

CHAPTER 71

Statistics of wave group parameters

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Abstract

The statistical properties of four spectral shape parameters (Q_p , Q_e , κ and ν) and the correlation coefficient between succeeding wave heights (ρ_{HH}) are investigated using field data and numerically simulated data. The effects of spectral smoothing, integration range and duration of the data record on estimates of these parameters are discussed. The relation between spectral shape and wave grouping is discussed in relation to Kimura's theory for group length statistics. A group length distribution independent method for computing this mean group length is introduced. Further, a comparison is made between time and frequency domain estimates of the correlation coefficient between successive wave heights. Observed discrepancies between them are analyzed and an improved method for the spectral computation of this coefficient is suggested.

1 Introduction

The statistical analysis of random wave groups has received much attention in the last years. These studies can be divided into analyses in terms of individual wave heights or in terms of wave envelopes. This paper concentrates on wave group analysis in terms of individual waves, where a wave group is defined as a sequence of waves all exceed a certain height. The most successful model for the statistical description of group lengths has been given by Kimura (1980). Principal parameter in this model is the correlation between successive wave heights. Various parameters have been developed to relate wave group length statistics to the spectral shape, e.g. the well known peakedness parameter Q_p , introduced by Goda

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(1970), or the κ parameter, used by Battjes and Van Vledder (1984). This κ parameter links the spectral width via the correlation coefficient ρ_{HH} between successive wave heights to group length statistics. Other measures for the spectral width are the parameter ν , introduced by Longuet-Higgins (1975), or Q_e , introduced by Medina and Hudspeth (1987).

Most of these parameters lack a theoretical basis linking wave group statistics and spectral width. Only the κ parameter has such a basis, although it underestimates group lengths. Medina and Hudspeth (1990) have theoretically analyzed the relation between the spectral shape parameters Q_p , Q_e , κ and the correlation coefficient between successive wave heights ρ_{HH} . They used a three-axes representation to show that these parameters are interrelated. They argue, that because of this interrelationship, only one of these parameters is required in order to evaluate wave groupiness. Although they note the possible effect of statistical variability on the estimates of these parameters, they do not pursue the consequences of this variability on the interrelationship between these parameters.

The purpose of this paper is to analyze the statistical properties of four spectral shape parameters (Q_p , Q_e , ν and κ) and their usefulness in relation to wave grouping. Also, the effects of spectral smoothing, sensitivity to integration range and duration of the underlying wave record on estimates of these parameters are investigated. Finally, assumptions in the spectral computation of this coefficient are reviewed and improved where possible.

2 Wave group analysis in terms of individual waves

In this paper wave groups are defined in terms of individual zero-up crossing waves. A wave group is defined as a sequence of succeeding waves with heights that all exceed a preset threshold level (e.g. the mean wave height). The length of the wave group is equal to the number of waves in a group. The mean group length in a wave record is considered as the measure for the amount of wave grouping.

Models for the probability distribution of group lengths have been given by Goda (1970) and Kimura (1980). The model of Goda underestimates group lengths since it neglects the correlation between succeeding wave heights. As was shown by Rye (1974) and others, consecutive wave heights are positively correlated. This correlation is quantified by means of the coefficient of linear correlation:

$$\rho_{HH,t} = \frac{1}{\sigma_H^2} \frac{1}{N-1} \sum_{i=1}^{N-1} (H_i - H_m)(H_{i+1} - H_m) \quad (2.1)$$

in which σ_H is the standard deviation and H_m the mean wave height and N the number of waves in a record. The subscript t refers to time domain.

These correlations are considered in the model of Kimura (1980), in which it is assumed that succeeding wave heights form a Markov-chain. To compute the probability of a sequence of high waves with a certain length, Kimura used the conditional probability p_{22} that a wave height exceeds the threshold level, H_c , given that the previous wave also exceeds H_c :

$$p_{22} = \text{Prob} \{ H_{i+1} > H_c \mid H_i > H_c \} \quad (2.2)$$

The group length distribution function is:

$$P_1(j) = (1 - p_{22}) p_{22}^{j-1} \quad (2.3)$$

The mean group length can be computed as:

$$\bar{j}_1 = E\{j\} = \sum_{j=1}^{\infty} j P_1(j) = \frac{1}{1 - p_{22}} \quad (2.4)$$

and the standard deviation of group length can be computed as:

$$\sigma_1(j) = \{E\{j^2\} - E\{j\}^2\}^{1/2} = \frac{\sqrt{p_{22}}}{1 - p_{22}} \quad (2.5)$$

The probability p_{22} is computed from the joint probability density function $p(H_1, H_2)$ of succeeding wave heights:

$$p_{22} = \int_{H_c}^{\infty} \int_{H_c}^{\infty} p(H_1, H_2) dH_1 dH_2 / \int_{H_c}^{\infty} \int_0^{\infty} p(H_1, H_2) dH_1 dH_2 \quad (2.6)$$

where $p(H_1, H_2)$ is the bi-variate Rayleigh distribution:

$$p(H_1, H_2) = \frac{\pi^2}{4} \frac{H_1 H_2}{H_m^4 (1 - \kappa^2)} \exp \left(-\frac{\pi}{4} \frac{H_1^2 + H_2^2}{H_m^2 (1 - \kappa^2)} \right) I_0 \left(\frac{\pi}{2} \frac{\kappa}{(1 - \kappa^2)} \frac{H_1 H_2}{H_m^2} \right) \quad (2.7)$$

In Eq. (2.7) κ is a correlation parameter, H_m the mean wave height, and I_0 the modified Bessel function of zeroth order. The relation between the correlation parameter κ and the coefficient of linear correlation is given by:

$$\rho_{HH} = \frac{E(\kappa) - \frac{1}{2}(1 - \kappa^2)K(\kappa) - \pi/4}{1 - \pi/4} \quad (2.8)$$

in which K and E are the complete elliptic integrals of the first and second kind, respectively. An accurate approximation of Eq. (2.8) has been given by Battjes (1974):

$$\rho_{HH} \approx \frac{\pi}{16 - 4\pi} \left(\kappa^2 + \frac{\kappa^4}{16} + \frac{\kappa^6}{64} \right) \quad (2.9)$$

The truncation error involved in approximation (2.9) is less than 0.1 % for $0 \leq \kappa < 0.7$ and less than 1 % for $0.7 \leq \kappa < 0.95$.

The model of Kimura has been verified against field measurements by Goda (1983), Battjes and Van Vledder (1984), and others. The present study also supports Kimura's model for predicting the mean group length. To that end, numerically simulated data have been used, see section 7, Fig. 1, panel a).

3 The mean group length

The standard method of deriving the mean group length $\bar{j}_1$ is via Eq. (2.4) on the basis of the theoretical group length distribution (2.3). The mean group length, however, is independent of this group length distribution (Van Vledder, 1983).

Consider a wave record with N_w waves of which N_h waves are higher than the threshold level H_c . Further, the wave record contains N_g groups of high waves. The mean group length $\bar{j}_1$ can then be computed as:

$$\bar{j}_1 = \frac{N_g}{N_h} \quad (3.1)$$

The end of each wave group can be identified as a sequence of a high wave followed by a low wave. Consequently, the number of wave groups is given by:

$$N_g = N_w \times \text{Prob}\{H_i > H_c \wedge H_{i+1} \leq H_c\} \quad (3.2)$$

The number of high waves in the wave record is given by

$$N_h = N_w \times \text{Prob}\{H_i > H_c\} \quad (3.3)$$

Thus, the mean group length $\bar{j}_1$ can be computed as:

$$\left. \begin{aligned} \bar{j}_1 &= \frac{\text{Prob}\{H_i > H_c\}}{\text{Prob}\{H_i > H_c \wedge H_{i+1} \leq H_c\}} \\ &= \frac{1}{1 - \text{Prob}\{H_{i+1} > H_c \mid H_i > H_c\}} \end{aligned} \right\} \quad (3.4)$$

which by virtue of Eq. (2.2) is equal to expression (2.4). This result implies that the mean group length $\bar{j}_1$ is directly related to the correlation coefficient between successive wave heights $\rho_{HH,i}$, through the Eqs. (2.6), (2.7) and (2.8). It also implies that the mean group length does not depend on correlations between non-successive wave heights.

4 Spectral shape parameters

It is well known that the spectral width is related to the amount of wave grouping. Sea states with narrow spectra show a higher amount of wave grouping than those with broad spectra (Rye, 1974, and others). Below, four spectral width parameters (Q_p , Q_e , ν and κ) are described that have been suggested in relation to wave grouping. The Q_p parameter has been introduced by Goda (1970) as a measure for the peakedness of the wave spectrum. It is defined as:

$$Q_p = \frac{2}{m_0^2} \int_0^{\infty} f S(f)^2 df \quad (4.1)$$

in which $S(f)$ the frequency spectrum and m_0 its zeroth moment. The Q_p parameter is frequently used by many authors in relation to the amount of wave grouping of wind waves. Recently, Medina and Hudspeth (1987) proposed the spectral peakedness parameter Q_e , similar to Goda's peakedness parameter, it is defined as:

$$Q_e = \frac{2m_1}{m_0^3} \int_0^{\infty} S(f)^2 df \quad (4.2)$$

with m_0 and m_1 the zeroth and first spectral moment of $S(f)$, respectively. Another spectral width parameter was introduced by Longuet-Higgins (1975),

$$\nu = (m_0 m_2 / m_1^2 - 1)^{\frac{1}{2}} \quad (4.3)$$

and applied by Longuet-Higgins (1984) and Chandler and Masson (1992) to wave group statistics.

Above three parameters have been proposed on intuitive grounds rather than on theoretical ones. A fundamental approach to relate the spectral shape with the amount of wave grouping is based on Rice's (1944) theoretical results on envelope statistics. Rice (1944) has derived the joint probability of two values R_1 and R_2 of the wave envelope $R(t)$ for a narrow-banded Gaussian process, separated by a time lag τ . This distribution is the bi-variate Rayleigh distribution, given by Eq. (2.7), but with the parameters H_1 , H_2 and H_m replaced by R_1 , R_2 and R_m , respectively. This bi-variate Rayleigh distribution contains a correlation parameter that depends on the lag τ and the spectral shape. The definition for this parameter has been rewritten by Battjes (1974) as:

$$\kappa^2(\tau) m_0^2 = \left[\int_0^{\infty} S(f) \cos(2\pi f\tau) df \right]^2 + \left[\int_0^{\infty} S(f) \sin(2\pi f\tau) df \right]^2 \quad (4.4)$$

For narrow spectra, Rice's result can be used to derive the joint distribution of two consecutive wave heights H_1 and H_2 by substituting $H_1 = 2R_1$ and $H_2 = 2R_2$, and using

$\tau = T_m$, where T_m is the mean wave period that can be computed from the wave spectrum (Arhan and Ezraty, 1978):

$$T_m = T_{m02} = \sqrt{m_0/m_2}. \quad (4.5)$$

Using relation (2.8) a frequency domain estimate of the correlation coefficient between successive wave heights can be obtained. Such an estimate is denoted by $\rho_{HH,f}(\tau)$.

5 Wave data and analysis

Field data were collected in the North Sea using a Waverider buoy in swell and wave growth situations. These data consist of 33 wave records and include some JONSWAP data as well as data from the severe storm of January 3, 1978 (Bouws, 1979). The wave records consist of time series of surface elevation (sampling rate 2 Hz) with a duration of approximately 20 minutes.

The random Fourier coefficient method (Tucker et al., 1984) was applied to generate relatively long time series of sea surface elevation. In this method, the sea surface $\eta(t)$ consists of N values sampled at discrete times t_m with intervals Δt :

$$\eta(t_m) = \sum_{n=0}^{N/2} \left\{ a_n \cos(2\pi f_n t_m) + b_n \sin(2\pi f_n t_m) \right\} \quad (5.1)$$

in which $f_n = n/(N\Delta t)$ and where the random Fourier coefficients, a_n and b_n , each are independent variables taken from a normal distribution with zero mean and variance $S(f_n)\Delta f$ with $S(f)$ the frequency spectrum. An inverse Fourier transform of the set of coefficients a_n and b_n then leads to the desired time series.

A total of 161 time series were generated, each with a time step of 0.5 s and a duration of 2 hours and 16 minutes. A JONSWAP spectrum was used to compute the random Fourier coefficients. The peak enhancement factor γ varied from 1.0 to 20 with a step of 0.125, and the peak period was 5.0 s. Typically, each wave record contained 2000 individual waves.

6 Sampling properties

It is well known that raw (unsmoothed) estimates of the spectral density $S(f)$, based on a single record, have a relatively large sampling variability. This is generally reduced by applying some smoothing at the expense of resolution. The four spectral parameters, considered here, all depend on integrals over the entire spectrum and therefore have a relatively small sampling variability (random error), regardless of the degree of smoothing. The same is true for the bias in the estimates of ν and κ , because $S(f)$ appears linearly in the integrals. For κ this is also because the cosine and sine terms in (4.4) vary slowly

compared to the unsmoothed estimate of $S(f)$. The parameters Q_p and Q_e , however, are proportional to an integral of the square of $S(f)$. Therefore, sampling variability in $S(f)$ causes a positive bias in estimates of Q_p and Q_e .

As shown by Elgar et al. (1984), the expected value of Q_p is given by

$$E\{Q_p\} = Q_p(1 + 1/n) \quad (6.1)$$

in which n is the effective number of frequency bands. A similar analysis has been performed for the Q_e parameter, with similar result. The parameters κ and ν are not affected by any smoothing of the spectrum, because they depend linearly on $S(f)$. For smoothly varying spectra (such as analytically expressed spectra), Q_p and Q_e are measures of peakedness in the sense of concentration of energy near a single frequency (which is the conventional interpretation), but for estimated spectra it is just as much a measure of all local peaks and thus of spectral roughness due to sampling variability. Therefore, the parameters Q_p and Q_e are no suitable spectral width parameters, and not even useful in relation to wave grouping (Van Vledder and Battjes, 1992).

The sampling properties of above parameters are given in Table 1, based on the analysis of a typical North Sea wind wave record.

n	Q_p	Q_e	κ	ν
1	6.23	7.17	0.622	0.346
3	4.02	4.59	0.620	0.347
5	3.50	3.97	0.624	0.346
7	3.57	4.04	0.620	0.344
9	3.33	3.82	0.609	0.351

Table 1 Computed values of Q_p , Q_e , κ and ν as a function of the number n of raw frequency bands used in the spectral smoothing.

The results confirm that estimates of the parameters ν and κ are practically free of bias, whereas estimates of the parameters Q_p and Q_e are strongly biased. The results for the latter two parameters are nearly proportional to $(1 + 1/n)$, which is in agreement with the theoretical result of Elgar et al. (1984). The dependence of estimates of Q_p and Q_e on the amount of smoothing makes them unsuited as measure for spectral width, especially when the amount of smoothing is not known.

7 Effect of time series duration

Nelson (1987) and Medina and Hudspeth (1990) argue that long data records are required in order to reduce the variability in the estimates of wave group parameters to an acceptable level, since the standard deviation of group lengths is of the same order as their mean. Nelson (1987) recommends to use data records of at least 2 hours duration. Since such long data records are difficult to collect, time series of sufficient duration were generated numerically.

The effect of record duration on the variability of group length statistics was analyzed by using 153 simulated time series of 2 hours and 16 min duration, and by using 153 short time series with a duration of approximately 20 minutes. The results are shown in Fig. 1. Shown are the mean group lengths $\bar{j}_1$ as a function of $\rho_{HH,t}$, together with the relation according to Kimura's theory.

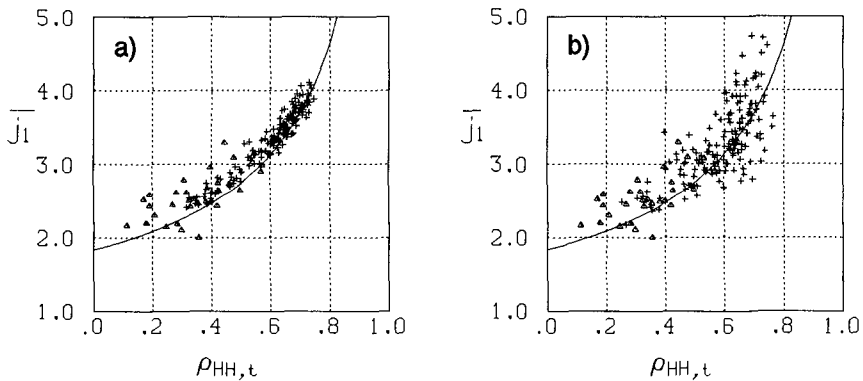


Fig. 1 Observed mean group lengths $\bar{j}_1$ as a function of the correlation between successive wave heights $\rho_{HH,t}$. Triangles (field data), crosses (simulated data), solid line (relation 2.8). Panel a), simulated time series of 2 hour and 16 minutes duration, panel b) simulated time series of 18 minute duration.

These results show that long time series of surface elevation reduce the variability in estimates of the mean group length and correlation coefficient to an acceptable level (i.e. the data points cluster around the theoretical line). The results shown in panel b), are both based on time series of approximately 20 minute duration. As can be seen, the variability around the theoretical line is of the same order, both for field data and simulated data.

8 Effect of varying upper integration limit

The effect of changing the upper integration limit on the estimates of above four spectral parameters is shown in Fig. 2. Based on a simulated JONSWAP spectrum, the dimensionless upper integration limit f_{up}/f_p was varied over the range 1 to 4.

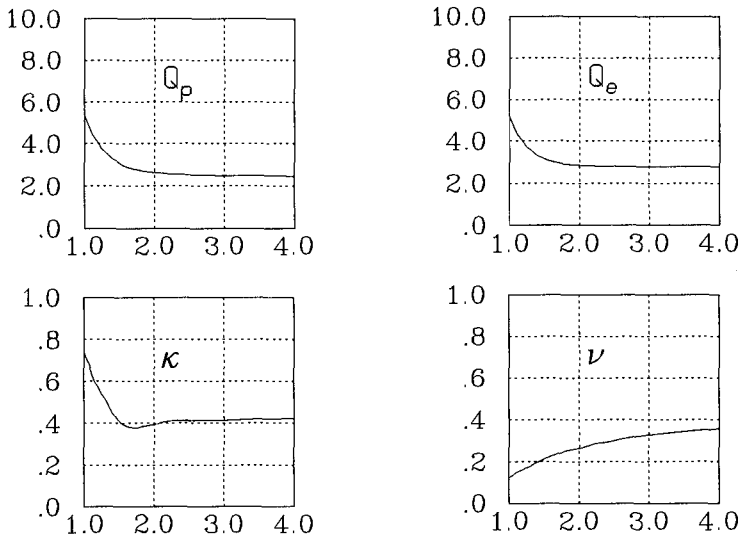


Fig. 2: Variation in the estimates of Q_p , Q_e , κ and ν as a function of the dimensionless upper integration limit f_{up}/f_p .

The results indicate that for $f_{up}/f_p > 2$ the κ parameters is converged to a limiting value, whereas for $f_{up}/f_p > 3$ the Q_p and Q_e are converged to their final value. The ν parameter, however, is still increasing for $f_{up}/f_p = 4$. These results indicate that the κ is least sensitive to the choice of the upper integration limit.

9 Spectral computation of correlation coefficient

As noted by Battjes and Van Vledder (1984), IAHR (1992), Chandler and Masson (1992), the spectrally computed coefficient of correlation between successive wave heights $\rho_{HH,f}(T_{m02})$ is consistently smaller than its time domain estimate $\rho_{HH,t}$. This is illustrated in Fig. 3. Possible reasons for this underestimation have been considered by Stam (1988),

who identified 3 assumptions used in the derivation of the joint distribution of succeeding wave heights:

- 1 The underlying stochastic process is Gaussian,
- 2 the frequency spectrum is narrow, and
- 3 the joint distribution of two values of the amplitude envelope $R(t)$ and $R(t+\tau)$ is translated into the joint distribution of succeeding wave heights by defining succeeding wave heights as twice the values of $R(t)$ and $R(t+T_{m02})$, respectively.

The first, Gaussian, assumption implies that the sea surface can be considered as a linear sum of mutually independent harmonic components. Possible non-linearities would increase the difference between the time and frequency domain estimates of the correlation coefficient. A possible effect of non-linearities has been investigated by Stam (1988) by analyzing wave flume experiments with different values of the ratio of water depth d over the deep water wave length L_0 . These investigations indicate that non-linearities have a negligible effect. Thus, the linear assumption is not inconsistent with Stam's experiments. The linear assumption is also supported in the literature (e.g. Elgar et al., 1984; Chandler and Masson, 1992).

The second assumption is related to the existence of a well defined envelope. As argued by Battjes (1974), the assumption of a narrow spectrum is not necessary for the validity of the bi-variate Rayleigh distribution. The validity of this distribution was verified by computing the κ parameter directly in the time domain and comparing it with $\rho_{HH,t}$. As noted by Battjes (1974), κ^2 is equal to the coefficient of linear correlation between squared succeeding wave heights:

$$\kappa_{HH,t}^2 = \frac{\frac{1}{N-1} \sum_{i=1}^{N-1} (H_i^2 - \bar{H}^2) (H_{i+1}^2 - \bar{H}^2)}{\frac{1}{N} \sum_{i=1}^N (H_i^2 - \bar{H}^2)^2} \quad (9.1)$$

The relation between $\rho_{HH,t}$ and $\kappa_{HH,t}$ is illustrated in Fig. 4, together with the theoretical relation (2.8). The agreement is good, which supports the validity of the bi-variate Rayleigh distribution.

The third assumption, a wave height is twice the amplitude at the time of a wave crest, is only valid for narrow spectra. For broader spectra, the use of wave envelope can under-estimates, as well as over-estimate computed wave heights. This will affect the correlation between succeeding wave heights in case wave heights are based on wave amplitude values. Following Stam (1988), this was inspected by computing the correlation between succeeding wave crests (or maximum amplitudes). To that end the correlation coefficient between succeeding wave crests (or maximum amplitudes) $\rho_{AA,t}$ was computed:

$$\rho_{AA,t} = \frac{\frac{1}{N-1} \sum_{i=1}^{N-1} (A_i - \bar{A})(A_{i+1} - \bar{A})}{\frac{1}{N} \sum_{i=1}^N (A_i - \bar{A})^2} \quad (9.2)$$

in which $\bar{A}$ is the mean crest elevation. Estimates of this correlation coefficient have been compared with time domain estimates of the κ parameter, defined similarly as $\kappa_{HH,t}$, but now in terms of wave amplitudes:

$$\kappa_{AA,t}^2 = \frac{\frac{1}{N-1} \sum_{i=1}^{N-1} (A_i^2 - \bar{A}^2)(A_{i+1}^2 - \bar{A}^2)}{\frac{1}{N} \sum_{i=1}^N (A_i^2 - \bar{A}^2)^2} \quad (9.3)$$

Inspection of the relation between $\rho_{AA,t}$ and $\kappa_{AA,t}$ (not shown here) gives an even better agreement with theory (Eq. 2.8) than between $\kappa_{HH,t}$ and $\rho_{HH,t}$. These results suggest that the bi-variate Rayleigh distribution is better suited to describe the joint distribution of succeeding wave amplitudes than of succeeding wave heights. Based on this notion, the relation between $\rho_f(T_{m02})$ and $\kappa_{AA,t}$ was investigated. The result thereof is shown in Fig. 5.

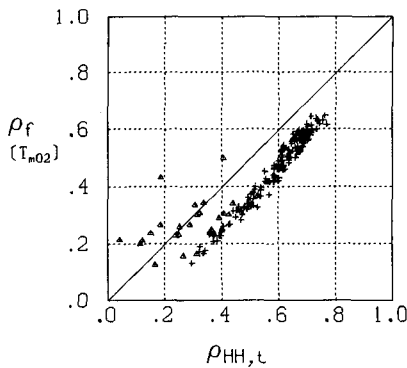


Fig. 3: Relation between $\kappa_{HH,t}$ and $\rho_{HH,t}$. Crosses (simulation data), triangles (field data).

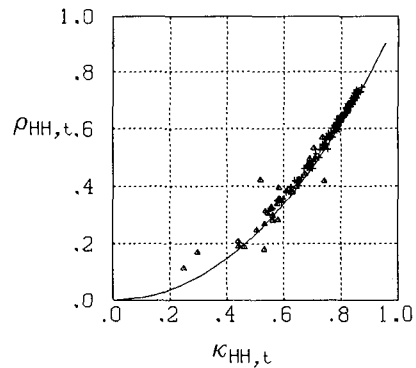


Fig. 4: Relation between $\kappa_{AA,t}$ and $\rho_f(T_{m02})$. Crosses (simulation data), triangles (field data),

solid line (relation 2.8).

Fig. 4 shows good agreement between $\kappa_{HH,t}$ and $\rho_{HH,t}$ according to theory. The agreement between $\kappa_{AA,t}$ and $\kappa_f(T_{m02})$ is rather good which implies that theory of envelope statistics is applicable to wave heights, but not to wave heights. The difference between $\kappa_{HH,t}$ and

$\kappa_f(T_{m02})$ can be attributed to the transformation of amplitudes to wave heights by a factor 2.

The assumption that the wave height is twice the value of the amplitude envelope at the time of a wave crest was considered by Tayfun (1990). An improved estimate of the wave height is to define a wave height as the sum of two values of the wave envelope separated by a time lag of half a wave period T :

$$H = A(t) + A\left(t + \frac{1}{2}T\right) \quad (9.4)$$

Substitution of Eqs. (9.4) into the definition of the correlation coefficient gives (Van Leeuwen, 1988):

$$\hat{\rho}_{HH} = \frac{\text{cov}\left(A(t_i) + A(t_i + \frac{1}{2}T), A(t_i + T) + A(t_i + \frac{3}{2}T)\right)}{\sigma\left(A(t_i) + A(t_i + \frac{1}{2}T)\right)\sigma\left(A(t_i + T) + A(t_i + \frac{3}{2}T)\right)} \quad (9.5)$$

Elaboration of Eq. (9.5) leads to:

$$\hat{\rho}_{HH,t} = \frac{\rho_{HH,f}(\frac{1}{2}T) + 2\rho_{HH,f}(T) + \rho_{HH,f}(\frac{3}{2}T)}{2 + 2\rho_{HH,f}(\frac{1}{2}T)} \quad (9.6)$$

In addition, the effect of finite bandwidth on the mean zero-crossing wave period is considered. Such a correction was given by Tayfun (1990):

$$\hat{T} = T_{m02}\left(1 - \frac{1}{2}v^2\right) \quad (9.7)$$

with v the spectral width parameter proposed by Longuet-Higgins (1975). Replacing the wave period T in Eq. (9.6) by the corrected expression of Eq. (9.7) gives:

$$\hat{\rho}_{HH,t} = \frac{\rho_{HH,f}(\frac{1}{2}\hat{T}) + 2\rho_{HH,f}(\hat{T}) + \rho_{HH,f}(\frac{3}{2}\hat{T})}{2 + 2\rho_{HH,f}(\frac{1}{2}\hat{T})} \quad (9.8)$$

The results of the improved method of computing the correlation coefficient between successive wave heights from the spectrum are shown in Fig. 6.

The effect of these corrections is to remove almost all of the bias in the spectral computation of the correlation coefficient between succeeding wave heights.

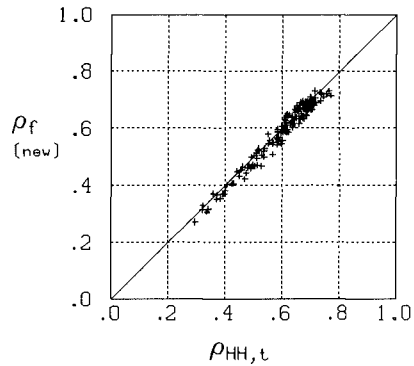
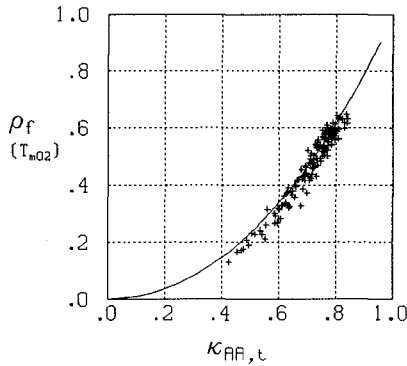


Fig. 5 Relation between $\kappa_{AA,t}$ and $\rho_f(T_{m02})$ Fig. 6: Relation between $\rho_{HH,t}$ and $\hat{\rho}_f$

10 Conclusions

- the mean group length is independent of the group length distribution.
- statistical variability distorts the inter-relationship between the parameters Q_p , Q_e , κ and $\rho_{HH,t}$, as proposed by Medina and Hudspeth (1990).
- the Q_p , Q_e and ν parameters are not suited as spectral group parameters.
- the κ parameter is a good parameter to relate wave grouping with the spectral shape.
- the Q_p and Q_e parameter are not suited as measures for the spectral width.
- long data records are necessary to decrease the variability in group length statistics to an acceptable level.
- the κ parameter is not sensitive to spectral smoothing, and least sensitive to the choice of the upper integration limit.
- the improved method of computing the spectral correlation coefficient removes practically all bias with respect to previous computations. This improved method enhance the applicability of the κ parameter in relation to wave grouping.

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