CHAPTER 61

LOW FREQUENCY WAVES IN THE SURF ZONE

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ABSTRACT

The generation of low-frequency waves (LFW, also known as 'infragravity waves') within a two-dimensional surf zone is investigated numerically using a short-wave resolving model. In this simplified model, based on the nonlinear shallow-water equations, breaking waves are represented by 'bores', at which there are jumps in both water depth and velocity. Some idealized trains of modulated waves are then used to investigate how LFW may be generated by forcing within the surf zone, as opposed to the mechanisms of bound wave reflection and moving break point forcing. In this way, the process of LFW generation may be examined in some detail. The model is also compared with some measurements of irregular waves in a flume: good agreement is obtained.

INTRODUCTION

Low-frequency waves (LFW) are generated by the transfer of energy from modulated high-frequency waves (short waves) when they propagate into shallow water near the shore (Hamm et al., 1993). The energy transfer may be thought of as being brought

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about by the variations in short-wave momentum flux (radiation stress) as the short waves of varying amplitude propagate over changes of water depth and break (Longuet-Higgins & Stewart, 1964). Past work on modelling LFW generation has used linear theory for the short waves and their radiation stresses. Empirical assumptions about wave amplitudes within the surf zone are used to estimate the radiation stress forces (Gallagher, 1971; Symonds et al., 1982; Nakaza & Hino, 1991; Schäffer, 1993; List, 1992; Roelvink et al., 1992).

Three particularly significant aspects of the generation process have been discussed:

(1) "Bound" LFW are generated with the short-wave groups and these grow as they propagate shorewards (Longuet-Higgins & Stewart, 1962; Agnon, 1993). The bound waves are released to propagate freely when the short waves lose their energy by breaking, or when they propagate over depth changes such as bars.

(2) Modulated short waves break in different depths. The radiation stress gradient is negative to shoreward of the break point and positive to seaward. LFW are generated as the break point moves (Symonds et al., 1982; Schäffer, 1993).

(3) Within the surf zone, the wave set-up fluctuates in response to fluctuations in incident wave amplitude. This rising and falling mass of water at the shoreline generates LFW. If the surf zone is saturated this effect is directly related to (2) but in general, modulations will penetrate into the surf zone and cause a complex time-varying radiation stress field (List, 1991).

These investigations have suffered from the disadvantage that questionable assumptions are made about the validity of linear theory for the propagation and radiation stresses of breaking waves within the surf zone. An alternative to this 'wave-averaged' approach is to use short-wave-resolving models to study the generation processes in more detail, without the need for such assumptions. Here we report studies of LFW generation using the nonlinear shallow-water equations. These are particularly appropriate in thc inner surf zone, where the waves have formed into turbulent bores (Packwood, 1980) and they have been proved adequate for modelling breakers on a shallow beach (below, also Cox et al., 1992). It is in the surf zone that LFW have their largest amplitudes and the above generation mechanisms act. Other nonlinear equations such as the Boussinesq equations are only valid for non-breaking waves (although recent efforts have been made to extend their validity into the surf zone: see Schäffer et al., 1992). The shallow-water equations are thus best suited to the study of mechanism (3) above, and it is this which is discussed below.

Initially, our attention has been confined to one horizontal dimension and a plane beach. The situation is thus simplified by the exclusion of edge waves and longshore currents. Bottom friction has been neglected, because it introduces an unnecessary empirical element into the model. Previous work has suggested that it has no qualitative effects, and that quantitative differences are mainly important in the very shallow swash zone (Packwood, 1980). Beach porosity has also been neglected for simplicity.

MATHEMATICAL MODEL

The motion of a shallow layer of water, if the length scale of the motion is much greater than the water depth, may be described by the shallow-water equations for the conservation of mass and momentum,

$$d_t + (ud)_x = 0 \tag{1}$$

$$u_t + uu_x + g(d-h)_x = 0 \tag{2}$$

where u is the depth-averaged flow velocity, d the water depth, g the acceleration due to gravity, $h(x)$ the undisturbed water depth and $-h_x$ the local bottom slope (assumed small). Subscripts indicate differentiation. The surface elevation is $\eta = d-h$. The variables are illustrated in figure 1.

In appropriate conservation form, the equations are:

$$\frac{\partial}{\partial t}\begin{pmatrix} d \\ ud \end{pmatrix} + \frac{\partial}{\partial x}\begin{pmatrix} ud \\ u^2 d + \frac{1}{2}gd^2 \end{pmatrix} = \begin{pmatrix} 0 \\ gdh_x \end{pmatrix} \tag{3}$$

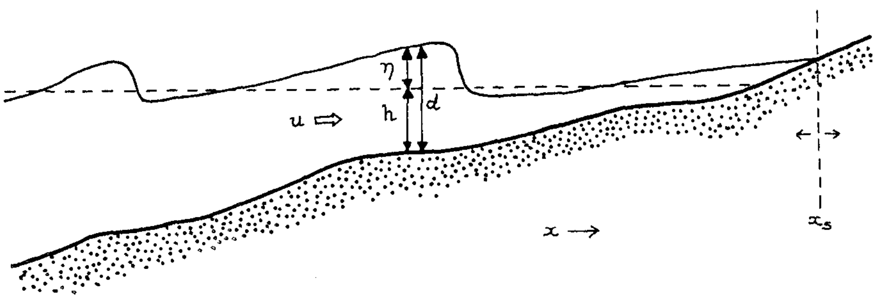

Figure 1: Sketch showing variables referred to in the text.

$$\text{or,}\quad \underline{U}_t + \underline{F}_x = \underline{S}. \tag{4}$$

$\underline{U}$ is a vector of the conserved quantities mass and momentum, $\underline{F}$ is the flux of those quantities and $\underline{S}$ is a source term due to the bottom slope. These nonlinear, hyperbolic partial differential equations admit the development and propagation of discontinuities which represent bores at which both mass and momentum are conserved. These fundamental conservation properties are important to ensure that the basic physics is correct. They mean that no empirical terms are required, even for wave breaking, although this does not give a detailed model. The conservation of mass and momentum at a bore may be demonstrated mathematically by integration of eq. (4) to yield the hydraulic jump relations.

A useful way in which to write eqs. (1) and (2) is in the characteristic form. This is obtained by making the substitution $c^2 = gd$ (c is the local long-wave speed), rearranging, and expressing in terms of time derivatives along certain trajectories:

$$\frac{d}{dt}(u+2c) = gh_x \quad \text{on} \quad \frac{dx}{dt} = u+c \tag{5}$$

$$\frac{d}{dt}(u-2c) = gh_x \quad \text{on} \quad \frac{dx}{dt} = u-c. \tag{6}$$

In terms of the Riemann invariants, $R^+ = u+2c$ and $R^- = u-2c$, these are

$$R^+{}_t + (u+c)R^+{}_x = gh_x \tag{7}$$

$$R^-{}_t + (u-c)R^-{}_x = gh_x. \tag{8}$$

Eqs. (5) and (6), or (7) and (8), indicate that the quantities R^+ and R^- propagate along the characteristics at speeds $u+c$ and $u-c$ respectively, changing at a rate gh_x as they do so. $u+c$ and $u-c$ are equal to the local long-wave speeds of shoreward- and seaward-propagating waves respectively, advected by the local flow velocity u. R^+ and R^- thus specify the shoreward- and seaward-propagating waves, respectively, a fact which is very useful in analysing results. At bores, there are jumps in R^+ and R^- (Peregrine, 1974).

The boundary conditions to be satisfied are as follows. The shoreline boundary conditions are that the water depth becomes zero and its position $x_s(t)$ moves such that it has the same velocity as the water:

$$d(x_s[t], t) = 0, \qquad u(x_s[t], t) = \frac{dx_s}{dt}. \tag{9}$$

The appropriate seaward boundary conditions depend on the particular situation being studied. For wave modelling on a real beach, we need to prescribe the incident waves and yet permit outgoing waves to escape without reflection. This is done using the Riemann invariants discussed above. As long as the flow is subcritical ($|u|<c$) at all times, R^- propagates into the domain at speed $u+c$ and R^- propagates out of it at speed $u-c$. R^- at the seaward boundary must thus be computed from the solution just inside, using eq. (8). Incident waves are specified by prescribing $R^+(t)$. In supercritical conditions, both R^+ and R^- would be specified if $u>c$, but neither need be specified if $u<-c$. Supercritical conditions do not occur in the cases studied here.

For wave tank experiments, the correct seaward boundary conditions must be chosen to fit the data that are available. In the case discussed below, the water depth $d(t)$ is set equal to that measured at a wave probe, with R^- computed as before. Unfortunately this permits non-physical reflections at the seaward boundary, which must be borne in mind when interpreting the results. If the entire flume is to be modelled, the mode of generation must also be modelled correctly.

NUMERICAL METHOD

A new numerical scheme, the weighted average flux method, was adopted for the solution of these equations. Invented primarily for aerodynamics, it is a development of currently favoured methods such as Godunov's and Roe's. Toro (1989) introduced the method for a simple advection equation and for the Euler equations of compressible gas dynamics. Toro (1992) applied it to the shallow-water equations for water of uniform depth. Watson, Peregrine & Toro (1992) adapted it for use with a moving shoreline and variable depth.

The method is 'shock-capturing', in that discontinuities (bores) are automatically treated correctly without the need for a special tracking algorithm. Bores are followed very well and for a given accuracy less discretization points are required than with most methods. It is found to be more efficient and robust than previously used methods such as the Lax-Wendroff scheme used by Hibberd &

Peregrine (1979).

The essence of the method consists of solving the initial-value Riemann problem for the shallow-water equations in each cell, with constant data in each half of the cell and a jump at the mid-point. This is done analytically using Riemann invariants and the mass and momentum conditions at bores. In this way the average mass and momentum fluxes in each cell are estimated one half-timestep in advance. A Total Variation Diminishing (TVD) adjustment is then made to the flux average in order to eliminate spurious oscillations near bores. This is done by means of upwinding, using a flux limiter to reweight the flux average (hence the name, 'Weighted Average Flux'). The TVD procedure effectively makes the scheme somewhere between first and second-order, so as to achieve a compromise between accuracy and stability. An explicit finite difference scheme is used for advancing in time. Each time step Δt_n must be less than the time taken for the fastest wave in the solution to propagate one grid point (the 'CFL' condition).

The shoreline boundary conditions (9) are not solved explicitly, but are approximately satisfied in the model. Any negative values of d are reset to zero and a dry-bed Riemann problem is used at the next timestep (Toro, 1990). As the depth becomes very small near the shoreline, large errors would result if the unmodified scheme were used. This is because small errors in the momentum variable ud become large errors in u when divided by a small value of d. In order to avoid such errors, an alternative approximation is used for u wherever the depth is less than a suitable small depth tolerance d_{tol}. To plot the position of the moving shoreline, another small depth d_s is chosen and the position of that depth is plotted.

The seaward boundary conditions were implemented along the lines mentioned above, using a simple finite difference approximation to eq. (8). Note that it is necessary to check whether the flow is in fact subcritical before using this scheme.

RESULTS

Before proceeding with more complicated cases, the numerical scheme was tested against an analytic solution for non-breaking shallow-water motion on a beach (Carrier & Greenspan, 1958). The test showed good agreement, except for a small error in velocity very close to the shoreline.

Some Idealized Illustrations.

Figures 2 and 3 illustrate the generation of a single low-frequency cycle by an idealized wave group. The input $R^+(t)$ consists of fully modulated sinusoidal waves cut off after one group of ten waves. Figure 2 shows a perspective view of the surface elevation solution in space-time. All variables are dimensionless, with the beach slope scaled out of the problem. In this example, a slope of 1/30 and an offshore depth of 1 m would correspond to a wave period of 5 s and a wave height of 0.8 m. The waves steepen into bores as they travel towards the beach, decreasing in amplitude and slowing down as they do so. In the first half of the group, each successively larger wave pushes more water up the beach face. As the wave amplitude decreases in the second half of the group, this water recedes back down the beach. The inertia of the backwash of these waves pulls the shoreline water level down beneath the still water level and it finally rises rather rapidly to its initial level. This rising and falling motion, on the time-scale of the wave group, shows up clearly in the shoreline position (thick line). It generates a low-frequency wave which propagates offshore, and which can just be seen in the latter half of the plot.

In figure 3 the incident and outgoing waves are separated by means of the Riemann invariants. $2c \pm u$ $(-R^{\pm})$ has been plotted rather than $u \pm 2c$ so that higher values always correspond to deeper water. These are plotted at different offshore distances after subtraction of the undisturbed value. At $x = 0.2$ the beach is normally dry, but values become defined when a wave runs up past this position.

The incident invariant shows the waves steepening and decreasing in amplitude as they approach the shore, and the raising of the mean level in the middle of the group. This corresponds to the set-up which is forced by the wave group. It also shows the modulation of the group becoming weaker as the waves saturate. The outgoing invariant shows the almost complete absence of short waves travelling away from the beach, because they have dissipated their energy and are not reflected. An asymmetric low-frequency pulse is seen to propagate away from the beach, decreasing in amplitude as it does so. Note that at x=0, positive elevation is approximately in phase with the peak of the incident group (but this is expected to depend on group length).

The incident wave group used in figure 2 is not very realistic, since such large waves are not in reality sinusoidal. Also, large-amplitude sine waves have a net mass transport associated with them because the water is deeper when the velocity is onshore (wave crest) and shallower when it is offshore (wave trough). This transport will be an increasing function of wave height and thus will cause a LFW to be generated by the group. Although a real wave group will have a mass transport associated with it, it will not necessarily be the same as that of these sine waves, and it may be thought of as being part of the bound wave driven by the group. In this example the LFW may thus be too large: an incident wave group is normally accompanied by a bound wave of depression, whereas ours is not. The bound wave

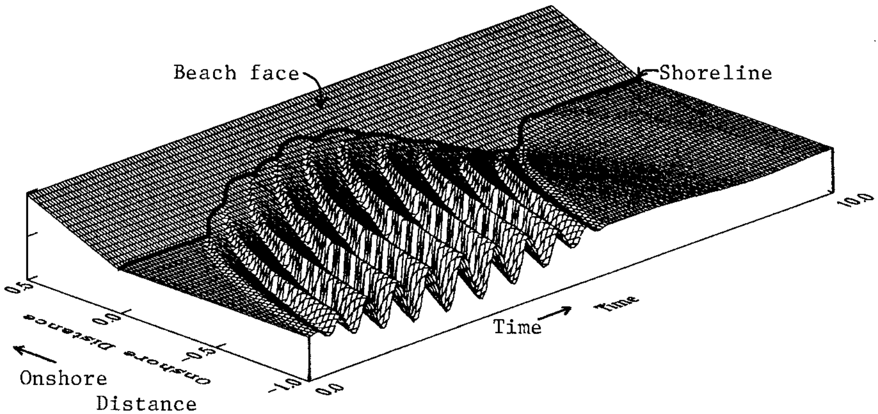

Figure 2: Response to a single wave group. Perspective view of space-time plot of surface elevation, including shoreline motion.

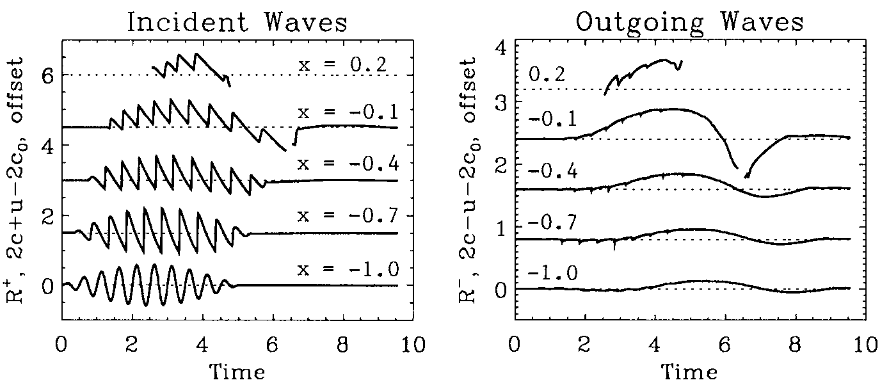

Figure 3: Plots of incident and outgoing wave signals (Riemann Invariants) at various distances offshore, for the waves in figure 2.

will depend on the offshore topography, in a way which is being investigated. A theory for bound waves in moderately shallow water, where the theory of Longuet-Higgins & Stewart (1962) fails, has recently been derived by Agnon (1993).

In the mean time we look to the other extreme and present an example where there is no mass transport associated with the wave group. The wave shape is also modified to the form of a 'sawtooth', representing waves which have already broken. The mass transport in each wave is forced to be zero by choosing appropriate values of the peak and trough water depths. The results from this wave group are shown in figures 4 and 5, which are equivalent to figures 2 and 3 and have the same scales.

In these results also, a similar LF pulse is generated. Its amplitude is about half that in the previous case. The trough of the wave is deeper, indicating that the

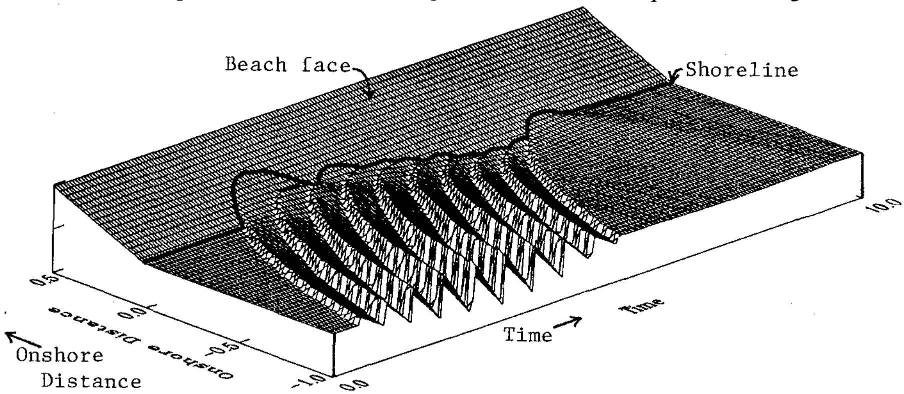

Figure 4: As figure 2, but for a group of breaking waves with no mass transport.

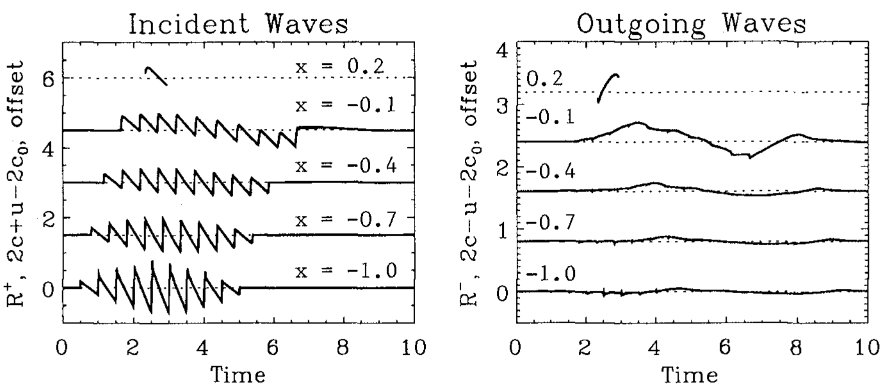

Figure 5: As figure 3, but for a group of breaking waves with no mass transport.

outgoing wave, like the incident wave, carries little mass. The phase of the pulse at x=0 relative to the incident wave goup is also slightly different, with the peak occurring about one fifth of a cycle (70°) earlier and being more sharply defined. Note that this corresponds closely to the different shape of the incident pulse at thestill-water shoreline (x=0). In both cases, the shape of the outgoing pulse is close to the shape of the low-frequency component of the incident group at this position.

We conclude that even for a wave group where the bound wave component is small, a significant LFW will be generated if the groupiness persists inside the surf zone. Note also that in both cases, the amplitude of the LF pulse is such that the propagation velocity of the incident short waves is modified significantly. This interaction is not usually treated in wave-averaged models, nor is the substantial shoreline excursion.

Experiments are under way to verify these results for single wave groups in a wave flume. The effect on the LFW of changes in incident wave amplitude, period and group shape remains a subject for further research.

Comparison with Wave Flume Experiments.

In order to assess the relevance of the model to real waves, comparison is made with data from some wave flume experiments. The measurements are supplementary to those reported by Hansen & Svendsen (1979), and were made in the same flume. The measurements consisted of a series of depth gauges within the surf zone on a slope of 1/34.26. Data from the furthest offshore of these gauges were used to specify the waves at the seaward boundary of the model. Since velocity measurements were not available, the incident Riemann invariant R^+ could not be found exactly at the boundary. Instead, the boundary condition was approximated by setting the depth equal to the measured value, and using the outgoing invariant to compute the velocity. As already noted, this is not ideal.

The result from one such run is given in figure 6. Surface elevation is plotted against time, at each offshore distance where a wave gauge was located. The first wave reaches the offshore probe at about 20s after startup from rest. The measured data are plotted with a solid line and the model result with a dashed line. The two are identical at the furthest station offshore (x=-2.81 m), which was the seaward boundary for the numerical model. As the shore is approached, differences begin to appear between the two.

Except for the two gauges closest to shore, these differences are small and the

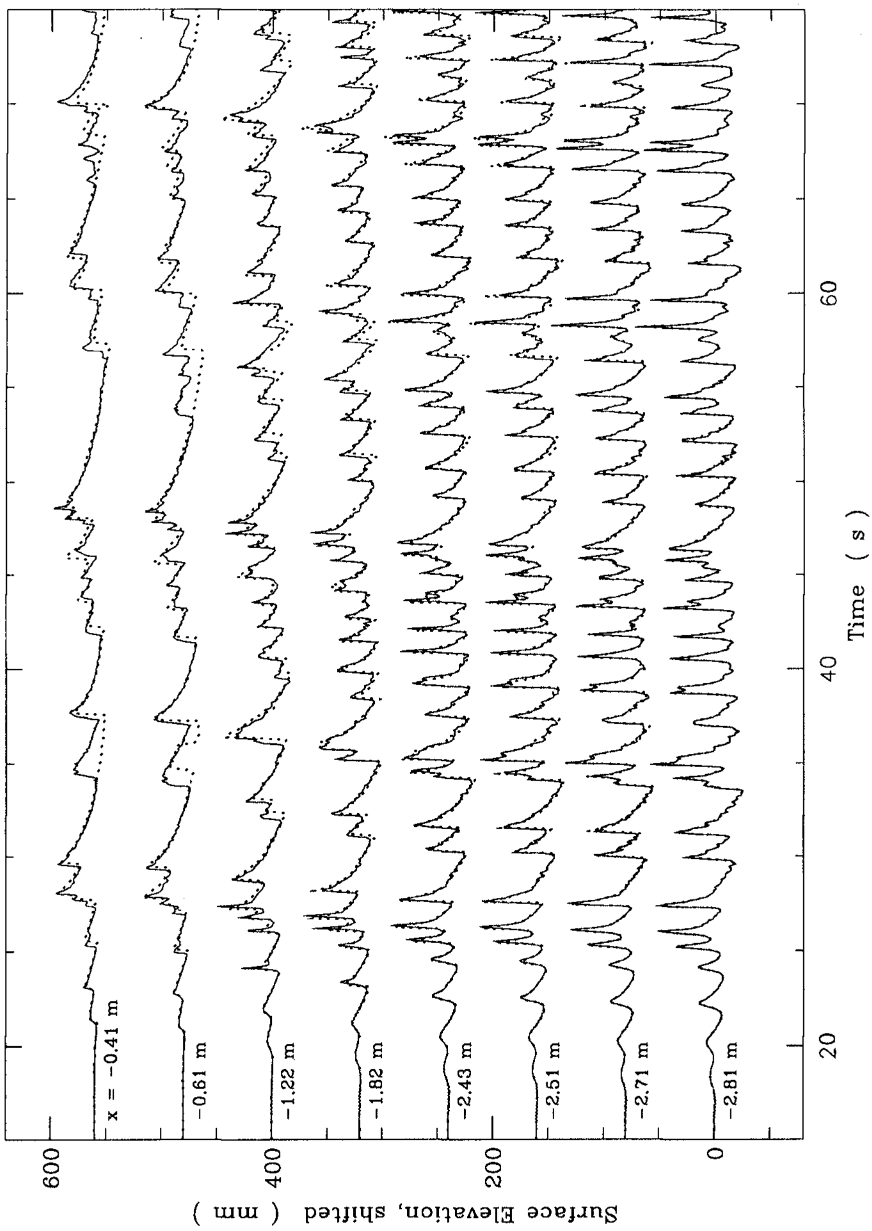

Figure 6: Comparison between measured and modelled surface elevation for irregular waves in a flume, data courtesy of J.B. Hansen and I.A. Svendsen.

agreement is very good. Propagation speeds, and changes in wave amplitude and shape, are reproduced well by the model. This is true even for the small waves at the beginning of the record. The onset of breaking is reproduced well by these equations, as was also found by Packwood (1980). At the two shallowest gauges, larger differences begin to appear. Some waves are missing altogether in the model result. This is because the waves did not reach this location in the experiment. At t=36s in the data from x=-0.61, there is a negative jump in surface elevation which was not present in the data. This results from a bore being forced to move back down the beach by the backwash from the preceding wave. The differences in this very shallow water may be due to the neglect of friction in the model. As in the above example, small changes in the position of a bore relative to a wave gauge may produce a large change in the time series at the gauge.

Despite these differences in high-frequency detail, the low-frequency component of the motion is very well reproduced in the shallow water. This manifests itself as changes in surface elevation on a timescale of about 10 seconds. This confirms that the nonlinear shallow-water equations used by the model contain all the essential components necessary for the LFW generation process to be modelled quite accurately.

CONCLUSIONS

The nonlinear shallow-water equations have been shown to provide a good basis for the modelling of LFW generation in the surf zone. Comparison with wave flume data indicates a good reproduction of the long-wave motion everywhere, and of the short-wave properties except in very shallow water. Runs using idealized wave groups illustrate the process by which LFW are generated by forcing within the surf zone, as distinct from breakpoint forcing or bound wave reflection.

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