

CHAPTER 6

BAR/TROUGH EFFECTS ON WAVE HEIGHT PROBABILITY DISTRIBUTIONS AND ENERGY LOSSES IN SURF ZONES

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Abstract

Laboratory measurements of irregular wave heights across a bar/trough beach profile are being studied to develop improved probability distributions including the subset of breaking waves. The energy dissipation in breaking waves modeled as a periodic bore is inversely proportional to the wave period and this may explain why the bore model appears to underestimate measured average energy dissipation rates. The study is ongoing and preliminary results for one wave period are presented in this paper.

1. Introduction

The transformation of irregular waves across surf zones is dominated by wave breaking. In contrast to regular (single frequency) waves, irregular (multiple frequency) waves can break almost anywhere depending upon the wave characteristics (heights, periods) and the water depth. Thus some waves break because of their steepness, i.e., they behave as if in "deep" water while others break due to a depth limiting criteria for "shallow" water. The nearshore bathymetry, especially the presence of bars dominates the wave transformation process.

The traditional, wave transformation method assumes a slowly-varying, quasi-uniform wave field as represented by the time averaged and depth-integrated wave energy flux per unit surface area and the wave energy balance equation including rate of energy dissipation per unit area. For irregular waves, two different approaches are possible for solving the multiple frequency, wave

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energy balance equation.

One approach randomly selects offshore waves from a known joint (height, period) distribution, transforms individual waves with the energy balance equation and then reassembles the wave heights into probability distributions across the surf zone. This approach is called the wave-by-wave or Monte-Carlo method and requires computation for hundreds of waves. Dally (1990,1992) presents an excellent review paper and application to field data of this approach.

A second approach, herein called the probability density function (pdf) approach, assumes *a priori*, the wave height distribution functions for all the waves and the subsets of breaking waves. The energy balance equation is then solved only once for the transformation of a single, statistical descriptor wave through the surf zone.

This paper focus on the pdf approach. For coastal engineering applications, especially for two-dimensional wave transformations, it is felt that this approach is more practical. Section 2 briefly reviews the literature and summarizes the objectives of an ongoing study to learn more about the pdf's for bar/trough beach profiles and methods to estimate the breaking wave energy dissipation rates. The laboratory facilities and experimental design is presented in Section 3. Early test results are presented in Section 4. Section 5 gives some conclusions and future directions for the research.

2. The Probability Density Function(pdf) Approach

2.1. Literature Review

Battjes and Janssen(1978) were the first to integrate the energy flux balance equation using the pdf approach and calculate wave heights over non-monotonic bottom profiles. Previous random wave transformation models (see Thornton and Guza, 1983 for review) used a cut-off model for the pdf when waves broke so that calculated wave heights depended only on the local water depth. On bar/trough beaches this produced physically unrealistic lower energy levels over the bar and energy gains in the adjacent trough.

Thornton and Guza(1983) extended the work of Battjes and Janssen(1978) by including a semi-empirical expression for the breaking wave distribution, $p_b(H)$ that was a *subset* of the theoretical, Rayleigh distribution, $p(H)$ for all the waves. The average rate of energy dissipation $\bar{e}_b$, in each breaking wave is modeled after a periodic bore (Stoker, 1956) in both these models. However, Thornton and Guza (1983) derived an analytical expression for the ensemble average, $\langle \bar{e}_b \rangle$ for an irregular wave train by integrating the product $\bar{e}_b * p_b(H)$ for all the waves. The final expression for $\langle \bar{e}_b \rangle$ includes two coefficients (γ , B) that require field verification as discussed later in this paper. The ensemble

average, $\langle \bar{e}_b \rangle$ is also inversely proportional to the wave period and the implications of this result are also reviewed at the end of this paper.

These modelers use the root-mean-square wave height, H_{rms} , as the statistical, descriptor wave and both models give good prediction's for the H_{rms} transformations occurring over real (field) beach profiles (see Battjes and Stive, 1985 for laboratory and field calibrations with Northsea waves, $T_{max} < 8.7$ sec; and Thornton and Guza, 1983 for West Coast swell waves, $T = 13-17$ sec). However, this agreement between calculated and observed H_{rms} required the use of physically unrealistic coefficients as discussed below and also does not mean that the underlying pdf's are correct as noted by these researchers.

2.2. Research Objectives

A laboratory study is underway that focuses on the wave height probability distributions for all the waves, $p(H)$ and the breaking waves as a subset, $p_b(H)$ as they pass over a synthetic, bar/trough beach profile. One objective is to compare the measured pdf's with those predicted by the Thornton and Guza (1983) theory and to make improvements in the theory, if warranted.

The second objective is to investigate the apparent *underestimation* of the average rate of wave breaking energy dissipation when using the periodic bore model as claimed by some researchers (Svendsen et al., 1978; Stive, 1984).

3. Laboratory Facility and Experiments

Experiments are being conducted in the new 18m long by 0.9m deep by 0.9m wide wave tank in the Coastal Engineering Laboratory at Old Dominion University. This facility is equipped with a random wave generator as designed by the Danish Hydraulic Institute (DHI) that includes an automatic wave absorption system at the wave board to remove reflected wave energy in the tank. Wave generation, recording, and analysis is accomplished through a special software package also developed by DHI. Wave heights up to 30cm and a period range between 0.7-3.5sec is possible in this facility.

Fig. 1 schematically shows the 1:20 beach slope with 1:10 artificial bar and dimensions such that the bar crest lies 22cm below the SWL in 60cm water depth. Seven wave gauges are positioned as shown across this synthetic bar/trough beach profile with all dimensions in meters. Four additional locations for wave gauges are shown in the trough region (vertical dashed lines) to provide details in this location.

A JONSWAP spectrum with standard shape parameters is generated at the wave board. The irregular wave train is constructed by a random number generator that always begins with the same seed to permit repetition of the time history of sea surface variation. For the results reported in this paper, the

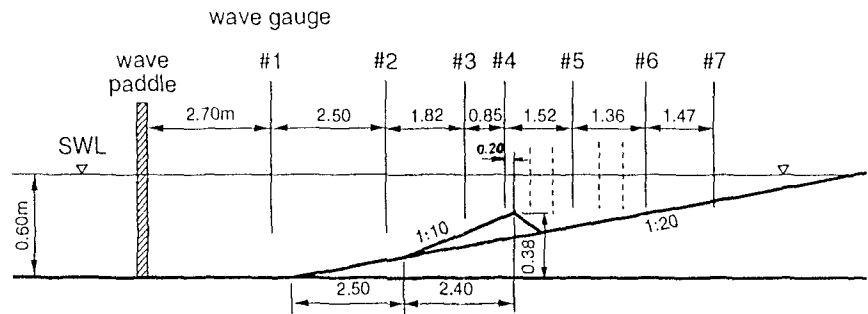


FIG 1. SCHEMATIC OF BEACH PROFILE AND TEST ARRANGEMENT

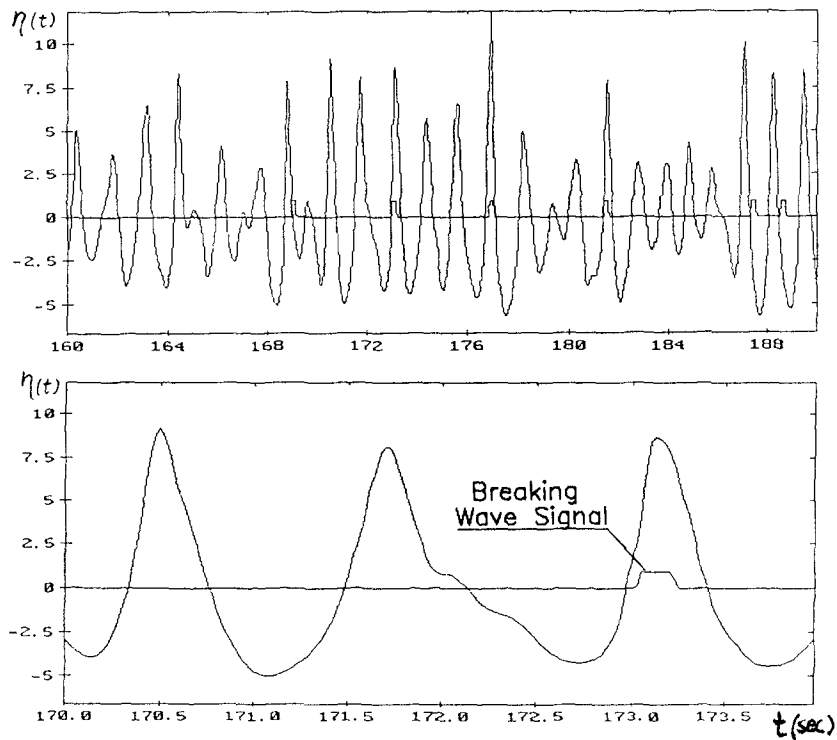


FIG 2. MEASURED WAVE PROFILES & BREAKING WAVE SIGNALS

spectral significant wave height, H_{m0} was 15.0cm with peak period, $T_p = 1.30$ sec. These input conditions were verified at wave gauge no. 1 and convert to a relative water depth (d/L) ratio of 0.249 and wave steepness of 0.0569 at this position as given from linear wave theory.

Waves were generated for six minutes and approximately 300 were distinguished by the zero-down-crossing method. The number of waves passing each wave gauge was relatively constant. A spectral analysis of the sea surface at each wave gauge location produced similarly-shaped spectrums with reduced energy content. Therefore, the bar/trough shape, position, water depth and wave spectrum chosen did *not* generate higher frequency wave energy after the bar.

Fig. 2 shows an example of the wave train measured over a 30 second interval with an enlargement below it for a 4 second span. Also shown is the impulse voltage signal generated manually into the record by an observer whenever a breaking or broken wave event passed the gauge. This somewhat subjective observation of which waves were classified as "breaking waves" was confirmed by running replicate sets and using different observers. A special, software program has been developed to automatically distinguish the broken waves as a subset of all the waves identified in the record. Note that the largest waves are not always the breaking/broken waves identified in the record.

4. Test Results

4.1. Probability Distributions - All Waves

Wave height histograms were calculated using the DHI software package and checked with specially developed software that also computed histograms for the breaking waves identified as a subset of all the waves. Fig. 3 shows the measured histograms for *all the waves* at four, representative gauge locations, namely: Gauge 2 on the horizontal bottom before the beach slope; Gauge 4 near the bar crest; Gauge 5 in the trough; and Gauge 7 on the plane beach slope.

Also shown are the theoretical, probability distribution functions for all the waves, $p(H)$ as given by the Rayleigh distribution (dotted line) and the Beta-Rayleigh distribution (solid line). Thornton and Guza (1983) showed that the Rayleigh distribution produced an reasonable description of all surf zone waves (mean period about 14 seconds) transforming over a monotonically decreasing profile (Torry Pines, CA). These authors also reviewed the works of others attempting to explain why the Rayleigh distribution *overpredicts* the number of large waves in the tail when compared with observations.

The Beta-Rayleigh distribution has been offered by Hughes and Borgman, 1989 to "... better characterize the wave height distributions for shallow water waves." It is semi-empirical and requires that the $p(H)$ be bounded by H_{max} ; be

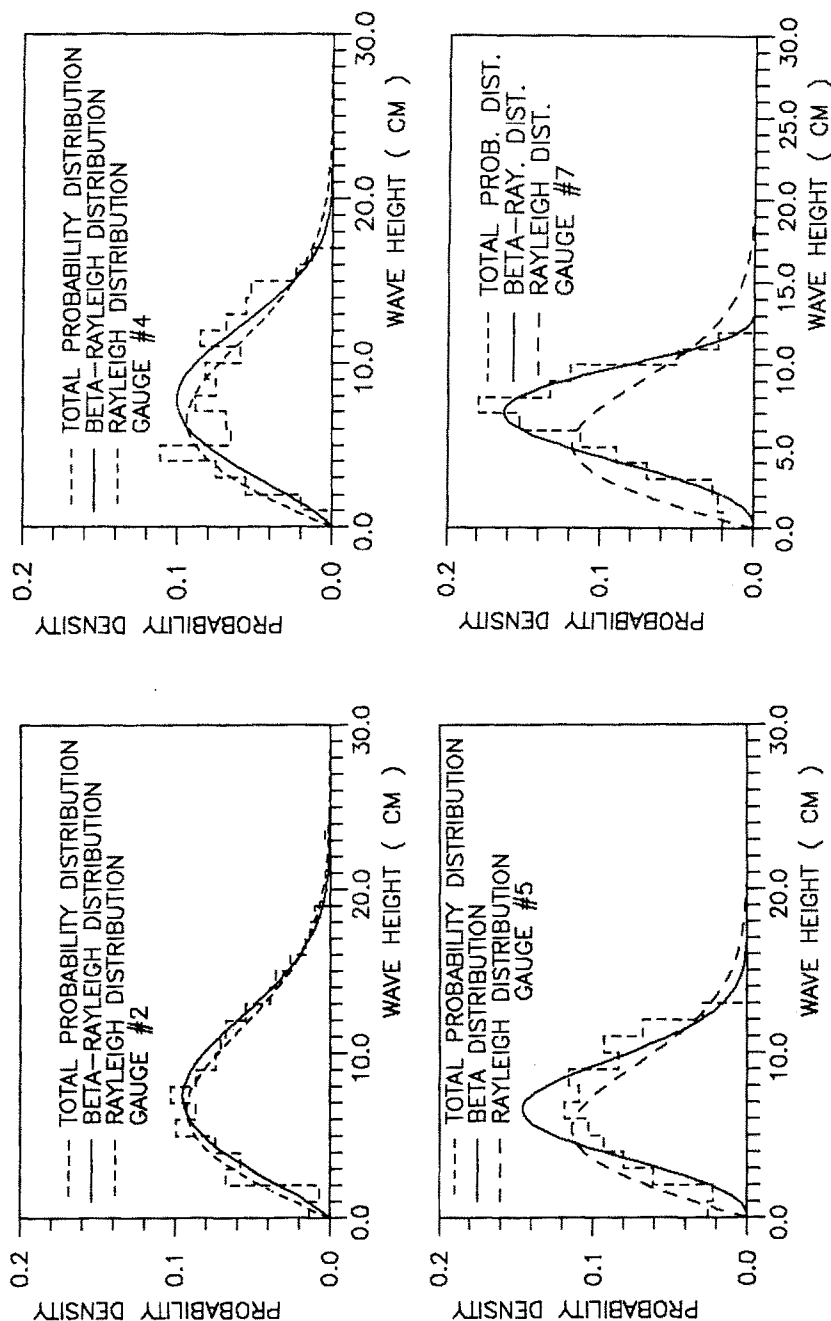


FIG 3. WAVE HEIGHT PROBABILITY DENSITY DISTRIBUTION OF ALL WAVES

skewed toward the larger waves; become the Rayleigh distribution in deep water and; be physically justified. Using the calibration parameters identified by Hughes and Borgman, 1989, we have also presented the Beta-Rayleigh, $p(H)$ distributions in Fig. 3.

At Gauge 2 in "deep" water, the Rayleigh distribution is found to give a good fit to the measured data. Near the bar crest (Gauge 4) the measured distribution flattens out (or becomes double peaked) and both theoretical curves give a reasonably good fit. In the trough (Gauge 5) both theories give a relatively inaccurate shape compared with the measured distribution, but in different ways. Finally, on the plane beach slope, the Beta-Rayleigh distribution is found to give an excellent fit to the measured histogram, especially for the peak value and in the tail where the Rayleigh distribution *overpredicts* the number of larger waves as discussed above.

4.2. Probability Distributions - Breaking Waves

The measured, breaking wave histograms are plotted in Fig. 4 for the same four gauge location (2, 4, 5, and 7) as discussed above. The scales are constant for each location and reveal that most waves break near the bar and on the plane beach, as expected. Many waves breaking on the bar *continue* through the trough region as broken waves to account for the totals displayed at Gauge 5.

The theoretical, breaking wave distributions, $p_b(H)$ shown have been computed from the theory of Thornton and Guza (1983). Here, $p_b(H) = W(H) \cdot p(H)$ where $p(H)$ is the Rayleigh distribution and $W(H)$ is a semi-empirical, weighting distribution (model M2) that includes two coefficients (γ , n). Field calibration values of these coefficients ($\gamma = 0.42$, $n = 2$) are used to determine the theoretical curves shown in Fig. 4. Except for Gauge 7 on the plane beach where the shape of $p_b(H)$ is satisfactory but too large, the theoretical $p_b(H)$ appears to *under predict* the large wave heights that are measured to be breaking. However, ensemble average, breaking wave energy dissipation, $\langle \bar{e}_b \rangle$ is determined from the integration of the $p_b(H)$ distribution so that, at least qualitatively, the areas under the distributions are modeled correctly, except for Gauge 7 on the plane beach.

The length of the test run was doubled to 12 minutes giving approximately 600 waves but no significant difference in the measured histograms for all the waves and the breaking wave subsets were noted.

4.3. Energy Dissipation and H_{rms} Distribution

The measured variation of the root-mean-square wave height, $H_{rms}(x)$ for eleven positions across the bar/trough profile is shown in Fig. 5(a). The $H_{rms}(x)$ variation is relatively flat up to the bar crest (near Gauge 4), drops rapidly in the trough region, then recovers again and afterwards drops again as all the wave

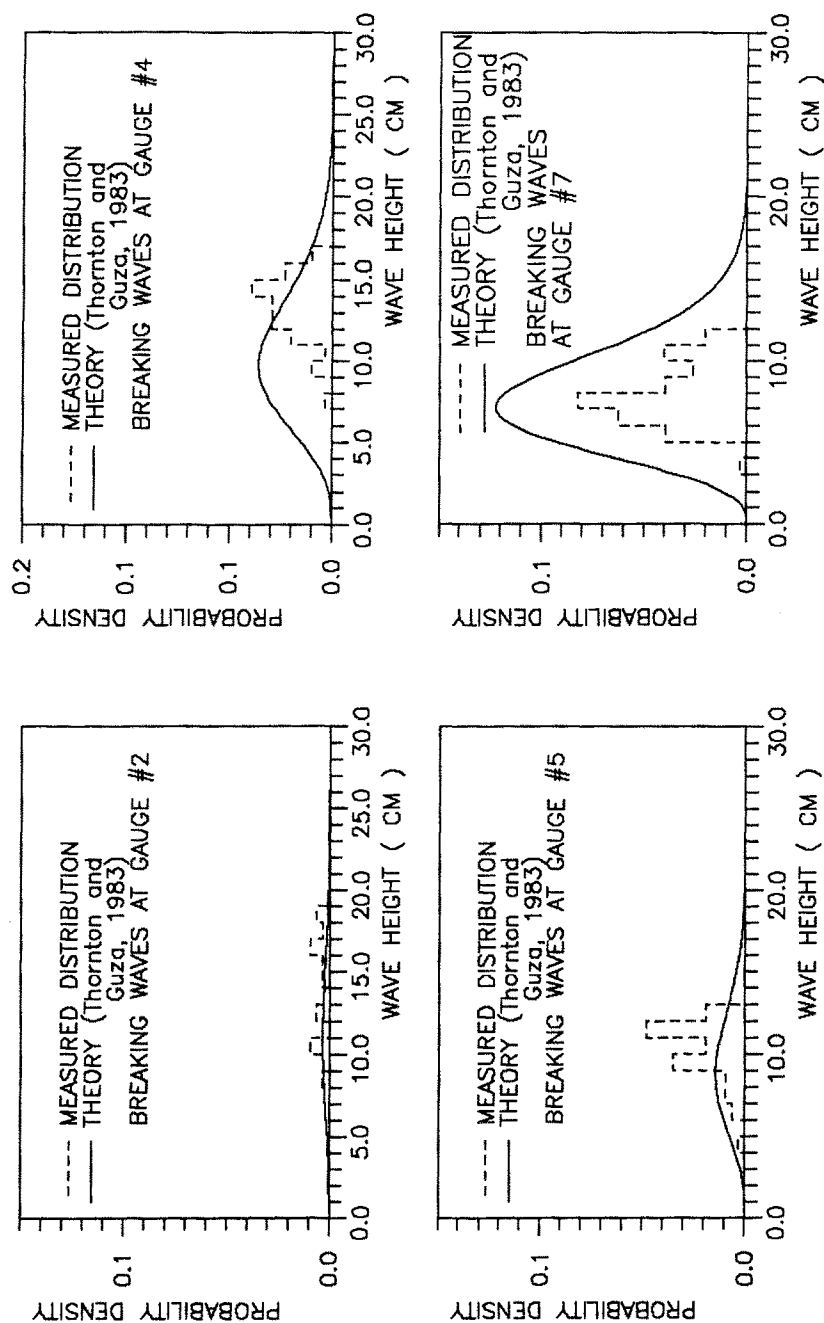


FIG 4. WAVE HEIGHT PROBABILITY DENSITY DISTRIBUTION OF BREAKING WAVES

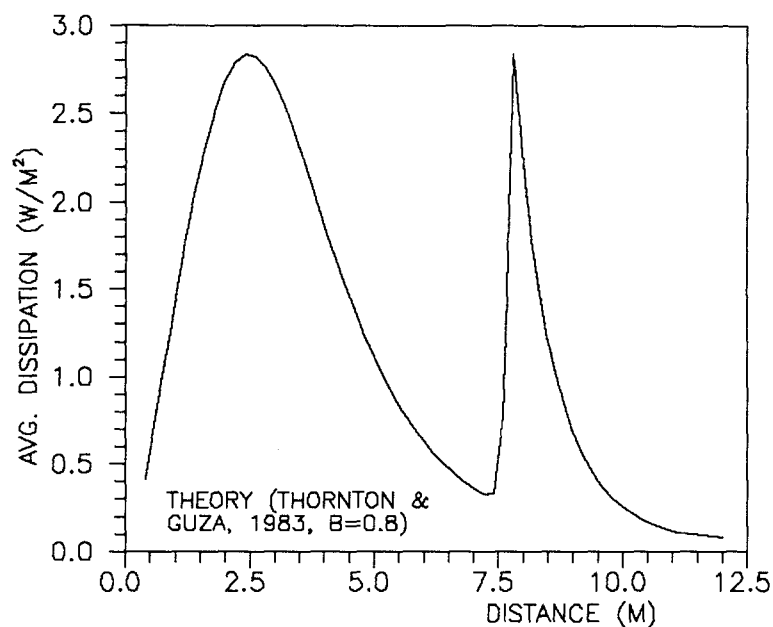


FIG 5b. WAVE ENERGY DISSIPATION ACROSS BEACH

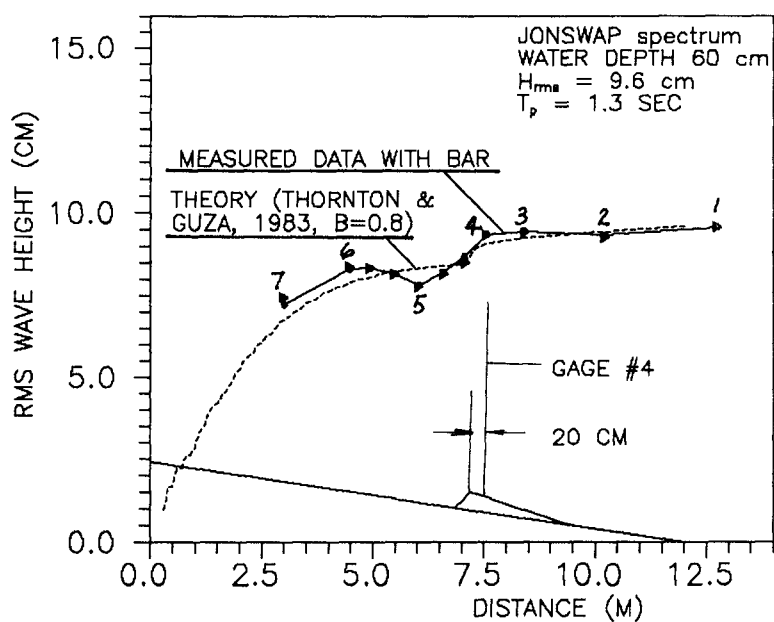


FIG 5a. WAVE HEIGHT DISTRIBUTION ACROSS BEACH

energy is dissipated on the plane beach. Shoaling of individual waves on the slope approaching the bar takes place but so does wave breaking for some of the waves so that the $H_{rms}(x)$ variation actually decreases slightly as the waves approach the bar crest.

Numerical integration of the energy flux balance equation by a simple, finite-difference approximation using the ensemble average, breaking wave energy dissipation, $\langle \bar{e}_b \rangle$ from Thornton and Guza (1983) gives the theoretical curve (dotted line) also shown in Fig. 5(a) for the laboratory bar/trough beach profile. The coefficient chosen to give the fit shown for this laboratory data is $B = 0.8$ where B is the ratio of the broken wave height to the height of turbulence on the front of the broken wave. Thus $B \leq 1$ is physically realistic for this laboratory-scale model. Local wave energy flux is computed from linear wave theory for both the average wave energy, E per unit surface area (i.e. $\frac{1}{2}\rho g H_{rms}^2$) and the group celerity, C_g for the local water depth. The theory with $B = 0.8$ and all other approximations ($\gamma = 0.42$, $n = 2$) gives a "smoothed" fit to the data for $H_{rms}(x)$ but misses the more rapid changes occurring in the trough and wave recovery region on the plane beach. Therefore, even though the theories for the probability distributions $p(H)$ and $p_b(H)$ were somewhat inadequate, the results translated into the $H_{rms}(x)$ distribution are fairly representative of the measured data.

Fig. 5(b) shows the calculated distribution of $\langle \bar{e}_b \rangle$ across the bar region. The large spike in energy loss at the bar is apparent. The modeled energy loss term then drops way off again in the trough region, as physically expected. However, the theory as presently formulated does not appear to permit the $H_{rms}(x)$ variation to *increase* again in the trough region as wave shoaling takes place on the plane beach. The $\langle \bar{e}_b \rangle$ theory extracts too much wave energy in the inner surf zone region on the plane beach and this is postulated to be due to the excessively large area beneath the $p_b(H)$ theory (see Fig. 4, Gauge 7) as compared to that actually measured.

5. Future Directions

5.1. Bores and Breaking Waves

Following LeMehauté (1962) many researchers use the theory for the energy dissipation across a moving hydraulic jump (bore) to approximate the rate of energy dissipation in a single breaking wave, e_b per unit width. Thornton and Guza (1983) took

$$e_b = \frac{1}{4} \rho g C_b \frac{(BH)^3}{d} \quad (1)$$

where C_b is the wave celerity, B is the breaker coefficient as discussed above and d is the local water depth. The average rate of energy dissipation per unit surface area of each wave of length L is then determined as

$$\bar{e}_b \equiv \frac{e_b}{L} = \frac{1}{4} \rho g \frac{C_b}{L} \frac{(BH)^3}{d} \quad (2)$$

or since $c \equiv L/T$

$$\bar{e}_b = \frac{1}{4} \rho g \frac{1}{T} \frac{(BH)^3}{d} \quad (3)$$

so that $\bar{e}_b$ is inversely proportional to wave period, T .

Consider three waves all of equal height but with periods, T of 5, 10, and 15 seconds. After breaking in shallow water due to depth-limiting effects, Equation (3) says average wave energy is dissipated but only at one-third the rate for the 15 second wave compared with the 5 second wave. This seems physically unrealistic because the energy dissipation is concentrated between the trough and crest regions and for trough Froude numbers relative to a moving observer, the lengths of hydraulic jumps, L_j are far less than the wave length, L .

Fig. 6 is taken from the field calibration efforts of Thornton and Guza (1983) for their theory discussed above. Long period swell waves ($T = 13 - 18.2$ seconds) were present and each required a different calibration coefficient, B ranging between 1.3 - 1.7 to get the best fit between measured $H_{rms}(x)$ in the field and their model computation. In essence, the longer period waves reduced $\bar{e}_b$ in (3) and consequently require a larger B coefficient to extract enough energy to get a proper fit for $H_{rms}(x)$. Note also that B values greater than unity are physically unrealistic by definition and that B^3 values needed for "calibration" ranged from 2.2 to 4.9 for these field results. Thornton and Guza (1983) also report setting $B = 0.8$ to calibrate the laboratory measurements of Battjes and Janssen (1972). For laboratory wave periods, L_j is closer to L .

Svendsen, et al. (1978), Stive (1984) and others have argued that the bore model *underestimates* the actual rate of energy dissipation in breaking waves because pressure, momentum and energy distribution coefficients are neglected along with the flux of turbulent energy into and out of the control volume. These factors may account for some of the discrepancy in the use of the bore theory, but using L to define $\bar{e}_b$ may be the most important reason.

5.2. Current Research

Tests are being conducted with a peak wave period of 2.3 seconds to study longer period effects on $\bar{e}_b$ and hence B in the theory. The ratio (L_j/L) used as

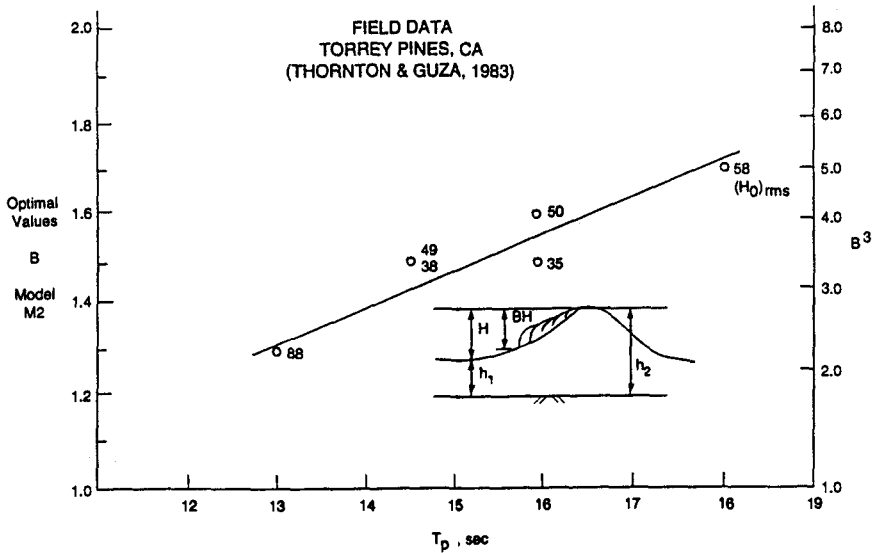


FIG 6. Variation of Calibration Coefficient, B , with Wave Period (from Thornton & Guza, 1983)

a coefficient (keeping $B \approx 1$) will remove the strong dependency of $\bar{e}_b$ on the wave period. Various ways to estimate L_j are being investigated.

Modifications to the probability density functions for all the waves and the breaking waves are also being made to more closely represent shallow water effects on $p(H)$ and deep water effects (wave steepness) on $p_b(H)$.

Field data sets from Duck, NC at the Corps', Field Research Facility (Ebersole and Hughes, 1987) are also being used in this effort.

6. Summary and Conclusions

The transformation of an irregular wave train across a bar/trough beach profile by using the (1) energy flux balance equation including the bore model for breaking waves and (2) a single, statistical wave height, H_{rms} , together with probability functions for all the waves and those breaking as a subset, is a powerful tool for coastal engineers. Laboratory measurements reveal that refinements in the underlying theories are necessary to improve the $H_{rms}(x)$ prediction especially in the wave height recovery region in the trough. These improvements include incorporating physically realistic coefficients for both laboratory and field data sets that are used to modify and verify the theory.

The completed results will be reported in the Proceedings of the next Conference on Coastal Engineering.

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