CHAPTER 49

ON SPECTRAL INSTABILITIES AND DEVELOPMENT OF NON- LINEARITIES IN PROPAGATING DEEP-WATER WAVE TRAINS

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Abstract

Modulation instabilities in surface waves on deep water are investigated. Experimental wave elevation records from laboratory tests are analysed, based on simultanous records at different distances from the wavemaker. Initially monochromatic and biohromatic wave trains are seen to gradually change with the propagation, by splitting up into smaller groups. After further propagation, large individual waves, and wave breaking, may occur. Results from tests with irregular (random) waves indicate that the skewness of the record, which is mainly a measure of 2nd order nonlinearities, is closely connected to the wave steepness during the whole propagation, more or less independently of other wave processes going on. The non-linear wave group formation, however, which is related to higher-order instabilities, show a development during the propagation. This may lead to frequency down-shift of the wave energy, combined with energy dissipation and wave breaking.

1. INTRODUCTION

Initially monochromatic wave trains (or more correctly: periodic Stokes waves) are unstable (Benjamin & Feir 1967). After a certain propagation distance, slow modulations will occur and grow with further propagation. This non-linear phenomenon, also known as side-band instabilities in the frequency domain, depends further on the wave steepness and is therefore coupled to the initial 2nd order (Stokes) non-linear component of the waves. The modulation itself, however, is a

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higher-order non-linear effect, and may be described by a 3rd order theory through the non-linear Schrødinger equation (Zakharov 1968). A 4th order correction is necessary in order to account for an asymmetric behaviour of the 2 sidebands (Dysthe 1979), resulting in a net frequency downshift of the wave energy, which is an important practical consequence of this phenomenon.

Such modulational instabilities will also occur in irregular (random) wave trains, although a complete theoretical description then may become rather complex. Tulin & Li (1991) have proposed a theoretical model where these effects are coupled to the development of large and breaking waves in wave groups, even in moderate sea states. Similar results have been shown by Cointe & Boudet (1991). These ideas may, together with experimental studies, lead to interesting contributions in the general problem of modelling random wave trains, and in particular problems like the prediction of the extreme wave of a given sea state.

The main purpose of the present work is to show some observations from laboratory tests with irregular waves, and thus provide experimental documentation of the nature of such higher-order non-linearities. The interpretation of the results will basically be done by spectral and statistical analysis of the measured wave elevation. Prior to the presentation of these results, however, results from an experimental verification of these modulational instabilities in monochromatic and bichromatic wave trains is given, as a first step to understand the problem.

2. INSTABILITES DEVELOPING IN MONOCHROMATIC AND BICHROMATIC WAVE TRAINS

Results from laboratory tests with initially monochromatic & bichromatic waves are shown in the following. The monochromatic test was performed in MARINTEKs Ocean Basin, measuring 50mx80m, with the waves generated in the longitudinal direction. The bichromatic test was performed in MARINTEKs long towing tank, measuring 10mx270m. Wave elevation measurements were simultaneously made at a number of distances from the wavemaker, on the central longitudinal axis of the basin/tank. The water depth in the Ocean Basin tests was 2.8m, while for the long tank test it was 5.0m. These depths correspond to deep water for the tests considered. All input to the wavemakers were generated as linear wave signals.

The measured time series samples in fig. 1 as well as fig. 3 illustrate how the wave trains gradually develope from more or less undisturbed linear wave signals into wave trains breaking up into smaller groups with relatively large extreme waves. Wave breaking was also observed at the largest distances from the wavemaker. The power spectra in figs. 2 & 4 show how the energy is spread out from the initial frequencies into neighbouring side bands. Some loss of energy is also observed.

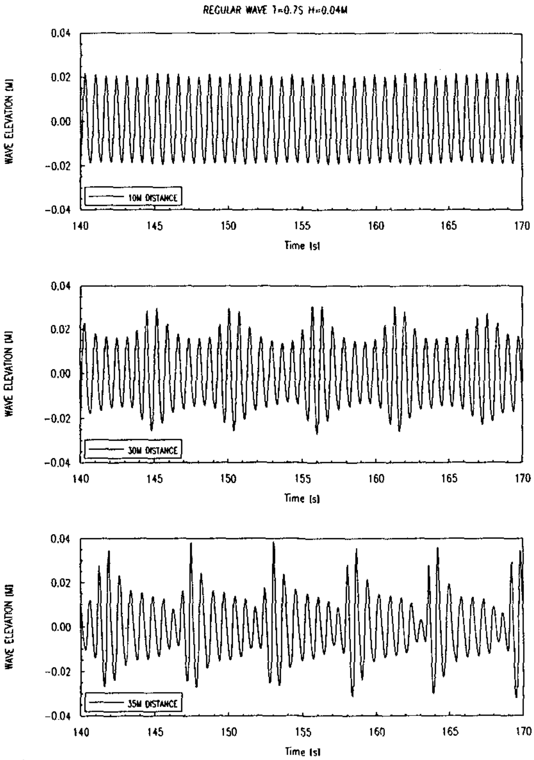

Figure 1. Time series samples of an initial monochromatic wave train, measured simultaneously at 3 locations.

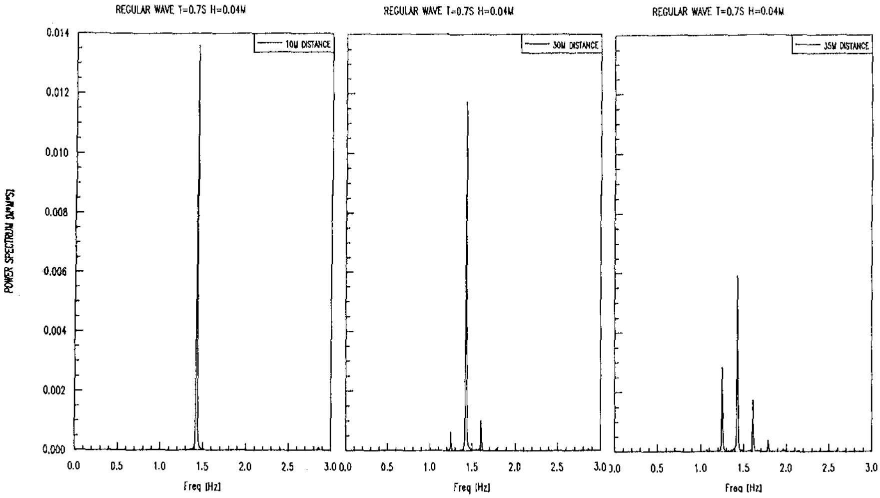

Figure 2. Power spectra of the wave records in figure 1.

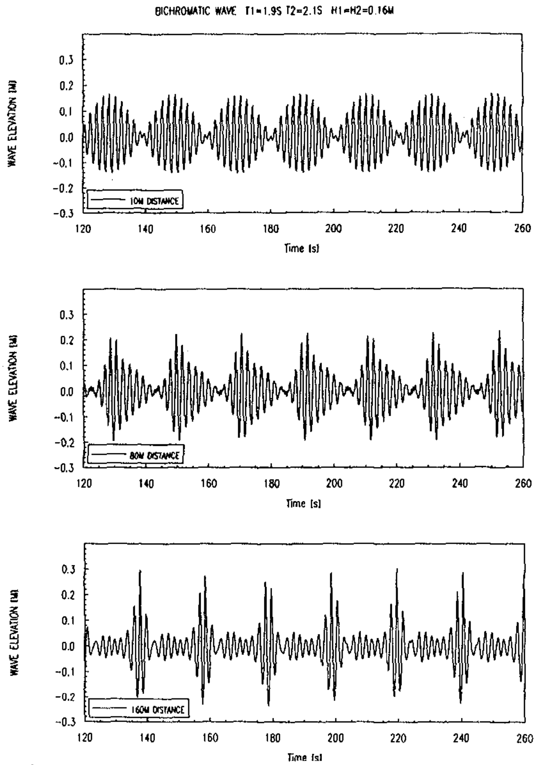

Figure 3. Time series samples of an initial bichromatic wave train, measured simultaneously at 3 locations.

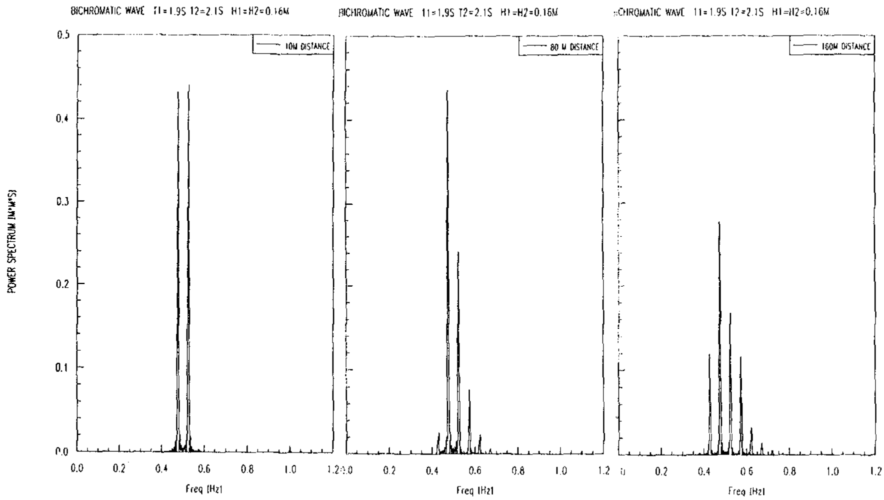

Figure 4. Power spectra of the wave records in figure 3.

3. SPECTRAL AND STATISTICAL PROPERTIES OF IRREGULAR WAVE TRAINS.

The influence from wave steepness and propagation length on the characteristics of longcrested irregular wave trains is experimentally investigated in the following. Measured records from laboratory tests in MARINTEKs Ocean Basin (size & depth: see Chapter 2) are used. The irregular input signal to the wavemaker was synthesized as a linear superposition of harmonic components by use of inverse Fast Fourier Transform with 4096 components. JONSWAP spectra with a peak enhancement factor, Gamma, equal to 3.0 were generated. A more detailed description of the irregular wave generation procedure is given in Stansberg (1990). Wave elevation measurements were made simultaneously at a number of distances from the wavemaker, in the same manner as described in Chapter 2. The total duration of each record was 20 minutes. In the analysis, we shall distinguish between the measurements made close to the wavemaker (only some wavelengths), and those made further away.

3.1 Observations at short distances (< 10 wavelengths).

Fig. 5 shows selected time series samples from 2 tests with spectral peak period Tp=1.8s, both measured at 35m distance (i.e. 7 wavelengths) from the wavemaker. The selected time windows include the largest waves of the records. The 2 tests were run with the same input signal apart from a scaling factor of 2. The main difference observed is, apart from the scaling, the asymmetric and peaked shape of the extreme crests in the highest sea state. This is also reflected in the values obtained for the statistical skewness, based on the 3rd order statistical moment of the wave elevation record:

$$\text{skewness} = (1/N\sigma^3)\cdot\sum_{i=1}^{N}(x_i-\bar{x})^3 \tag{1}$$

(expected to be zero for a Gaussian process)

Here $\bar{x}$ = mean value, σ = standard deviation, N = no. of samples.

It is seen that for the high sea state, the skewness value is twice the value for the low sea state. We may interprete this result as a direct result of steepness-induced 2nd order (Stokes) processes in the waves (Marthinsen & Winterstein 1992, Stansberg 1993). On the other hand, the values for the kurtosis, which is based on the 4th order statistical moment of the record:

$$\text{kurtosis} = (1/N\sigma^4)\cdot\sum_{i=1}^{N}(x_i-\bar{x})^4-3 \tag{2}$$

(expected to be zero for a Gaussian process)

is relatively low in both cases. The kurtosis is a measure of group formation, i.e. of enhanced modulation, in the signal. Thus an increased steepness of the waves does not seem to induce any increase in higher-order instabilities at this distance.

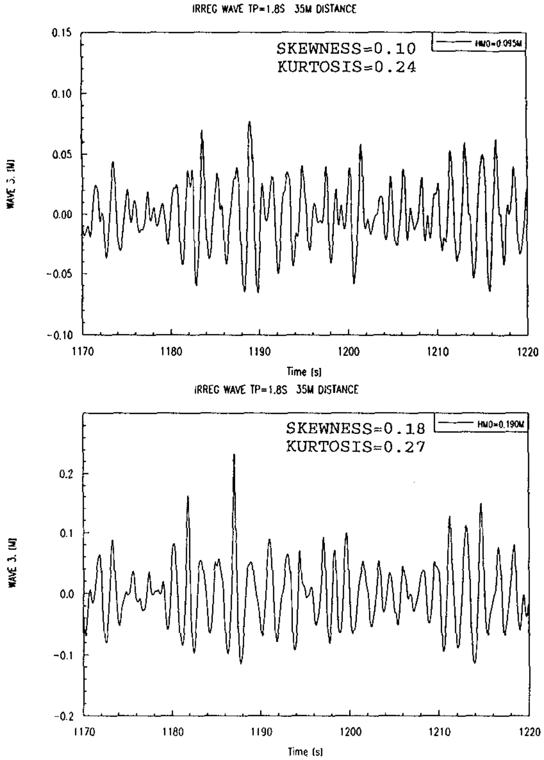

Figure 5.Time series samples from 2 irregular wave tests with Tp=1.8s. Significant wave heights Hm0: 0.095m and 0.19m Both measured at 35m.

The influence from the wave steepness on the statistical skewness and kurtosis is also reflected in fig. 6, where results from several test runs are presented.

The power spectra in fig.7 show that a small part of the wave energy has dissipated at high frequencies for the steepest wave condition. No significant changes have taken place, however, in the frequency distribution of the energy.

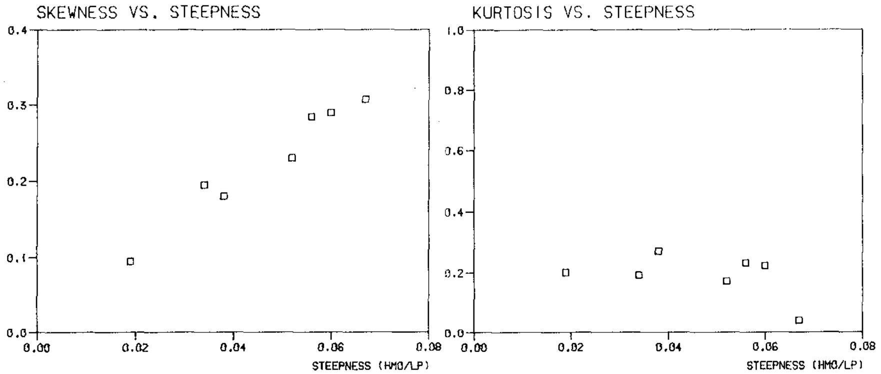

Figure 6. Measured skewness vers. wave steepness (Lp=wavelength of spectral peak frequency).

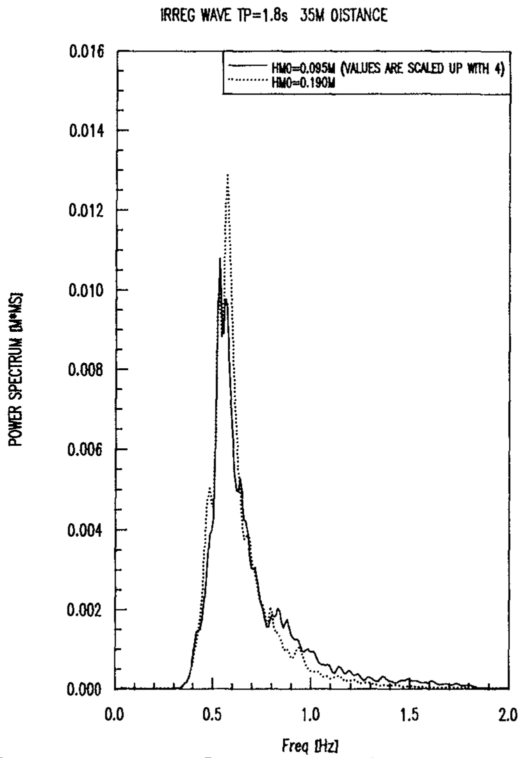

Figure 7. Power spectra of the waves in figure 5 (based on full 20 minutes records).

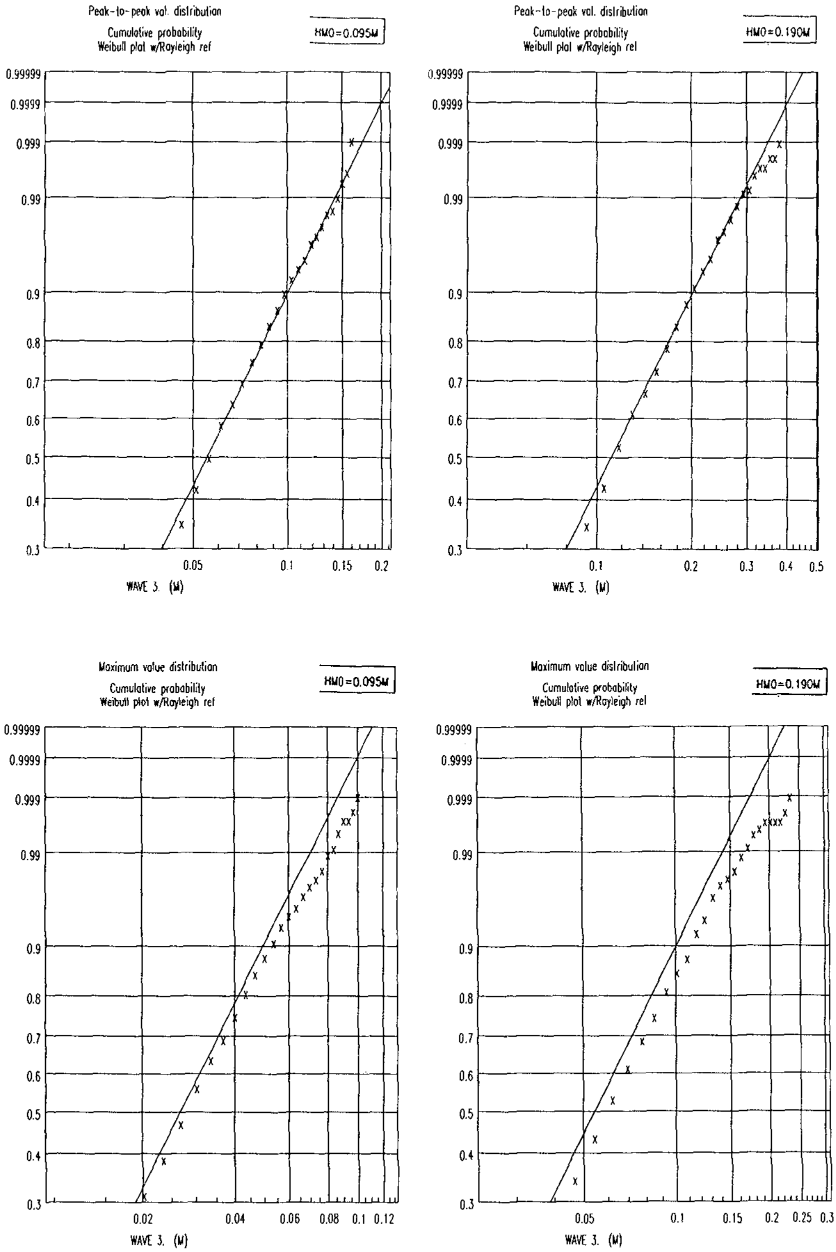

Figure 8. Cumulative distributions of wave and crest heights of the wave records in figs. 5 and 7.

The statistical distribution of wave and crest heights from the tests shown in figs. 5 & 7, are presented in fig. 8. In both wave conditions, the peak-to-peak wave heights seem to follow the commonly used Rayleigh model reasonably well, while the highest crests somewhat exceed the Rayleigh estimates. The latter observation is also discussed and reported in Stansberg (1991,1993), and is connected with 2nd order non-linearities commented on above.

3.2. Observations at a longer distance (>10 wavelengths).

The influence from the wave propagation on steep irregular waves is illustrated in fig. 9, where time series samples from a test with Tp=1.0s is shown. Wave elevation records measured simultaneously at 10m & 35 m distance (i.e. at 7 & 20 wavelengths, respectively) are shown. The selected samples include the largest waves of the records. The first location corresponds to the case discussed in section 3.1, while propagation-dependent effects may be observed in the latter. The shown samples illustrate how a wavetrain with steep, large waves may develope into a wavetrain

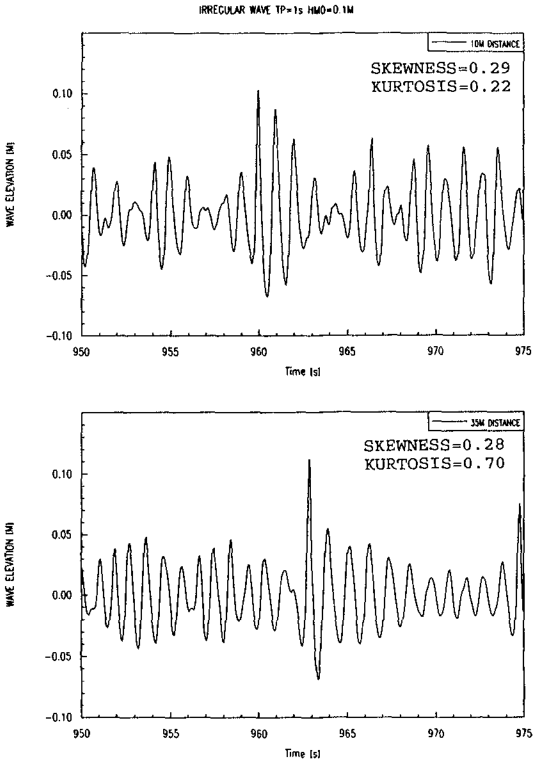

Figure 9. Time series samples recorded at 2 locations, for steep irregular wave with Tp=1s, Hm0=0.1m.

with high group formation and a few particularly high waves. A similarity between the recorded extreme wave group and the unstable groups in Chapter 2 is observed. The increase in wave group formation may also be identified through the kurtosis value, which increases from 0.2 to 0.7 as the wave propagates. This development is additional to the 2nd order skewness effect seen in in the previous section, and may probably be connected with the higher-order modulational instabilities demonstrated in Chapter 2. It is interesting to note that the skewness value is the same at both locations.

From the spectral plots in fig. 10, a net energy transfer from high to lower frequencies is observed. In addition, a 10% wave energy loss occurs due to dissipation by wave breaking and other processes.

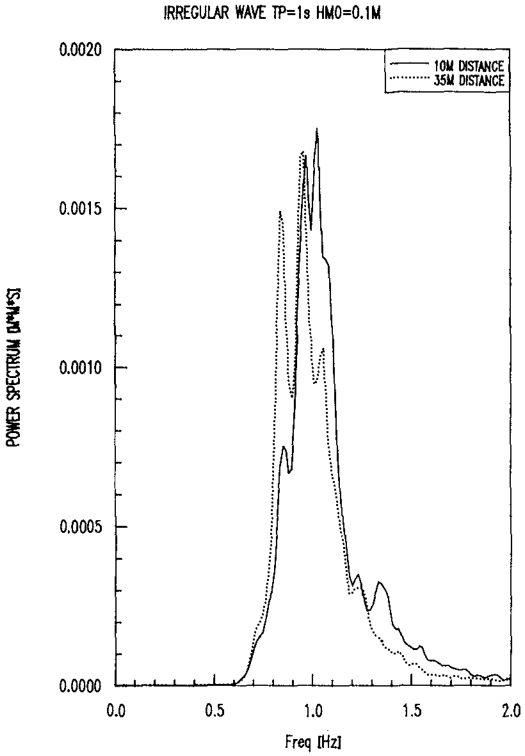

Figure 10. Power spectra of the wave in figure 9 (based on full 20 minutes record).

The influence from the propagation distance on the statistical wave parameters is shown in fig. 11, where results from several tests with different wave periods and wave heights are included. The propagating distance is normalized as D/Lp, where D is the distance from the wavemaker, and Lp is the "peak wavelength of the spectrum":

$$Lp = (g/2\pi) \cdot Tp^2 \tag{3}$$

It is seen that up to a certain point, the kurtosis (i. e. the non-linear wave group formation) increases with D/Lp. Then energy dissipation occurs by breaking etc., and the wave steepness decreases. A connected decrease in the kurtosis is then also observed at a far distance. The skewness, assumed to be related to 2nd order effects, seems to be given by the steepness (as in fig.6), regardless of the other higher-order wave processes.

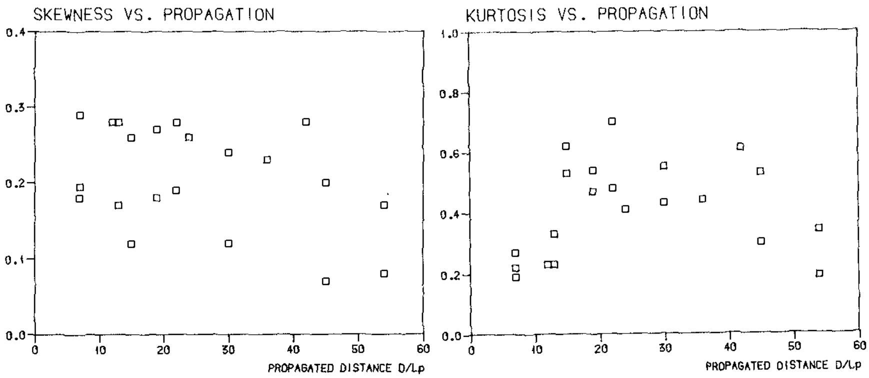

Figure 11. Measured statistical parameters vers. normalized propagation distance.

The statistical distribution functions of the full 20-minutes records illustrated in fig.9, are shown in fig. 12. As expected, the record closest to the wavemaker shows similar properties to the records in section 3.1, and may probably be described by 2nd order effects. At the far location, however, the increased modulation is observed through larger deviations from the Rayleigh estimates for large wave heights as well as large crest heights. This is connected with the high kurtosis value discussed above.

4. DISCUSSION WITH CONCLUSIONS.

The tests with initially monochromatic and bichromatic waves verify the expected efects: After a certain propagation length (> 10 wavelengths), the wave trains get unstable, and with further propagation they gradually form smaller groups. This may lead to groups with some asymmetric and pronounced individual waves. Wave breaking may also occur, even in wave trains with initially moderate steepness. The

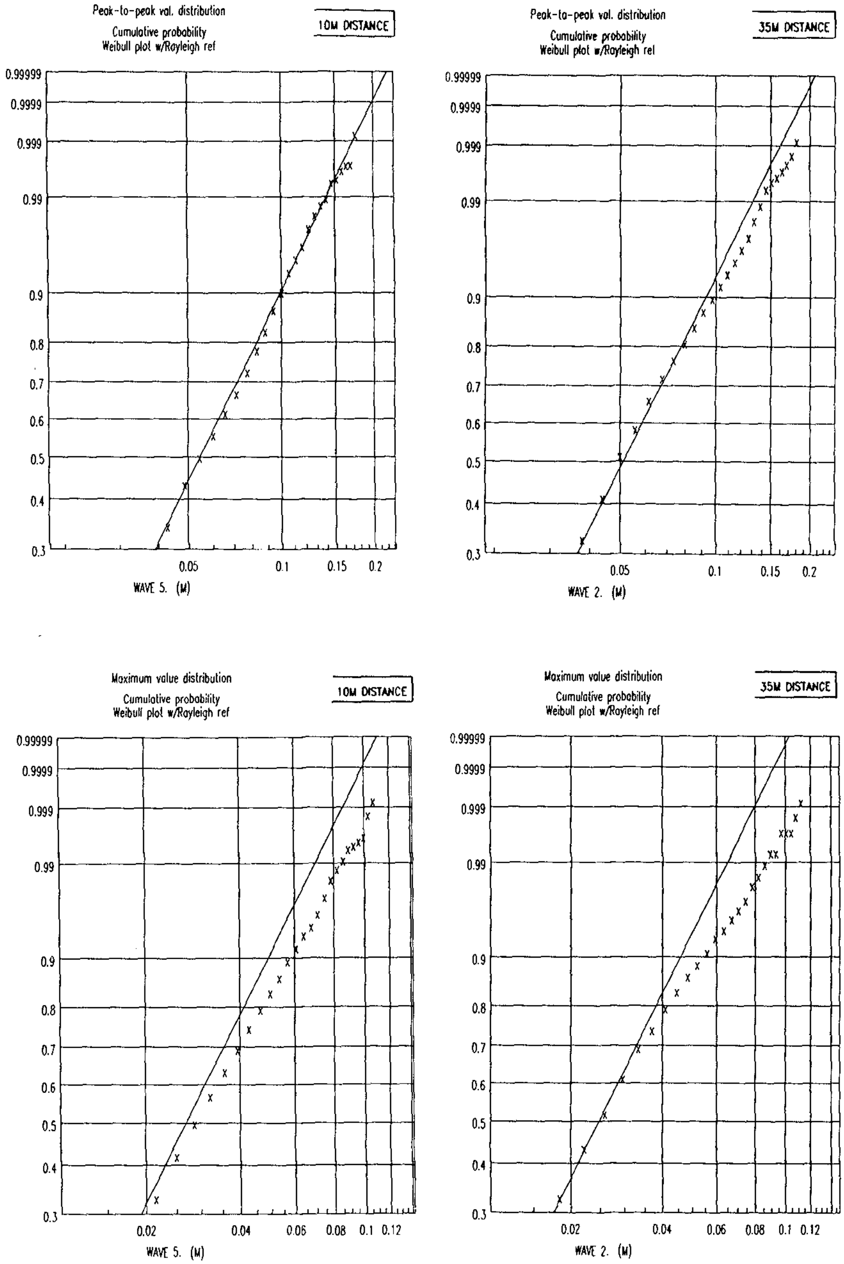

Figure 12. Cumulative distributions of wave and crest heights of the wave records in figs. 9 and 10.

instabilities are connected with a continous formation of side bands in the wave spectrum, i.e. the wave energy spreads out to lower as well as to higher frequencies. The net effect, as observed from a number of different tests (also including tests that are not shown in figs. 3 & 4), seems to be an energy transfer to lower frequencies, although the high-frequency side-bands seem to decrease rather slowly with the propagation. This confirms the results of Dysthe (1979) and others. Energy dissipation also occurs.

From the irregular wave tests it is observed that the statistical skewness of the records, interpreted as a consequence of 2nd order (Stokes) non-linearities, occurs already at a few wavelengths from the wavemaker. With further propagation, the skewness is still determined mainly by the steepness of the sea state. On the other hand, the statistical kurtosis, interpreted as a consequence of higher-order modulational instabilities, grows gradually with the propagation until the instabilites lead to energy dissipation (wave breaking and other processes) in the same manner as observed for monochromatic and bichromatic waves. The growth of the instabilities also seem to depend on the wave steepness, as expected. After the dissipation, however, the results seem to indicate that the wave train "reorganizes" to a more stable process with lower energy, lower skewness and lower kurtosis values.

For practical purposes, we may interprete the above results in the way that for proper future modelling of random wave fields, one should probably include at least 2nd order non-linearities, if the occurrence of large waves is an essential matter. Whether the higher-order instability effects should also be considered, is more uncertain, since the rise and decrease of such effects may possibly occur more as local events rather than permanent, homogenous properties. This is, however, a field where more studies are needed.

5. REFERENCES

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