

CHAPTER 57

A Method for Estimating Reflection Coefficient in Short-Crested Random Seas

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Abstract

A method for estimating the reflection coefficient is developed for practical uses. In the method, a parametric expression of directional spectra is adopted and the parameters in it are estimated by the maximum likelihood method. The validity of the method is verified by applying it to simulated data. The method is also applied to field data, obtained at Oarai Port, Japan.

1 Introduction

The randomness of sea waves has recently become accounted for in the design of coastal and ocean structures. Directional spectra have often been used to describe multi-directional random sea waves. However, to evaluate the reflection coefficient of structures, a theory for uni-directional random waves has usually been applied with a slight modification. This is because a theory for multi-directional random waves has not yet been established for practical uses.

The purpose of this paper is to derive a method to estimate the reflection coefficient of structures for multi-directional random waves and to examine its validity by applying it to simulated and field data.

In various methods to estimate the directional spectrum, the maximum likelihood method (MLM) is the one which has a high resolution spectrum (Capon,

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Table 1: Definitions of variables in Eq. (1)

$A(\mathbf{k}, \sigma)$	Amplitude (Eq. 1),
$A(d\mathbf{k}, d\sigma)$	Complex amplitude (Eq. 2)
\mathbf{k}	Wave number vector of incident waves
\mathbf{k}_r	Wave number vector of reflected waves
σ	Angular frequency
ϵ	Phase

1969). Isobe and Kondo (1984) proposed the modified maximum likelihood method (MMLM) to estimate the directional spectrum in a combined incident and reflected waves field, taking into account the fact that there is no phase difference between the incident and reflected waves at the reflective wall. Recently for practical purposes, Isobe (1990) proposed a method to estimate the directional spectrum of a standard form in which the spectrum is expressed in terms of a few parameters. In this study, this method is modified to estimate the directional spectrum and reflection coefficient in an incident and reflected wave field.

2 Theory

2.1 Parametric expression of directional spectrum and cross-power spectrum

In a monochromatic wave field which consists of incident and reflected waves, the water surface fluctuation, $\eta(\mathbf{x}, t)$, at the position, \mathbf{x} , are represented by Eq. (1):

$$\eta(\mathbf{x}, t) = A(\mathbf{k}, \sigma) \{ \cos(\mathbf{k}\mathbf{x} - \sigma t + \epsilon) + r \cos(\mathbf{k}_r\mathbf{x} - \sigma t + \epsilon) \} \quad (1)$$

The definitions of the variables in the above equation are given in Table 1. We integrate Eq. (1) with respect to \mathbf{k} and σ , and get the expression of $\eta(\mathbf{x}, t)$ for multi-directional random waves as Eq. (2):

$$\eta(\mathbf{x}, t) = \int_{-\infty}^{\infty} \int_{\mathbf{k}} A(d\mathbf{k}, d\sigma) \{ \exp [i(\mathbf{k}\mathbf{x} - \sigma t + \epsilon)] + r \exp [i(\mathbf{k}_r\mathbf{x} - \sigma t + \epsilon)] \} \quad (2)$$

where complex variables are introduced to represent the amplitude and phase. From Eq. (2), the directional spectrum (wavenumber-frequency spectrum), $S(\mathbf{k}, \sigma)$, is defined as Eq. (3):

$$S(\mathbf{k}, \sigma) d\mathbf{k} d\sigma = \langle A^*(d\mathbf{k}, d\sigma) A(d\mathbf{k}, d\sigma) \rangle \quad (3)$$

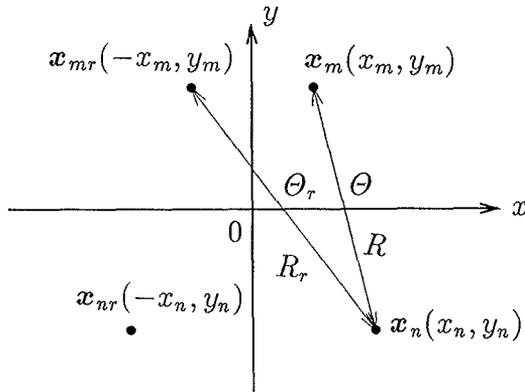


Figure 1: Definitions of variables

where $\langle \rangle$ represents the ensemble average and A^* the complex conjugate of A . Then, the cross-power spectrum, $\Phi_{mn}(\sigma)$, can be defined by Eq. (4) (Horikawa, 1988, Isobe and Kondo, 1984):

$$\Phi_{mn}(\sigma) = \int_{\mathbf{k}} S(\mathbf{k}, \sigma) \{ \exp(i\mathbf{k}\mathbf{x}_m) + r \exp(i\mathbf{k}\mathbf{x}_{mr}) \} \times \{ \exp(-i\mathbf{k}\mathbf{x}_n) + r \exp(-i\mathbf{k}\mathbf{x}_{nr}) \} d\mathbf{k} \tag{4}$$

in which the variables are defined in Fig. 1.

In the present study, the directional spectrum, $S(\mathbf{k}, \sigma)$, is assumed to be expressed by using the directional spreading function proposed by Mitsuyasu *et al.* (1975). Then, $S(\mathbf{k}, \sigma)$ is expressed by Eq. (5):

$$S(\mathbf{k}, \sigma) = P(f) \frac{2^{2s-1} \Gamma^2(s+1)}{\pi \Gamma(2s+1)} \left[\cos \frac{\theta - \theta_0}{2} \right]^{2s} \tag{5}$$

where $P(f)$ is the frequency spectrum or the power spectrum, s the degree of directional concentration, θ_0 the peak wave direction and Γ the Gamma function.

We rewrite Eq. (4) by using the transformation of variables as follows. The definitions of variables are indicated in Fig. 1 as before.

$$\begin{aligned} \mathbf{k} &= (k \cos \theta, k \sin \theta) \\ \mathbf{x}_m - \mathbf{x}_n &= (R \cos \Theta, R \sin \Theta) & (= \mathbf{X}_1) \\ \mathbf{x}_{mr} - \mathbf{x}_{nr} &= (R \cos(\pi - \Theta), R \sin(\pi - \Theta)) & (= \mathbf{X}_2) \\ \mathbf{x}_{mr} - \mathbf{x}_n &= (R_r \cos \Theta_r, R_r \sin \Theta_r) & (= \mathbf{X}_3) \\ \mathbf{x}_m - \mathbf{x}_{nr} &= (R_r \cos(\pi - \Theta_r), R_r \sin(\pi - \Theta_r)) & (= \mathbf{X}_4) \end{aligned}$$

Table 2: Directional spectrum parameters

s	Degree of directional concentration
θ_0	Peak wave direction
r	Reflection coefficient
$P(f)$	Frequency or power spectrum
ε	Ratio of the noise component to the power

Substituting Eq. (5) into Eq. (6) and using the above variables, Then we get the expression of the cross-power spectrum, $\Phi_{mn}(\sigma)$:

$$\Phi_{mn}(\sigma) = P(f) \frac{2^{2s-1} \Gamma^2(s+1)}{\pi \Gamma(2s+1)} \int_{\mathbf{k}} \left[\cos \frac{\theta - \theta_0}{2} \right]^{2s} \times \left\{ \exp(i\mathbf{k}\mathbf{X}_1) + r^2 \exp(i\mathbf{k}\mathbf{X}_2) + r \exp(i\mathbf{k}\mathbf{X}_3) + r \exp(i\mathbf{k}\mathbf{X}_4) \right\} d\mathbf{k} \quad (6)$$

To rewrite Eq. (6) in a simpler form, we define a new function F_j as follows:

$$F_j(k, s, \theta_0, R_j, \Theta_j) = \frac{2^{2s-1} \Gamma^2(s+1)}{\pi \Gamma(2s+1)} \int_{-\pi}^{\pi} \exp [ikR_j \cos(\theta - \Theta_j)] \left[\cos \frac{\theta - \theta_0}{2} \right]^{2s} d\theta \quad (7)$$

By using the above definition, Eq. (6) can be written as Eq. (8):

$$\Phi_{mn}(\sigma) = \begin{bmatrix} F_1(k, s, \theta_0, R, \Theta) \\ +r^2 F_2(k, s, \theta_0, R, \pi - \Theta) \\ +r F_3(k, s, \theta_0, R_r, \Theta_r) \\ +r F_4(k, s, \theta_0, R_r, \pi - \Theta_r) \end{bmatrix} \times P(f) \quad (8)$$

Here the noise component of the power spectrum is assumed to be $\varepsilon P(f)$, ε being the ratio of the noise component to the power. Finally we get the expression of the cross-power spectrum in terms of the five parameters which are summarized in Table 2 and called the directional spectrum parameters in this paper:

$$\Phi_{mn}(\sigma) = \begin{bmatrix} F_1(k, s, \theta_0, R, \Theta) \\ +r^2 F_2(k, s, \theta_0, R, \pi - \Theta) \\ +r F_3(k, s, \theta_0, R_r, \Theta_r) \\ +r F_4(k, s, \theta_0, R_r, \pi - \Theta_r) \end{bmatrix} \times P(f) + \delta_{mn} \varepsilon P(f) \quad (9)$$

The integral on the right-hand side of Eq. (7) can be expressed by using the integral expression of the Bessel function of the first kind (*e.g.* Abramowitz and

Stegun, 1972):

$$F_j = a_0 J_0(kR_j) + 2 \sum_{n=1}^{\infty} \{(i)^n a_n J_n(kR_j) \cos n(\theta_0 - \Theta_j)\} \tag{10}$$

where J_n is the Bessel function of the first kind and a_n is defined as Eq. (11):

$$\begin{cases} a_0 = 1 \\ a_n = \left(1 - \frac{1}{s+1}\right) \times \left(1 - \frac{3}{s+2}\right) \times \dots \times \left(1 - \frac{2n-1}{s+n}\right) \end{cases} \tag{11}$$

In the present study, we use Eqs. (10) and (11) to calculate F_j .

2.2 Definition of likelihood

The maximum likelihood method is used to get the most probable values of the directional spectrum parameters. The likelihood, L , is defined as Isobe (1990):

$$\begin{aligned} L(A^{[j]}; \Phi) &= \{p(A^{[1]}) \times p(A^{[2]}) \times \dots \times p(A^{[J]})\}^{1/J} \\ &= \frac{1}{(2\pi \Delta f)^M |\Phi|} \exp\left(-\sum_{m=1}^M \sum_{n=1}^M \Phi_{mn}^{-1} \hat{\Phi}_{nm}\right) \end{aligned} \tag{12}$$

where $p(A^{[j]})$ is a joint probability density function of the Fourier coefficients of the time series data, Δf the frequency interval, and $|\Phi|$ the determinant of the matrix, Φ_{mn} . The quantity $\hat{\Phi}_{nm}$ which is represented by Eq. (13) corresponds to the periodogram with a rectangular filter and can therefore be called the power spectrum ($n = m$) or the cross spectrum ($n \neq m$) in the spectral analysis.

$$\hat{\Phi}_{nm} = \frac{1}{2J\Delta f} \sum_{j=1}^J \overline{A_n^{[j]}} A_m^{[j]} \tag{13}$$

where $\overline{}$ denotes the complex conjugate.

2.3 The most probable values of the parameters

In this Section, we show the procedure to estimate the directional spectrum parameters including the reflection coefficient by using the likelihood defined above.

The maximum likelihood method implies that the most probable values of λ_i are the solutions of the algebraic equation

$$\frac{\partial L}{\partial \lambda_i} = \sum_{k=1}^M \sum_{l=1}^M \frac{\partial L}{\partial \Phi_{kl}} \frac{\partial \Phi_{kl}}{\partial \lambda_i} = 0 \tag{14}$$

From Eq. (12), Eq. (15) is obtained.

$$\begin{aligned}
 \frac{\partial L}{\partial \Phi_{kl}} &= \frac{\partial}{\partial \Phi_{kl}} \left[\frac{1}{(2\pi \Delta f)^M |\Phi|} \exp \left(- \sum_{m=1}^M \sum_{n=1}^M \Phi_{mn}^{-1} \hat{\Phi}_{nm} \right) \right] \\
 &= - \frac{1}{(2\pi \Delta f)^M |\Phi|^2} \exp \left(- \sum_{m=1}^M \sum_{n=1}^M \Phi_{mn}^{-1} \hat{\Phi}_{nm} \right) \frac{\partial |\Phi|}{\partial \Phi_{kl}} \\
 &\quad + \frac{1}{(2\pi \Delta f)^M |\Phi|} \exp \left(- \sum_{m=1}^M \sum_{n=1}^M \Phi_{mn}^{-1} \hat{\Phi}_{nm} \right) \\
 &\quad \times \left(- \sum_{m=1}^M \sum_{n=1}^M \frac{\partial \Phi_{mn}^{-1}}{\partial \Phi_{kl}} \hat{\Phi}_{nm} \right) \\
 &= -L \times \left(\frac{1}{|\Phi|} \frac{\partial |\Phi|}{\partial \Phi_{kl}} \right) + L \times \left(- \sum_{m=1}^M \sum_{n=1}^M \frac{\partial \Phi_{mn}^{-1}}{\partial \Phi_{kl}} \hat{\Phi}_{nm} \right) \tag{15}
 \end{aligned}$$

Also, the following relations are obtained from the theorem of the matrices.

$$\frac{\partial |\Phi|}{\partial \Phi_{kl}} = |\Phi| \Phi_{lk}^{-1} \tag{16}$$

$$\frac{\partial \Phi_{mn}^{-1}}{\partial \Phi_{kl}} = -\Phi_{ln}^{-1} \Phi_{mk}^{-1} \tag{17}$$

By using Eqs. (16) and (17), Eq. (15) is rewritten as follows:

$$\frac{\partial L}{\partial \Phi_{kl}} = L \times \left\{ -\Phi_{lk}^{-1} + \sum_{m=1}^M \sum_{n=1}^M \Phi_{ln}^{-1} \hat{\Phi}_{nm} \Phi_{mk}^{-1} \right\} \tag{18}$$

Substituting Eq. (18) into Eq. (14) and considering that $L \neq 0$, we obtain:

$$\sum_{k=1}^M \sum_{l=1}^M \left\{ -\Phi_{lk}^{-1} + \sum_{m=1}^M \sum_{n=1}^M \Phi_{ln}^{-1} \hat{\Phi}_{nm} \Phi_{mk}^{-1} \right\} \frac{\partial \Phi_{kl}}{\partial \lambda_i} = 0 \tag{19}$$

The directional spectrum parameters, λ_i , which satisfy Eq. (19) for all i ($i = 1 \sim 5$) are the most probable values. Then the directional spectrum parameters, including the reflection coefficient, are estimated.

The solutions, λ_i , of Eq. (19) are obtained numerically by using the Newton-Raphson method. The left-hand side of Eq. (19) is first defined as a function of the directional spectrum parameters:

$$f_i(\lambda_{i'}) = \sum_{k=1}^M \sum_{l=1}^M \left\{ -\Phi_{lk}^{-1} + \sum_{m=1}^M \sum_{n=1}^M \Phi_{ln}^{-1} \hat{\Phi}_{nm} \Phi_{mk}^{-1} \right\} \frac{\partial \Phi_{kl}}{\partial \lambda_{i'}} \tag{20}$$

In the Newton-Raphson method, the value of $\lambda_i^{(j+1)}$ at the $(j+1)$ -th iteration of the calculation is expressed in terms of the previous values, $\lambda_i^{(j)}$, in the following equation:

$$\lambda_i^{(j+1)} = \lambda_i^{(j)} - \left[\sum_{i'=1}^I \left[\frac{\partial f_i}{\partial \lambda_{i'}} \right]^{-1} f_{i'} \right]_{\lambda_i = \lambda_i^{(j)}} \tag{21}$$

Table 3: Values of the directional spectrum parameters

s	14.0
θ_0	3.054 radian
r	0.4
$P(f)$	$0.1 \text{ m}^3/\text{Hz}$
ε	0.1

where $\partial f_i/\partial \lambda_{i'}$ is expressed by Eq. (22):

$$\begin{aligned} \frac{\partial f_i}{\partial \lambda_{i'}} &= \sum_{k=1}^M \sum_{l=1}^M \left\{ -\Phi_{lk}^{-1} + \sum_{m=1}^M \sum_{n=1}^M \Phi_{ln}^{-1} \hat{\Phi}_{nm} \Phi_{mk}^{-1} \right\} \frac{\partial^2 \Phi_{kl}}{\partial \lambda_{i'} \partial \lambda_i} \\ &- \sum_{k'=1}^M \sum_{l'=1}^M \sum_{k=1}^M \sum_{l=1}^M \frac{\partial \Phi_{k'l'}}{\partial \lambda_{i'}} \frac{\partial \Phi_{kl}}{\partial \lambda_i} \\ &\times \left[-\Phi_{l'k}^{-1} \Phi_{lk'}^{-1} + \left\{ \begin{array}{l} \Phi_{lk'}^{-1} \sum_{m=1}^M \sum_{n=1}^M \Phi_{l'n}^{-1} \hat{\Phi}_{nm} \Phi_{mk}^{-1} \\ + \Phi_{l'k}^{-1} \sum_{m=1}^M \sum_{n=1}^M \Phi_{ln}^{-1} \hat{\Phi}_{nm} \Phi_{mk'}^{-1} \end{array} \right\} \right] \end{aligned} \tag{22}$$

3 Application

3.1 Application to simulated data

To verify the validity of the present method, we first created a set of $\Phi_{mn}(\sigma)$ by Eqs. (9) and (10) for directional spectrum parameters given in Table 3. Then, the directional spectrum parameters, λ_i , were estimated by the present method.

However, since a converged solution of Eq. (19) could not be obtained for any set of initial values used, solutions were determined for various fixed values of r and ε .

Table 4 shows the results of the computations. It is seen that the likelihood, L , is maximum when $\varepsilon = 0.1$, $r = 0.4$, $s = 13.3$ and $\theta_0 = 3.1$, which agrees closely with the given values. Hence, the directional spectrum parameters were estimated adequately by the present method.

3.2 Application to field data

From November 7 to December 2, 1990, a field experiment was conducted at Oarai port in Ibaraki prefecture, Japan (Fig. 2). Four measuring points are arranged normal to the offshore breakwater as sketched in Fig. 3. An ultrasonic-

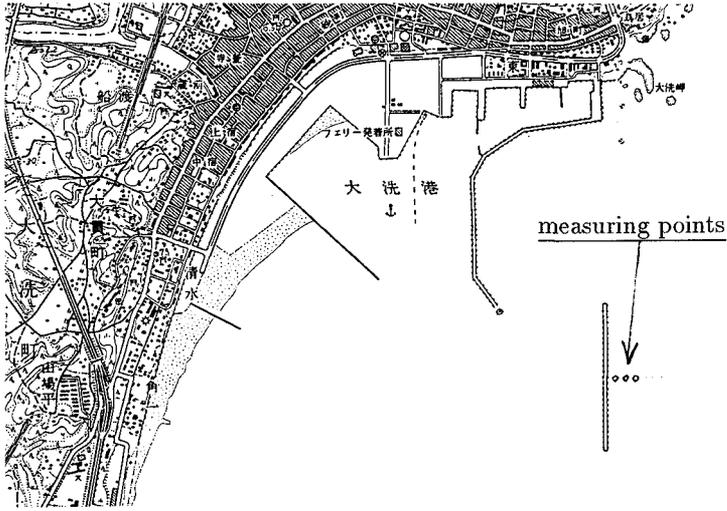


Figure 2: Map of Oarai port

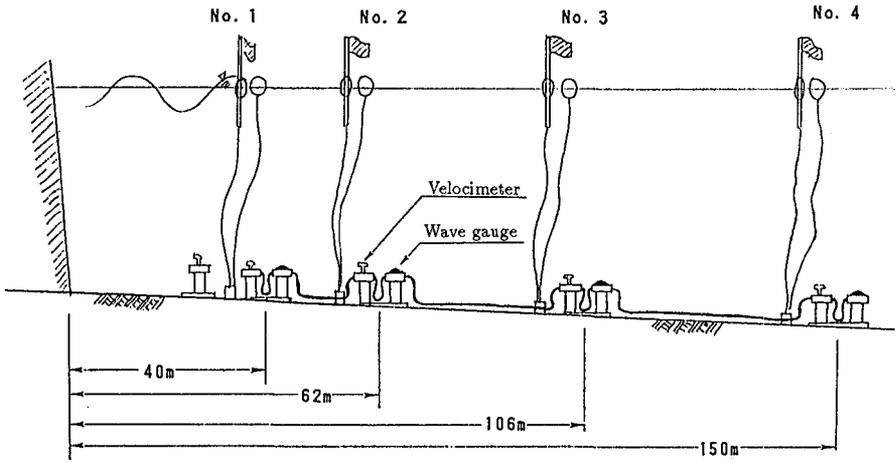


Figure 3: Sketch of measuring points

Table 4: Results of the estimation

ε	r	s	θ_0	$\ln L$	ε	r	s	θ_0	$\ln L$
0.00	0.2	4.570	3.141	19.944	0.10	0.2	12.378	3.139	20.660
0.00	0.3	5.694	3.139	20.504	0.10	0.3	12.753	3.139	20.824
0.00	0.4	6.476	3.139	20.706	0.10	0.4	13.311	3.140	20.862
0.00	0.5	7.088	3.139	20.722	0.10	0.5	13.875	3.140	20.818
0.00	0.6	7.601	3.139	20.651	0.10	0.6	14.367	3.140	20.728
0.05	0.2	8.079	3.141	20.387	0.15	0.2	15.504	3.140	20.708
0.05	0.3	8.587	3.142	20.702	0.15	0.3	15.670	3.140	20.813
0.05	0.4	9.130	3.141	20.798	0.15	0.4	16.135	3.140	20.826
0.05	0.5	9.648	3.140	20.775	0.15	0.5	16.654	3.140	20.776
0.05	0.6	10.109	3.140	20.690	0.15	0.6	17.127	3.140	20.687

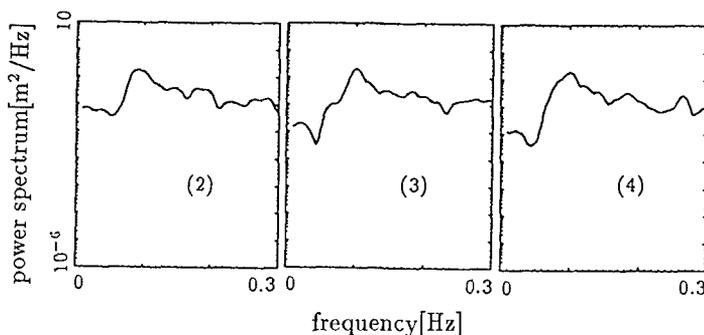


Figure 4: Power spectra

type wave gauge was installed on the sea bed of each measuring point. The four wave gauges were synchronized by a cable.

The sampling interval is 0.5 s and 2,046 time series data (about 17 min) were recorded every two hours. Because the wave gauge at the measuring point No. 1 did not work, the time series data at the three measuring points No. 2 to 4 were obtained and used.

The results of the spectral analysis are shown in Figs. 4 and 5. Time series data used were obtained at 14:00 on November 8. Figure 4 shows the power spectra at the measuring points, and Fig. 5 the cross spectra, the coherence squared and phase lag between the measuring points. In addition, the wave statistics at each measuring point are shown in Table 5.

Figures 4 and 5 indicate that both the power and cross spectra are maximum

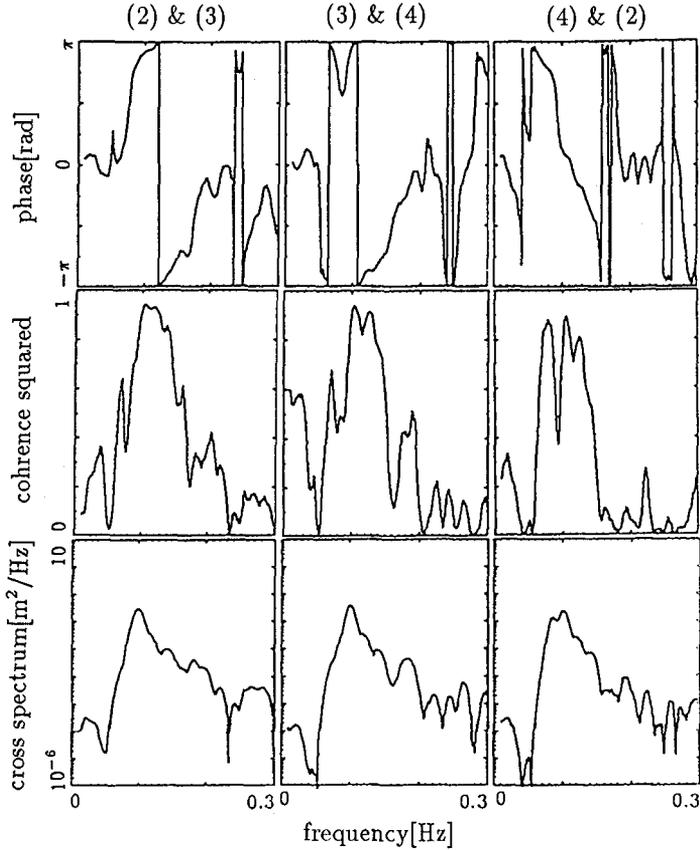


Figure 5: Cross spectra, coherence squared, phase

at the frequency of 0.1 Hz. Therefore, we calculated the directional spectrum parameters at this frequency.

The most probable values of the directional spectrum parameters as well as the likelihood are shown in Table 6. The parameters are estimated as $\epsilon \simeq 0.05$, $r \simeq 0.4$, $s \simeq 36$, $\theta_0 \simeq 3.14$ rad. Since concrete blocks are installed in front of the breakwater, the reflection coefficient obtained is considered to be a reasonable value.

Table 5: Wave statistics

Measuring point No.	h m	H_{\max} m	T_{\max} s	$H_{1/3}$ m	$T_{1/3}$ s	H_{mean} m	T_{mean} s
(2)	12.44	1.10	9.00	0.62	8.05	0.38	5.20
(3)	12.66	0.95	8.50	0.58	7.35	0.36	4.93
(4)	12.82	1.13	7.00	0.56	6.69	0.35	4.75
(5)	13.10	0.91	9.50	0.54	6.96	0.35	4.87

h : water depth;

H_{\max} : maximum wave height; T_{\max} : maximum wave period;

$H_{1/3}$: significant wave height; $T_{1/3}$: significant wave period;

H_{mean} : mean wave height; T_{mean} : mean wave period.

Table 6: Results of estimation

ε	r	s	θ_0	$\ln L$	ε	r	s	θ_0	$\ln L$
0.00	0.2	8.360	3.141	23.515	0.10	0.2	109.234	3.141	24.535
0.00	0.3	10.993	3.141	24.297	0.10	0.3	93.392	3.141	24.595
0.00	0.4	12.811	3.141	24.607	0.10	0.4	62.488	3.141	24.596
0.00	0.5	14.028	3.141	24.664	0.10	0.5	56.304	3.141	24.574
0.00	0.6	14.862	3.141	24.594	0.10	0.6	54.671	3.141	24.456
0.05	0.2	33.158	3.141	24.701	0.15	0.2	109.850	3.141	24.150
0.05	0.3	34.431	3.144	24.973	0.15	0.3	114.443	3.141	24.179
0.05	0.4	36.318	3.141	25.079	0.15	0.4	88.447	3.141	24.156
0.05	0.5	37.830	3.141	25.068	0.15	0.5	81.737	3.141	24.099
0.05	0.6	38.813	3.141	24.979	0.15	0.6	77.092	3.141	24.013

4 Conclusion

Based on the parametric expression of the directional spectrum, a method is proposed to estimate the directional spectrum parameters which include the reflection coefficient.

The validity of the method was confirmed by applying it to simulated data. The method was also applied to the data obtained by a field measurement at Oarai port. The result of estimation is considered to be reasonable.

Acknowledgement

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