (1969), Thornton (1970) and Longuet-Higgins (1970). These works clearly demonstrated that a lateral mixing (presumably due to turbulent fluctuations) needs to be included to give predictions of the cross-shore structure of the longshore currents that resemble measured currents in laboratory and field experiments. The works cited above, and the numerous models proposed later, differ primarily in the assumptions related to the lateral mixing due to turbulence.

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[1]Postdoctoral Fellow and Professor, respectively, Center for Applied Coastal Research, Department of Civil Engineering, University of Delaware, Newark, DE 19716, USA

Models to predict vertical structure of nearshore currents have been proposed by Svendsen (1984), Dally & Dean (1984), Stive & Wind (1986), Svendsen *et al.* (1987) and Svendsen & Hansen (1988) for the cross-shore current (the undertow) and by Svendsen & Lorenz (1989) for the three dimensional current structure.

Svendsen & Putrevu (1990) developed the principles for 3D nearshore circulation modelling using analytical solutions for the 3D current profiles in combination with a numerical solution of the depth integrated 2D horizontal equations. Like Svendsen & Lorenz (1989), however, they neglected the current-current and the wave-current interaction terms. They pointed out that there is an order of magnitude difference between the eddy viscosity required to model accurately the vertical profiles and the eddy viscosity required for lateral mixing to achieve realistic results for the longshore current on a long, straight coast. This contradicts all physical arguments which point to the two being the same order of magnitude.

Svendsen & Putrevu (1992; hereafter referred to as SP92) found that the current-current and wave-current interactions neglected in previous investigations provide the additional lateral mixing. The effect involves the vertical structure of both cross- and longshore currents and turns out to be a generalization of the mechanism for longitudinal dispersion found by Taylor (1954) for pipe flow, Elder (1959) for open channel flow and Fischer (1978) for currents on the continental shelf. It is found to exist even on a long, straight coast with no alongshore variations.

In the present paper we present an analysis of this effect under the simplest possible conditions in order to illustrate the core of the mechanisms involved. It is shown that the dispersion effect is crucially linked to the overall slope of the longshore current profile. It disappears completely for a depth uniform longshore current.

The paper is organized as follows: Section 2 deals with the basic equations and outlines the assumptions involved. Section 3 discusses the dispersion caused under the vertical structure of the currents assumed here. A numerical example is presented in section 4. In section 5 we give a qualitative interpretation of the what we believe to be the essential nature of the dispersion mechanism. The paper concludes with a summary and an discussion of the important results of the paper in section 6.

2. BASIC EQUATIONS FOR A LONG STRAIGHT COAST

SP92 derived the basic equations governing the wave averaged nearshore circulation. For a long, straight coast with no alongshore variation these equations are

Depth Integrated continuity

$$\frac{d}{dx}\left[\int_{-h_0}^{\bar{\zeta}} U\,dz + Q_w\right] = 0 \tag{1}$$

Depth Integrated Cross-Shore Momentum Equation

$$\frac{1}{\rho}\frac{dS_{xx}}{dx} = -g(h_0 + \bar{\zeta})\frac{d\bar{\zeta}}{dx} \tag{2}$$

Depth Integrated Alongshore Momentum Equation

$$\frac{1}{\rho}\left[\frac{d}{dx}\left(S_{xy} + S'_{xy}\right) + \tau_{by}\right] + \frac{d}{dx}\left[\overline{\int_{-h_0}^{\bar{\zeta}} UV\,dz} + \overline{\int_{\zeta_t}^{\zeta}(u_w V + v_w U)\,dz}\right] = 0 \tag{3}$$

In the above u_w and v_w are the wave induced velocities in the x (cross-shore) and y (alongshore) directions respectively.

These equations differ from the equations found in Phillips (1977) or Mei (1983) in that they allow for the currents to have a vertical variation. Dealing with wave averaged equations, one has to clearly define what is meant by a "current" above wave trough level where there is water only part of the time. When evaluating the terms in these equations, we assume that the mathematical expression defining the currents below trough level is valid above that level also. Such an assumption is implicitly made by Phillips (1977) and Mei (1983). For the case of depth uniform currents, the equations above reduce to the equations found in Phillips and differ slightly from those found in Mei (1983) owing to a difference in the definition of the current velocity.

In (1) - (3) the wave averaged quantities Q_w (volume flux), S_{xx} and S_{xy} (radiation stress components) are defined as

$$Q_w = \overline{\int_{-h_0}^{\zeta} u_w\,dz} = \overline{\int_{\zeta_t}^{\zeta} u_w\,dz} \tag{4}$$

$$S_{xx} = \overline{\int_{-h_0}^{\zeta}(\rho u_w^2 + p)\,dz} - \frac{1}{2}\rho g h^2 \tag{5}$$

$$S_{xy} = \overline{\int_{-h_0}^{\zeta} \rho u_w v_w\,dz} \tag{6}$$

Figure 1 shows the definitions of the geometrical parameters. S'_{xy} represents the turbulent radiation stress (depth integrated Reynolds' stress) and is defined by

$$S'_{xy} = \overline{\int_{-h_0}^{\zeta} \tau_{xy}\,dz} \tag{7}$$

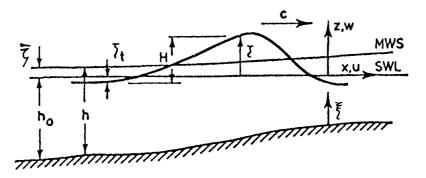where τ_{xy} represents the Reynolds' stress.

Figure 1: Definition sketch

Using a perturbation expansion based on slowly varying depth SP92 showed that at the lowest order the following equations govern the vertical variation of the cross- and alongshore currents
Depth Dependent Cross-Shore Momentum

$$\frac{\partial}{\partial z}\left(\nu_{tz}\frac{\partial U}{\partial z}\right) = g\frac{d\bar{\zeta}}{dx} + \overline{u_w\frac{\partial v_w}{\partial x}} \tag{8}$$

Depth Dependent Alongshore Momentum

$$\frac{\partial}{\partial z}\left(\nu_{tz}\frac{\partial V}{\partial z}\right) = \overline{u_w\frac{\partial v_w}{\partial x}} + U\frac{dV_b}{dx} \tag{9}$$

Of particular importance here is the last term in (9) which represents the lowest approximation to the current-current interaction.

SP92 solve these equations for arbitrary ν_{tz} and u_w, v_w distributions.

3. SOLUTION FOR DEPTH UNIFORM UNDERTOW

In the present paper we analyze the effect of the interaction terms in (3) for the simple situation where the undertow is constant over depth and the longshore current is quadratic. The longshore current profile corresponds to assuming that ν_{tz} and the wave induced velocities are depth uniform (cf. eq. 9). Hence, we assume the following

$$U(\xi) = U_0 = \frac{-Q_w}{h} \tag{10}$$

$$V(\xi) = V_b + b_v\xi + a_v\xi^2 \tag{11}$$

where $\xi = z + h_0$ is the distance from the bed (see figure 1). While simplifying the analysis, this situation captures the essential nature of the interaction term and the simpler algebra helps exposing the mechanisms involved.

The parameters a_v and b_v are then given by

$$a_v = \frac{1}{2\nu_{tz}} \left(\overline{u_w \frac{\partial v_w}{\partial x}} + U \frac{\partial V_b}{\partial x} \right)$$
(12)

as indicated by (9) and

$$b_v = \frac{f_w u_0}{\pi \nu_{tz}} V_b$$
(13)

where u_0 is the amplitude of the near bottom wave induced velocity. The equation for b_v is derived from the bottom boundary condition

$$\rho \nu_{tz} \frac{\partial V}{\partial z} \bigg)_{z=-h_0} = \tau_{by} = \frac{\rho f_w u_0 V_b}{\pi}$$
(14)

(see Svendsen & Putrevu 1990).

Using (10) and (11) we find that

$$\int_{-h_0}^{\overline{\zeta}} UV dz = -Q_w \left(V_b + \frac{b_v h}{2} + \frac{a_v h^2}{3} \right)$$
(15)

and

$$\overline{\int_{\zeta_t}^{\zeta} V u_w dz} \approx \overline{V(\zeta)} \overline{\int_{\zeta_t}^{\zeta} u_w dz} = Q_w \left(V_b + b_v h + a_v h^2 \right)$$
(16)

The Uv_w term in (3) is small for small angles of incidence which is the typical situation on long, straight coasts. Hence, this term is neglected in the following.

Equations 15 and 16 clearly show that the terms representing the interaction of the currents oppose one another and for the case of a depth uniform longshore current ($a_v = b_v = 0$) they exactly cancel one another. In total (15) and (16) give

$$\int_{-h_0}^{\overline{\zeta}} UV dz + \overline{\int_{\zeta_t}^{\zeta} V u_w dz} = \frac{Q_w h^2 \overline{u_w \partial v_w / \partial x}}{3 \nu_{tz}} + \frac{Q_w f_w u_0 h}{2 \pi \nu_{tz}} V_b - \frac{Q_w^2 h}{3 \nu_{tz}} \frac{dV_b}{dx}$$
(17)

Substituting (17) into (3) then leads to

$$\frac{1}{\rho} \frac{d}{dx} \left(S_{xy} + S'_{xy} \right) + \frac{\tau_{by}}{\rho} + \frac{d}{dx} \left(\frac{Q_w h^2 \overline{u_w \partial v_w / \partial x}}{3 \nu_{tz}} + \frac{Q_w f_w u_0 h}{2 \pi \nu_{tz}} V_b - \frac{Q_w^2 h}{3 \nu_{tz}} \frac{dV_b}{dx} \right) = 0$$
(18)

The solution of (18) requires the specification of the way in which we parameterize the turbulent radiation stress S'_{xy} (the bottom stress τ_{by} is given by equation 14). For the turbulent radiation stress we use

$$S'_{xy} = \rho h \nu_{tx} \frac{dV_b}{dx}$$
(19)

SP92 show that using V_b in (19) instead of the mean velocity V_m is consistent to the order of validity of (3). The calculation of V_b using (18) still requires the specification of Q_w and S_{xy}. These are related to the wave motion and are assumed to be known in wave averaged models.

Introducing the definitions

$$D_c = \frac{Q_w^2}{3\nu_{tz}} \tag{20}$$

$$F_1 = \frac{\alpha_y h^2 Q_w}{3\nu_{tz}} \tag{21}$$

$$F_2 = \frac{f_w Q_w u_0 h}{2\pi\nu_{tz}} \tag{22}$$

we may rewrite the equation governing the longshore current in this simplified situation as

$$\frac{d}{dx}\left\{ h\left(\nu_{tx} + Dc\right)\frac{dV_b}{dx} \right\} - \frac{d}{dx}\left(F_2 V_b\right) - \frac{f_w u_0}{\pi}V_b = \frac{1}{\rho}\frac{dS_{xy}}{dx} + \frac{dF_1}{dx} \tag{23}$$

which clearly demonstrates that the interaction provides mixing – this can be seen from the D_c term of (23). In addition to the dispersion effect the interaction also produces other effects that are reflected in the F_1 and F_2 terms. The general results for D_c, F_1 and F_2 were given by SP92.

Discussion of the dispersion effect

Before we proceed any further it is worthwhile to discuss the nature of the dispersion at this stage. We first notice from (20) that the dispersion coefficient D_c is proportional to the square of the volume flux due to the waves and inversely proportional to the vertical eddy viscosity. This result is analogous to the longitudinal dispersion in pipe flow found by Taylor (1954). The dispersion coefficient K is given by an expression of the form (see, e.g., Fischer et al. 1979, p. 94)

$$K = \frac{d^2 < u_d^2 >}{< \nu_{tz} >}I \tag{24}$$

where d is a characteristic length, u_d is the deviation of the velocity from its cross-sectional average, $< \nu_{tz} >$ is the cross-sectionally averaged mixing coefficient and I is a dimensionless integral of order 0.1. The angular brackets in (24) represent cross-sectional averaging.

Here u_d, d and $< \nu_{tz} >$ are analogous to our U_0, h and ν_{tz} and (20) is analogous to (24). In the present case the contributions to the dispersion also come from the fact that the waves and the fact that the currents are modified by the dispersion which is not the case in simple dispersion of contaminants analyzed by Taylor and others.

We may get a preliminary estimate of the size of the dispersion coefficient relative to ν_{tz} by noticing that we typically have (inside the surf-zone)

$$Q_w \sim 0.1 \; \frac{H^2}{h} \sqrt{gh} \tag{25}$$

$$\frac{H}{h} \sim 0.7 \tag{26}$$

and

$$\nu_{tx} \sim \nu_{tz} \sim 0.01 h \sqrt{gh} \tag{27}$$

(see, *e.g.*, Svendsen *et al.* 1987). These estimates imply that

$$D_c \sim 0.08 h \sqrt{gh} \tag{28}$$

Comparison of (28) and (27) indicates that with these simplifying assumptions the dispersive effect is about eight times stronger than the lateral mixing. The more detailed calculations of SP92 show that accounting for the actual vertical structure of the undertow enhances this effect approximately by a factor of two. Thus, we see that $D_c \gg \nu_{tz}$ and we can expect the mixing for the longshore current is dominated by the dispersion.

4. A NUMERICAL EXAMPLE

The numerical example presented below will demonstrate the conclusion of the previous paragraph. In the calculations we have used some simplifying assumptions which do not change the nature of the problem (though they do influence the accuracy of the predictions). These assumptions are:

- Use linear long wave theory to calculate S_{xy} and α_y.

- Use $H \propto h$ inside the surf-zone and $H \propto h^{-1/4}$ outside the surf-zone.

- Use $Q_w = 0.1(H^2/h)\sqrt{gh}$.

The eddy viscosity variation used is given by

$$\nu_{tx} = \nu_{tz} = \begin{cases} 0.01 h \sqrt{gh} & \text{inside surf zone} \\ [0.8(h/h_b)^4 + 0.2]\nu_{tb} & \text{outside surf zone} \end{cases} \tag{29}$$

where $\nu_{tb} = 0.01 h_b \sqrt{gh_b}$. The variation of the eddy viscosity outside the surf-zone represents an estimate based on the assumption that $\nu_t \propto l\sqrt{k}$ and the the measurements of Nadaoka & Kondoh (1982) of the turbulence outside the surf-zone. These measurements show that while the intensity of the turbulence decays seaward of the break point, there is still some residual turbulence even far seaward of the break point (see their figure 9).

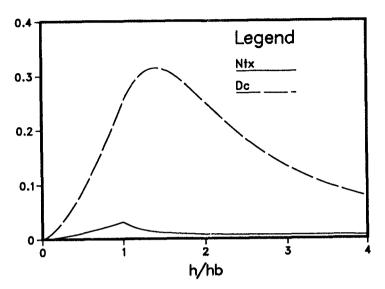Figure 2: Variation of ν_{tx} and D_c

Equation 18 is then solved with boundary conditions $V_b = 0$ at the shoreline and $V_b \to 0$ as $h/h_b \to \infty$. Since the RHS of (18) is discontinuous at the break point, the solution has to be matched at that point. The matching conditions used are

$$V_b(x_b+) \;\; = \;\; V_b(x_b-) \tag{30}$$

$$\left. \frac{\partial V}{\partial x} \right)_{x_{b+}} \;\; = \;\; \left. \frac{\partial V}{\partial x} \right)_{x_{b-}} \tag{31}$$

where x_b represents the breaker location.

Figure 2 shows the variations of the dispersion coefficient and the eddy viscosity with cross-shore location. As mentioned earlier, over the entire nearshore region the dispersion coefficient is significantly larger than the eddy viscosity showing that the dispersion due to current-current interaction totally dominates the nearshore mixing.

The resulting cross-shore distribution of the near bottom longshore current is shown in Figure 3. This figure also shows the solution obtained by neglecting the dispersive mixing. A comparison of the two solutions demonstrates, as expected from the enhanced mixing due to current-current interaction, that the dispersion has significant influence on the cross-shore structure of the longshore current. Specifically, we see that accounting for the current dispersion does bring the
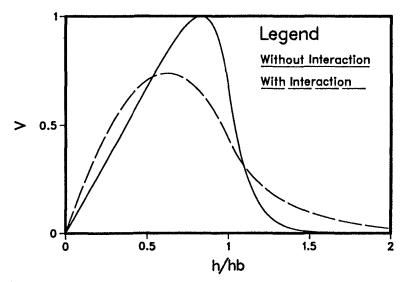
Figure 3: Cross-shore distribution of the near bottom longshore current

cross-shore distribution in line with what the measurements indicate without having to take recourse to a large mixing coefficient (see, *e.g.*, Visser 1984).

Once V_b is calculated, we may calculate the longshore current profiles using (11) and (13). The profiles so calculated are shown in figure 4. We first notice that the individual longshore current profiles do not show very strong variation with vertical location. This means that the cross-shore distribution of the near bottom longshore current shown in figure 3 is representative of all vertical locations. Second, we notice that the longshore current increases with distance from the bed in the region $0 < h/h_b < 0.7$ while it decreases with distance from the bed in the region $h/h_b > 0.7$. As demonstrated in the next section, this turns out to be the essential feature that is responsible for the dispersion.

The only experimental investigation that reports systematic measurements of the vertical structure of longshore currents is Visser (1984) for a long, straight beach. These measurements consistently confirm the pattern predicted above. As an example, we reproduce in figure 5 one set of Visser's measurements (experiment 2). The measurements clearly show a longshore current increasing with distance from the bed inside the surf-zone (the first four panels) and decreasing with distance from the bed outside (last four panels). Considering the crucial dependence of the dispersion on this trend we believe that the experimental confirmation of the trend predicted by the computations is extremely encouraging.
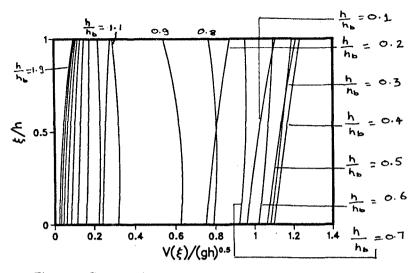
Figure 4: Computed vertical variation of the longshore current

5. QUALITATIVE INTERPRETATION OF THE DISPERSION MECHANISM

Equation 17 shows that the dispersion coefficient, D_c, originates from the $U \partial V/\partial x$ term in the forcing for the depth variation for the longshore currents. Since this term contributes to the curvature of the longshore current which, in turn, contributes to the overall slope of the longshore current profile it suggests that it is the overall slope of the longshore current that controls the dispersion. This is discussed further below.

As remarked in the discussion below (16) the contributions from below and above trough levels counteract one another. The overall effect depends on which of the two is stronger. We see that (15) and (16) may be written as

$$\int_{-h_0}^{\bar{\zeta}} UV dz = -Q_w \left(\overline{\frac{1}{h} \int_{-h_0}^{\zeta} V(z) dz} \right) \tag{32}$$

and

$$\overline{\int_{\zeta_t}^{\zeta} V u_w dz} = Q_w V(\bar{\zeta}) \tag{33}$$

Thus for the undertow variation used here the contribution from above trough level will dominate if

$$V(\bar{\zeta}) > \overline{\frac{1}{h} \int_{-h_0}^{\zeta} V(z) dz} \tag{34}$$

This corresponds to a longshore current that increases with distance from the bottom (figure 6a) which, as figure 4 shows, occurs for $h/h_b < 0.7$ in the ex-
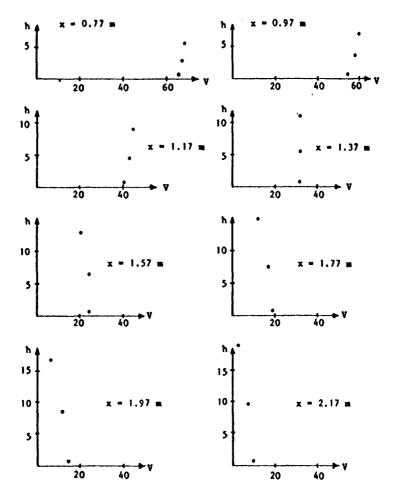
Figure 5: Measured vertical variation of the longshore current (from Visser 1984)
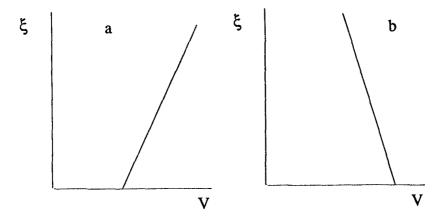
Figure 6: Sketch of the vertical variation of the longshore current
a) $V(\bar{\zeta}) > \frac{1}{h}\int_{-h_0}^{\zeta} V(z)dz$ b) $V(\bar{\zeta}) < \frac{1}{h}\int_{-h_0}^{\zeta} V(z)dz$

ample considered. Hence, in this region the net effect of the interaction is to convect the longshore current momentum shorewards and depending on the sign of $\partial V/\partial x$ this may increase or decrease the local value of V. Though the shoreward transport of the longshore momentum is at the surface with a smaller seaward transport below, the profile is maintained by the driving force and the vertical mixing. Conversely, in the region $h/h_b > 0.7$ (in the present example) the situation is reversed. In particular, outside the surf-zone this represents the major source of longshore momentum flux in the seaward direction which is equivalent to mixing.

Hence, although the cross-shore transport of the longshore momentum (equivalent to dispersion or mixing) is provided by cross-shore currents, the net effect depends crucially on the vertical variation of the longshore currents. This is also consistent with the fact that for depth uniform longshore currents we get no dispersion effects.

6. SUMMARY AND CONCLUSIONS

It has been demonstrated that the dispersive mixing caused by the interaction of the longshore currents and the undertow totally dominates the mixing in the nearshore region. Even inside the surf-zone it is an order of magnitude stronger than the direct turbulent mixing. In the example given in section 4, the effect was calculated for the simplest possible case which leads to a non-zero value, *viz.*, a depth uniform undertow and a longshore current that varies quadratically with

the vertical coordinate. It turns out that even though the example considered here underestimates the dispersive effect substantially in comparison to the more complete version of the theory given in SP92, it captures the essential nature of the dispersion mechanism and allows us to demonstrate the same in a rather simple manner.

The results show that the nearshore circulation on a beach is essentially a 3D phenomenon with the dispersion caused by the nonlinear interaction of the nearshore currents being a very important contribution. It depends crucially on the vertical profiles of the current, in particular in the longshore direction. While the turbulence contributes very little to the lateral mixing directly it is still very important because the turbulence has a strong influence on the shape of the current profiles.

The existing laboratory measurements support our predictions of the vertical structure of the currents which forms a part of the results. This is very encouraging considering the crucial dependence of the effect on the current profiles. More confirmation of the predictions, both in the laboratory and the field, would be particularly useful.

ACKNOWLEDGEMENTS

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