CHAPTER 81

The Movement of Submerged Bodies by Breaking Waves

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Abstract

This paper describes a mathematical model of the large but short-lived forces exerted on submerged solid bodies by a wave impacting on a plane impermeable surface nearby. We consider the forces on a hemispherical boulder situated close to a wave impact on (i) a vertical wall and (ii) a steep slope. We show that for certain positions of the body and for a sufficiently strong wave impact the impulsive force on the body can be much greater than either the flow drag or the weight of the boulder. For a body which is free to move under the wave impulse we compute the body's initial velocity.

Introduction

This paper describes recent work on the mathematical modelling of the large sudden forces exerted on submerged bodies by breaking waves. See figure 1. When a wave breaks against a wall the peak pressure at a point in the fluid can be ten times greater than hydrostatic, and the pressure can rise and fall in milliseconds. Richert (1968) measured high pressures on both walls and slopes. Nagai (1960) reports measurements in which the maximum peak pressure occurs at the bottom of the wall; this may have occurred because the bed was exposed before impact.

Cooker and Peregrine (1990a,1992) have shown theoretically that when a wave breaks against a vertical wall the peak fluid pressure can be significant all the way down the wall and along the bed. Grilli et al (1992) have measured impulsive wave impact pressures along the bed in front of a vertical wall. The theory and measurements show that for a given wave impact the peak pressure decreases along the bed with increasing distance from the wall. Suppose a small boulder lies on the bed. During the short time of wave impact the boulder experiences high pressure on the side near the wall and

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lower pressure on the side away from the wall. This impulsive fluid pressure must be treated differently to hydrostatic pressure: in order to find the net impulsive load on the body we must carry out calculations similar to those for finding added mass. In this work it is more convenient to work with the *pressure*

In this work it is more convenient to work with the *pressure impulse* than the peak pressure. The pressure impulse, P(x), at the point x due to a wave impact which begins at time t_b and ends at time t_a, is defined to be the time-integral of the pressure p(x,t):

$$P(\mathbf{x}) = \int_{\mathbf{t}_{\mathbf{b}}}^{\mathbf{t}_{\mathbf{a}}} p(\mathbf{x}, \mathbf{t}) \, \mathrm{dt}.$$
 (1)

For an incompressible liquid P satisfies Laplace's Equation (Lamb, 1932, §11). We expect the highest impact pressures to be generated when the fluid contains few air bubbles and so the water may be treated as incompressible. P is useful for finding the velocity after impact, $u_{\rm a}$, from the velocity before impact, $u_{\rm b}$:

$$\mathfrak{y}_{a}(\mathfrak{x}) = \mathfrak{y}_{b}(\mathfrak{x}) - \frac{1}{\rho} \nabla P(\mathfrak{x})$$
(2)

where ρ is the water density, and the flows u_a , u_b can contain

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vorticity. The peak pressure $p_{pk}(x)$ is connected to the pressure impulse by the approximate relation

$$p_{\mathbf{pk}}(\mathbf{x}) = 2P(\mathbf{x})/\Delta t \tag{3}$$

where the impact duration $\Delta t = t_a - t_b$ must be chosen appropriately. The observed decrease in peak pressure with distance along the bed in front of a wall which is undergoing wave impact, corresponds to a decrease in the pressure impulse along the bed. In general, where the water contains a gradient of pressure impulse it can exert an impulse on a solid body. We show below that the impulse on the body is directly proportional to the product of the volume of the body and the size of the pressure impulse gradient. The direction of the net impulse can be quite different to the direction of the pressure impulse gradient. The size of the impulse on the body also depends on its shape, and whether it is fixed or free to move. The calculation of the impulsive force on a body has two stages.

First the pressure impulse P_1 is calculated from a boundary-value problem (b.v.p.) appropriate to the shape and speed of the impacting wave at the instant it meets, for example, a vertical wall. A point X is chosen at which we will place a small body and there we find

the pressure impulse gradient $G = \nabla P_1$. In the second stage we solve

a second b.v.p. for P_2 , which is the pressure impulse close to the body and which accounts for the disturbance to P_1 caused by the presence of the body. Finally the net impulse is found from an integral of P_2 over the surface of the body. If the body is free to move we can also find its initial velocity.

The analysis can be used to find the force on a body in front of a vertical wall (see Cooker and Peregrine, 1992) or a steep slope. The measurements of Richert (1968) show that when a slope is subjected to wave impact significant pressure impulse gradients can occur. The impulsive forces estimated in this paper can be briefly much greater than the forces of flow drag or body weight. This work may explain the movement of large blocks seaward away from steeply sloping and vertical sea defences, when these same blocks are unmoved by the drag from the water motion of waves.

2. Pressure Impulse: A comparison with experiment.

Figure 2(a) is a sketch of a wave impact on a plane vertical The peak pressures are greatest when the incident wave crest wall. is parallel to the wall at the instant of impact, so the flow may be modelled in a two-dimensional plane perpendicular to the shore. The idealized boundary-value problem (b.v.p.) for the pressure impulse, P, is shown in figure 2(b). The total height of water is H and for simplicity we suppose the wave face impacts the wall between y = 0and $y = -\mu H$, where μ must be chosen between 0 and 1. The component of water velocity normal to the wall before impact is supposed the same at all points and denoted U₀. The bed and wall are impermeable.



FIGURE 2(a): Sketch of the wave before impact on a vertical wall.



FIGURE 2(b): Boundary-value problem for pressure impulse $P_{\tt 1}$.



FIGURE 3(a): Comparisons of measured pressure impulse with theory, for impact as in figure 2. μ = $\frac{1}{2}$, $U_{\rm 0}$ = 1m/s H = 0.1m .



FIGURE 3(b): As fig. 3(a). $\mu = \frac{3}{4}$, $U_0 = 1m/s$, H = 0.1m. The disagreement may be due to the impact being more prolonged and less intense than in fig. 3(a): peak pressures are only $\frac{4}{7}$ of those in fig. 3(a).

The b.v.p is solved by Cooker and Peregrine (1990a) and we compare this earlier result with the measurements of Hattori (1992). The values of μ , H, U₀ are estimated from high-speed video images: $\mu = 0.53$, H = 0.1m, and U₀ = 1m/s. Figure 3 shows two comparisons between the theory (with parameters chosen from the video) and the pressure measurements of Hattori. The close agreement of figure 3(a) occurs for a wave impact of short duration with a very high maximum peak pressure of 525gf/cm^2 . The disagreement in figure 3(b) is interesting because it shows the pressure impulse with a maximum near the bed, indicating that P (and its gradient) may be significant along the bed. Also this second comparison corresponds to a low impact pressure maximum measured to be 70gf/cm^2 , so pressure impulse theory may be inappropriate for this less sudden impact. Further note that the values of pressure impulse in figure 3(a) and figure 3(b) are similar even though the maximum peak peak pressures differ by a factor of 7. This accords with the repeated wave experiments of Bagnold (1939) who observed the constancy of pressure impulse compared with the very wide variation of the peak pressures.

3. The Impulse on a Hemispherical Boulder: Vertical Wall.

Figure 4 shows the distribution of pressure impulse on the wall and along the bed for the solution P_1 of the b.v.p. presented in figure 2(b). On the bed, the gradient of P_1 , denoted \mathcal{G} , is directed along the bed. $|\mathcal{G}|$ takes its greatest value of $0.11\rho U_0$ at $x = x_m =$ 0.48H, where x = 0 is at the wall. We now place a boulder in the shape of a hemisphere on the bed at $x = x_m$, so that it can experience the greatest gradient of pressure impulse. Locally we can model the variation of P_1 by a linear approximation:

$$P_1 = P_0 - G(x - x_m) \tag{4}$$

where P_0 and G are the positive constants $P_0 = P_1(x=x_m,y=-H)$ and $G = \left|\frac{\partial P}{\partial \bar{x}}(x=x_m,y=-H)\right|$. We now compute the effect on P_1 due to the presence of the boulder. P_2 is the pressure impulse on and near the boulder. Let r, θ, ϕ be spherical polar coordinates centred on the hemisphere, where θ is the angle subtended by the field point and the x-axis, at the centre of the hemisphere. P_2 is harmonic and must match the variation of P_1 , given by equation (4) (expressed in polar coordinates), at large distance from the boulder: i.e.

$$P_{2}(x) \longrightarrow P_{1}(x) = P_{0} - G r \cos \theta \text{ as } |x| \equiv r \longrightarrow \infty.$$
 (5)

On the boulder, r = a, we have the second boundary condition for P_2 :

$$\frac{\partial P_2}{\partial r} = -\rho V \cos \theta \tag{6}$$

where V is the as yet unknown velocity component along the x-axis



FIGURE 4(b): As figure 4(a). The pressure impulse on the bed for several values of μ . At points where $\partial P/\partial x$ is large the fluid can exert an impulsive load on a body.

acquired by the body due to the fluid impulse. Cooker and Peregrine (1992) show that the impulse on the hemisphere is

$$I = \frac{2}{3}\pi a^{3} \frac{1}{2} \left\{ 36 - \rho V \right\} .$$
 (7)

For a free boulder we may equate the impulse I with momentum gained by the body: if $\rho_{\rm b}$ is the boulder's uniform density then the initial speed of unconstrained motion is

$$V = \frac{3G}{2\rho_b + \rho} . \tag{8}$$

If the body is fixed then V = 0 in equation (7). If the body is free then equations (7) and (8) give

$$I = \frac{2}{3}\pi a^{3} \left\{ \frac{36\rho_{b}}{2\rho_{b} + \rho} \right\} .$$
 (9)

Equation (3) suggests that the peak force is related to the impulse:

$$\mathbf{F} = 2\mathbf{I}/\Delta \mathbf{t} \,. \tag{10}$$

EXAMPLE. Let H = 2m , U₀ = $\sqrt{(gH)}$ = 4.4m/s, and ρ = 1035kg/m³, $\overline{\rho_{\rm b}} = 2.7 \rho$. For impact on a vertical wall, with $\mu = \frac{1}{2}$, the position of maximum pressure impulse gradient is $x_m = 0.48H = 1.2m$ from the wall. $G = 0.1\rho U_0 = 0.44\rho = 460 \text{ Ns/m}$. Let the impact time $\Delta t = 0.01s$. From equation (8) V = 0.21 m/s and is the same for any

hemisphere radius a.

From equation (9) $l = 1.21 \text{ a}^3 \text{ kNs.}$ From equation (10) $F = 240 \text{ a}^3 \text{ kN.}$ We estimate the flow drag, D, from that for a sphere for Reynold's numbers between 10^4 and 10^6 with a drag coefficient $C_d = \frac{1}{2}$ and a typical flow speed of U_{0} (Batchelor, 1973, p341).

> $D = \frac{1}{2}C_{d}(\frac{1}{2}\pi a^{2})\rho U_{0}^{2}$ (11)

$$D = 7.9 a^{2} kN.$$
Finally the dry weight $W = \frac{2}{3}\pi a^{3} \rho_{b}g$ (12)
Hence $W = 57 a^{3} kN.$

The following table compares the forces for several hemispheres of different radius, a, all much less than the local depth H.

a (m)	F (N)	D (N)	W (N)		
0.05	30	19.7	7.2		
0.1	240	79	57		
0.2	1930	320	460		

The impulsive force F is directed away from the wall and the drag D is directed toward the wall, at the instant of impact. In each case the impulsive force F is much greater than either the drag, D, or the dry weight, W. Also D is directly proportional to the cross-sectional area of the body, whereas F is directly proportional to its *volume*. So for larger bodies we expect the impulsive force to be even more important than the other forces acting. If the body is fixed then the impulsive force F is 19% greater than that tabulated.

4. The Impulse on a Hemispherical Boulder: Steep Slope.

The effect of water impact is directly dependent on the inertia of the water, and during the short time of high impact pressures, gravity has no significant effect and so pressure impulse theory can be used for waves breaking directly onto a slope. For example figure 5 shows the distribution of P in a quarter-plane, adapted for a slope by simply turning it. We can choose any line P=constant as a free surface (P = 0), by subtracting the constant from the solution.

It is inadvisable to rely on pressure impulse theory near the impact region since the motion there is grossly simplified. Much energy is given to the small amount of water in the splash (see for example the jet flow computed in Cooker and Peregrine, 1990b). However, down the slope, the pressure impulse gradients are more reliably estimated. Proximity to the impact region leads to pressure impulse gradients which are much bigger than those on the bed in front of a wall.

In figure 5 the breaking wave face is modelled to strike the slope like a closing door which is hinged at y = -d, and has maximum speed U_o at y = 0. Figure 6 shows the variation on the slope of P and its derivative parallel to the slope. We place a hemisphere at y = -2d = -2m (d =1m), and we estimate $U_o = \sqrt{(dg)} = 3.13m/s$. At the hemisphere position $G = 0.065\rho U_o$. The impact time $\Delta t = 0.01s$. From equations (9,10,11,12)

The	impulsive	force is		F	=	111	a^{s}	kN.
The	flow drag	force is		D	=	4.0	a^2	kN.
The	weight of	the body	is	W	=	57	a^{3}	kN.

The following table compares the forces for several radii, a.

a (m)	F (N)	D (N)	W (N)
0.05	13.9	10	7.2
0.1	111	40	57
0.2	890	159	460

The impulsive force F is directed down the slope and the drag is directed up the slope. As a increases F becomes ever larger than the drag. The initial speed of each of the hemispheres is 0.09 m/s. The main difference between the two calculations of impulsive force on a body (in front of a wall and on a slope) is the value of G. Both F and V are directly proportional to G.



FIGURE 5: Contours of pressure impulse P for wave impact on a slope. A solution in a quarter-plane has been rotated to lie on the slope. By subtracting a constant from the solution a curved free surface can be obtained, (e.g. the bold contour shown). The impacting wave face is modelled as a closing door, hinged at y/d = -1, with speed U_0 = 1 at y = 0. The contour increment is 0.005 $\rho U_0 d$, with a maximum of 0.240 $\rho U_0 d$ on the slope at y/d = -0.4.



FIGURE 6: As figure 5. The distribution of P and its gradient on the slope. Note that $G = \partial P/\partial y$ acts to thrust bodies down the slope below the impact zone (which lies between y/d = 0 and y/d = -1.

5. Conclusions

The hemisphere is an idealized boulder. Cooker and Peregrine (1992) discuss the impulsive force on a hemi-ellipsoid and they show that a body broadside on to an impact region receives a much larger impulse than a body pointing toward the impact region.

A hemisphere which is free to move may be expected to have a fluid layer between its base and the bed. If the layer has a narrow width h(x,y) then the pressure impulse obeys a certain partial differential equation: $\nabla \cdot (h\nabla P)=0$. The distribution of pressure impulse in the layer may cause a net upthrust on the boulder. For the hemisphere, if the gap width is constant, it can be shown that the upthrust is equal and opposite to the downthrust on the upper curved surface. So in this special case there is zero net impulse normal to the bed (and hence no impulsive reaction between the boulder and the bed due to friction). Nevertheless the pressure impulse distribution on the base of the hemisphere causes an impulsive overturning moment. In general the impulsive uplift and overturning moment depend in a complicated way on h(x,y) and the shape of the boulder, and is the subject of future study.

This work suggests that the impulsive pressure field created inside a wave when it impacts a solid surface may be large enough to move nearby bodies, such as armour units, and may explain some of the damage to the Sines breakwater which had a wall at its crest (see Baird et al, 1980). For a wave impact which is sufficiently high-speed and short-lived the impulsive loads on a hemisphere can be much greater than the flow drag or the weight of the body. Further, the impulse increases with the volume of the body, whereas the drag increases with its cross-sectional area. So we expect impulsive loading to be most important for the biggest boulders. The impulse is greater for a fixed body than one free to move (19% greater for a hemisphere).

The theory must be modified for a body which is so large that it alters the incident wave flow. Here a b.v.p in a domain containing both the wave and the boulder must be solved with modified boundary conditions. Despite the increased complexity we still expect the impulsive forces to be significant compared with the other types of load.

The solution for a body on a slope shows that a boulder can be thrust seawards down the beach if it is below the impact zone. Inside the impact zone the predicted pressure impulse gradients suggest they would force a body up the slope, but here we can be less certain of the applicability of pressure impulse theory.

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REFERENCES

BAGNOLD, R.A. (1939) "Interim report on wave pressure research" Journal of the Institute of Civil Engineers (London), 12, 201-226.

BAIRD,W.F., J.M.Caldwell, W.L.Edge, O.T.Magoon, D.D.Treadwell (1980) "Report on the damages to the Sines breakwater, Portugal" Proc. 17th Intl. Conf. Coastal Engineering, ASCE, 3063-3077.

BATCHELOR, G.K. (1973) "An introduction to fluid dynamics" Cambridge University Press, 615pp.

COOKER, M.J., D.H. Peregrine (1990a) "A model of the shock pressures from breaking waves" Proc. 22nd Intl. Conf. on Coastal Engineering, ASCE, 1473-1486.

COOKER, M.J., D.H. Peregrine (1990b) "Violent water motion at breaking wave impact", Proc. 22^{nd} Intl. Conf. Coastal Engineering, Delft, ASCE, 164-176.

COOKER, M.J., D.H. Peregrine (1992) "Wave impact pressures and their effect upon bodies lying on the sea bed" Coastal Engineering (to appear), 25pp.

GRILLI, S.T., M.A. Losada, F. Martin (1992) "Wave impact forces on mixed breakwaters' ", Proc. 23rd Intl. Conf. Coastal Engineering.

HATTORI, M. (1992) Personal Communications.

LAMB, H. (1932) "Hydrodynamics" Cambridge University Press.

RICHERT, G. (1968) "Experimental investigation of shock pressures against breakwaters" Proc. 11th Conf. Coast. Eng. ASCE, 954-973.