CHAPTER 249

Influence functions

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Abstract

Influence functions combine part of the strong body of coastal engineering knowledge, the one-line theory, with the most reliable sources of data: position of the shoreline and geological information about the evolution of the shoreline. Influence functions yield a model of the shoreline erosion or -accretion, which is based upon the charteristic length $\sqrt{4at}$. This model can be used to reproduce the present shoreline and predict the changes of the shoreline after human interference. Examples are given of the Petrace delta in Italy and the port construction near the Kelantan Delta in Malaysia.

1. Introduction

Engineers must provide reliable answers based upon unreliable data. Coastal engineers are no exception to this rule. At the start of a project they are faced with scanthy wave data, unreliably or lacking bathymetric data and hardly any information on sediment yield of rivers. They have knowledge about the processes which take care of the longshore distribution of sediment. Clients however often have restricted time and money for the required advice. On these cross-roads the influence functions were born.

The first step on the mathematical modeling of curved shorelines has been made by Pelnard Considère (1956). He showed that the shoreline evolution could be described by the diffusion equation. This theme has been elaborated by various investigators e.g. Walton (1979), Grym (1960,1964) and Bakker and Edelman (1964).

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The application of the diffusion equation in engineering practice is often hampered by the fact that at the initial stage of the project the wave data is not available and in a later stage of the project that both wave data and sediment transportformulae appear to be somewhat unreliable. In the present study therefore a slightly different approach has been taken. A framework of analysis has been set up in which various sources of geological and geophysical information can be combined in the morphological study.

In chapter 2 the theory behind the diffusion equation is explained. Influence functions and some applications are worked out in chapter 3. A large scale study, dealing with the Kelantan delta, is presented in chapter 4, followed by a summary and conclusions.

2. Theory

Consider a stretch of beach as shown in Figure 1.

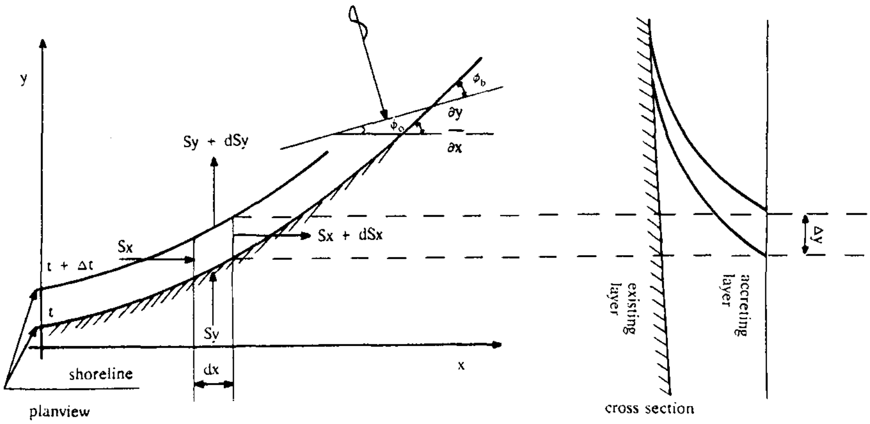

Figure 1: Definitions

It will be assumed that the time scale of changes in the beach profile in the direction normal to the shore is either much shorter or much longer than the time scale of changes in the direction parallel to the coast. This allows for the approximation of a parallel displacement of the beach profile. Furthermore it will be assumed that in all profiles similar sediment transport processes are taking place. Along the beach this leads to parallel depth contours allowing for simple refraction computations.

The erosion or accretion depth h is assumed to be constant along the beach. The continuity equation for the sedimenttransport S than reads:

$$\frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} + h \frac{\partial y}{\partial t} = 0 \qquad 2.1$$

In this study only sedimenttransport induced by wave driven longshore currents will be regarded. The effects of tidal currents on the sediment transport will be ignored. A similar restriction can be found in the work of M. Blosset (1935), where he presents the original conclusions, which form the basis of the famous 1956 Pelnard Considère study. The conclusions of Blosset were obtained from the morphological model studies of the port of Abidjan in Delft Hydraulics.

An important parameter in wave driven longshore transport is the angle ϕ_b between the wavecrest at the breakerline and the local depth contour (see Figure 1). The angle ϕ_b can be expressed in terms of the angle ϕ_o, (angle between the wavecrest and the positive x-axis) and $\partial y/\partial x$, (the angle between the shoreline and the positive x-axis) by:

$$\phi_b = \phi_o - \partial y/\partial x \qquad 2.2$$

In the present study it will be assumed that ϕ_o and $\partial y/\partial x$ are sufficient small, such that the longshore transport S may assumed to be equal to S_x and that S_y can be neglected ($S_y \ll S_x \simeq S$). In the discussion in chapter 5 this point will be taken up again.

Noting that with the chain rule $\partial S/\partial x$ can be expressed as $\partial S/\partial x = \partial S/\partial \phi_b . \partial \phi_b/\partial x$ it follows from formulea 2.1 and 2.2 that:

$$a \frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial t} = 0 \qquad 2.3$$

in wich

$$a = \frac{1}{h} \frac{\partial S}{\partial \phi_b} \qquad 2.4$$

3. Influence functions and applications

3.1 Influence functions

Pelnard Considère (1956) has presented solutions of the diffusion equation 2.3 for the accretion near a breakwater and sediment passing a breakwater. Furthermore Pelnard Considère obtained a solution describing the behaviour of a replenishment. These solutions are reproduced in Table 1 (see also Bakker, 1964).

	accretion near breakwater river delta 	sediment passing a breakwater 	sand replenishments 	definitions
y	$\operatorname{tg}\phi \sqrt{\frac{4at}{\pi}}\, F(u)$	$L\, G(u)$	$\frac{1}{\sqrt{\pi}} \frac{Q}{\sqrt{4at}} H(u)$	$F(u) = e^{-u^2} - u\sqrt{\pi}\, G(u)$
$\frac{\partial y}{\partial x}$	$\operatorname{tg}\phi\, G(u)$	$-\frac{2}{\sqrt{\pi}} \frac{L}{\sqrt{4at}} H(u)$	$-\frac{2}{\sqrt{\pi}} Q\, u\, H(u)$	$G(u) = 1 - \frac{2}{\sqrt{\pi}} \int_0^u e^{-u^2} du$
$\frac{\partial y}{\partial t}$	$\operatorname{tg}\phi \frac{1}{2t} \sqrt{\frac{4at}{\pi}}\, H(u)$	$\frac{1}{\sqrt{\pi}} \frac{L}{t} u\, H(u)$	$-\frac{1}{\sqrt{\pi}} \frac{Q}{\sqrt{4at}} \frac{1}{t} \{\frac{1}{2} - u^2\} H(u)$	$H(u) = e^{-u^2}$
Boundary conditions	$x = 0 \quad S = \text{constant}$ $x = \infty \quad S = 0$	$x = 0 \quad S = 0$ $x = \infty \quad S = \text{constant}$	$x = 0 \quad S = \text{constant}$ $x = \infty \quad S = \text{constant}$	

Table 1: Influence functions

As shown in Table 1 the solutions of the diffusion equation can be regarded as representing the influence in longshore direction of sources and sinks on respectively the shoreline displacements y, shoreline rotation $\partial y/\partial x$ and rate of displacement of the shoreline $\partial y/\partial t$.
It will be shown that complex morphological systems can rather simply be expressed in terms of influence functions. This expression can then be used to obtain an estimate of the parameter $\sqrt{4at}$, describing the shape of the shoreline, based upon a characteristic dimension of the existing shoreline.
The approach is based upon the property of the linear diffusion equation, that the superposition of two solutions of the diffusion equation forms a new solution of the diffusion equation. However this is only true if the boundary conditions for both initial solutions are also satisfied by the superposed solution.

3.2 The riverdelta consisting of one river

The evolution of a river delta, dominated by wave driven longshore transport, can according to the solutions given in Table 1, be described by

$$y = \operatorname{tg}\phi \sqrt{\frac{4at}{\pi}}\, F(u) \qquad 3.2.1$$

where ϕ is the angle between the incoming averaged wave field and the shoreline. The influence of the individual source on the displacement y of the shoreline at a distance x can be expressed in dimensionless terms, by rearranging 3.2.1, dividing by the distance x and noting that $u = x / \sqrt{4at}$.

$$\frac{y}{x \operatorname{tg} \phi} = \frac{1}{\sqrt{\pi}} \frac{1}{u} F(u) \qquad 3.2.2$$

The expression 3.2.2 is shown in Figure 2.

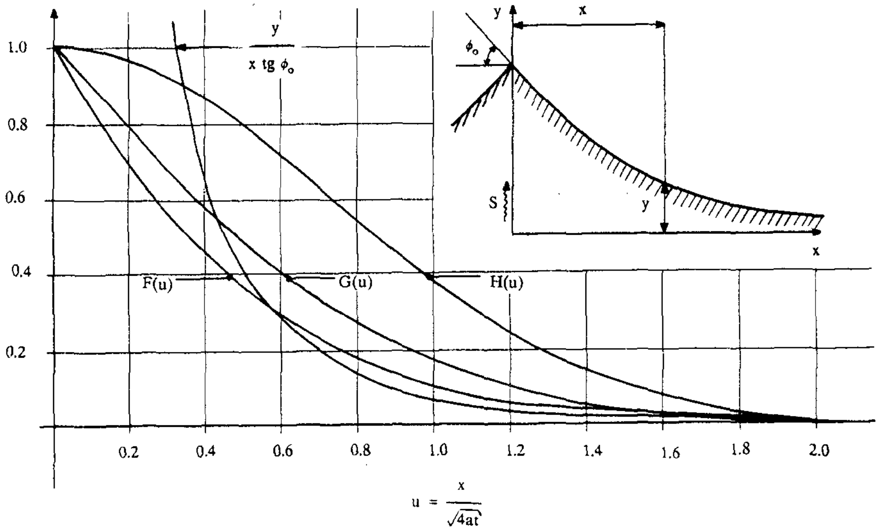

Figure 2: Influence functions

As can be seen from Figure 2 the influence of the source on the shoreline displacement is reduced to 10% of the displacement at the source at a distance of about 0,85 $\sqrt{4at}$. It is interesting to note that a similar influence on $\partial y/\partial x$, represented by G(u) in Figure 1, extends upto 1,2 $\sqrt{4at}$.

3.3 The riverdelta consisting of two rivers

3.3.1 Introduction

The second example is related to a coastal plain near Gioia Tauro in the South of Italy. This coastal plain is formed by two rivers: the Petrace and the Mesima. The Petrace and the Mesima will be represent by two sources with a sediment yield of γQ m^3/yr and Q m^3/yr respectively.

3.3.2 Determination of the value of $\sqrt{4at}$ if the baseline is known.

The evolution of the riverdelta due to a single source is described by 3.3.1. In case of two sources the contributions of the two rivers will be linearly superposed. The width of the delta repectively at the Petrace (x = 0) and the Mesima (x = L) becomes in that case respectively

$$y(o,t) = tg\,\phi \sqrt{\frac{4at}{\pi}} \{ 1 + \gamma F(u1) \} \qquad 3.3.1$$

$$y(L,t) = tg\,\phi \sqrt{\frac{4at}{\pi}} \{ \gamma + F(u1) \} \qquad 3.3.2$$

$$\text{where } u1 = \frac{L}{\sqrt{4at}} \qquad 3.3.3$$

and L is the distance in meters between both rivers.

From a map the angle ϕ_o between the shoreline at the mouth of the Mesima with a line parallel to the baseline can be obtained. This angle ϕ_o is formed by superposition of the angle ϕ in absence of the Petrace and the value of $\partial y/\partial x$ at the Mesima due to the Petrace alone or

$$tg\,\phi_o = \{ 1 + \gamma G(u1) \} \, tg\,\phi \qquad 3.3.4$$

The maximum width of the delta at x = L can be expressed in terms of u1 and known parameters, by combining 3.3.1, 3.3.3 and 3.3.4 rearranging and dividing by L.

$$\frac{y(L,t)}{L\,tg\,\phi} = \frac{1}{\sqrt{\pi}} \frac{1}{u1} \frac{1 + \gamma F(u1)}{1 + \gamma G(u1)} \qquad 3.3.5$$

For the application of Figure 3.3.5 it is essential that y(L,t), e.g. the distance between the baseline and the mouth of the river, is known. In the next example the baseline is unknown and in that case use will be made of a characteristic length of the shape of the shoreline in order to obtain an estimate of $\sqrt{4at}$.

4. Kelantan Minor Port Project

4.1 Introduction

The Kelantan Minor Port will be located in the north-eastern part of Malaysia, along the coast in between the mouths of Kemasin and Pengkalan Datu rivers (see Figure 3). The first part of the study was focussed upon the general morphology followed by an assessment of the morphological consequences of the projected harbour.

4.2 Description of the morhological system

The proposed harbour is located along the border of the delta of the Kelantan. This delta can be traced back upto 18 km inwards. The sediment yield of the remaining rivers on the delta is small relative to that of the Kelantan. West of the mouth of the Kelantan a spit is growing in N.W. direction, pointing towards a predominantly SE-NW sanddrift. East of the Kelantan, at a distance of about 50 km, an ondulation in the delta can be observed. This ondulation is caused by the presence of the offshore Perhentian islands leading to a kind of tombolo formation on the delta. At some stage of the delta formation, probably around 1700, the Kelantan has shifted its course, cutting off the sediment yield to the delta. Parts of the delta are presently eroding.

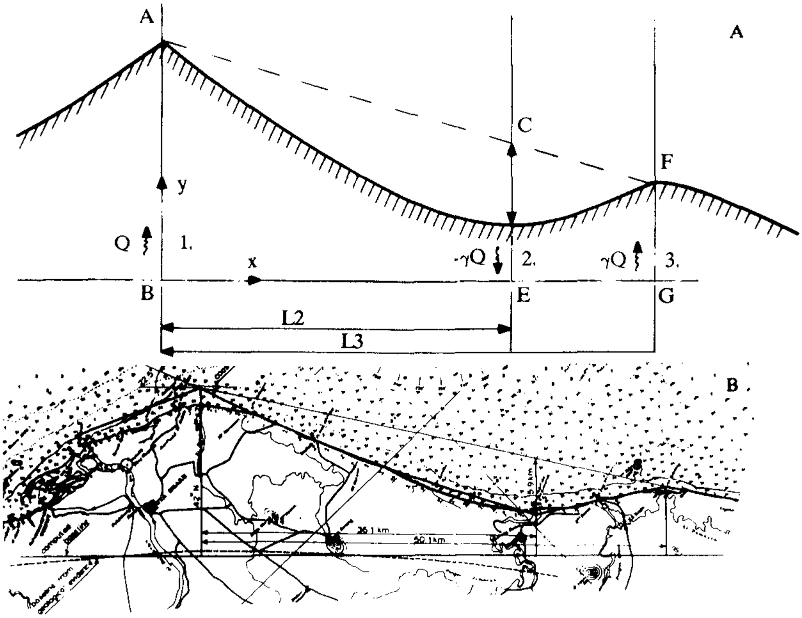

Figure 3: Before (A) and after (B) the change of the Kelantan.

4.3 Mathematical schematisation of the sediment balance

The delta formation has been represented by means of three rivers (Figure 3). The first river is the Kelantan with a sediment yield of Q m^3/yr. The second and third river are jointly representing the ondulation of the delta. Due to the Perhentian islands part of the coast is protected against the incoming wavefield, resulting along the shoreline in a reduced transport capacity a of the waves behind the islands. In order to allow for the continuous SE - NW sandtransport, an increased angle between coastline and incoming wave field in the area has resulted. This feature has been represented by two rivers, which together do not affect the overall sediment balance. The second river is located at a distance of L2 = 36,1 km from the Kelantan and acts like a sink with a negative sediment yield of γQ m^3/yr, where γ is a constant. The third river is located at L3 = 50,1 km from the Kelantan and has a sediment yield of γQ. In the mathematical formulation superposition of a continuous sanddrift from SE to NW does not change the shape of the delta and has therefore not been included. Loss of sediment in offshore direction, out of sediment balance, is not envisaged.

In the mathematical representation of the formation of the Kelantan delta, each point of the delta experiences the influence of the three rivers. These contributions can be expressed in terms of y1(x,t), y2(x,t) and y3(x,t), where the numbers refer to the numbers of the rivers. In terms of the influence functions of chapter 2 these contributions are:

$$y1(x,t) = \mathrm{tg}\,\phi \sqrt{\frac{4at}{\pi}}\; F(u1)\;;\; u1 = |x|/\sqrt{4at} \qquad 4.3.1$$

$$y2(x,t) = -\gamma\,\mathrm{tg}\,\phi \sqrt{\frac{4at}{\pi}}\; F(u2)\;;\; u2 = |x - L2|/\sqrt{4at} \qquad 4.3.2$$

$$y3(x,t) = \gamma\,\mathrm{tg}\,\phi \sqrt{\frac{4at}{\pi}}\; F(u3)\;;\; u3 = |x - L3|/\sqrt{4at} \qquad 4.3.3$$

The resulting delta is formed by superposition of these three contributions. For practical reasons use will be made of one parameter for instance u2 instead of the three parameters u1, u2 and u3. This can be achieved by simple algebra. The formation of the delta in dimensionless form follows from superposition of y1, y2, y3, dividing by a length scale, say L2, and rearranging:

$$\frac{y(x,t)}{\mathrm{tg}\,\phi\, L2} = \frac{1}{\sqrt{\pi}}\,\frac{1}{u22}\left\{ F\left(\frac{|x|}{L2}\,u22\right) - \gamma\, F\left(\frac{|x-L2|}{L2}\,u22\right) + \gamma\, F\left(\frac{|x-L3|}{L2}\,u22\right)\right\}$$

where $u22 = L2/\sqrt{4at}$ 4.3.5

In this representation of the Kelantan delta $tg\phi$ is the angle between the baseline and the shoreline near the mouth of the Kelantan in absence of rivers 2 and 3. The combined effects of the three rivers on $tg\phi$ results in $tg\phi_o$, where ϕ_o can be measured from a map near the mouth of the Kelantan.

$$tg\,\phi_o = tg\,\phi\,\{\,1 - \gamma\,G(u22) + \gamma\,G(\frac{L3}{L2}\,u22)\,\} \qquad 4.3.6$$

In this formulation the first term between brackets represents the influence of the Kelantan on $tg\phi_o$(note that $G(o) = 1$), the second term refers to river 2. The third term refers to river 3, but has been expressed in terms of 4.2.2.

Introducing 4.3.6 in 4.3.4 leads to expression 4.3.7.:

$$\frac{y(x,t)}{tg_0\,\phi\,L2} = \frac{1}{\sqrt{\pi}}\,\frac{1}{u22}\,\frac{\{\,F(\frac{|x|}{L2}\,u22\,) - \gamma\,F(\frac{|x-L2|}{L2}\,u22\,) + \gamma\,F(\frac{|x-L3|}{L2}\,u22\}}{\{\,1 - \gamma\,G(u22) + \gamma\,G(\frac{L3}{L2}\,u22)\,\}} \qquad 4.3.7$$

representing the shape of the delta in terms of known variables and one parameter u22. In the next paragraph the value of u22 or in fact the value of $\sqrt{4at}$ will be determined from a characteristic dimension of the shape of the delta.

4.4 Determination of the value of $\sqrt{4at}$ and γ from the shape of the delta.

As a characteristic dimension of the shape of the delta in Figure 5 the distance CD has been selected. Obviously the width of the delta AB could also have been selected, however at the beginning of the project the position of the baseline was not known and the influence of river 2 and 3 is rather small for a large range values of $\sqrt{4at}$.

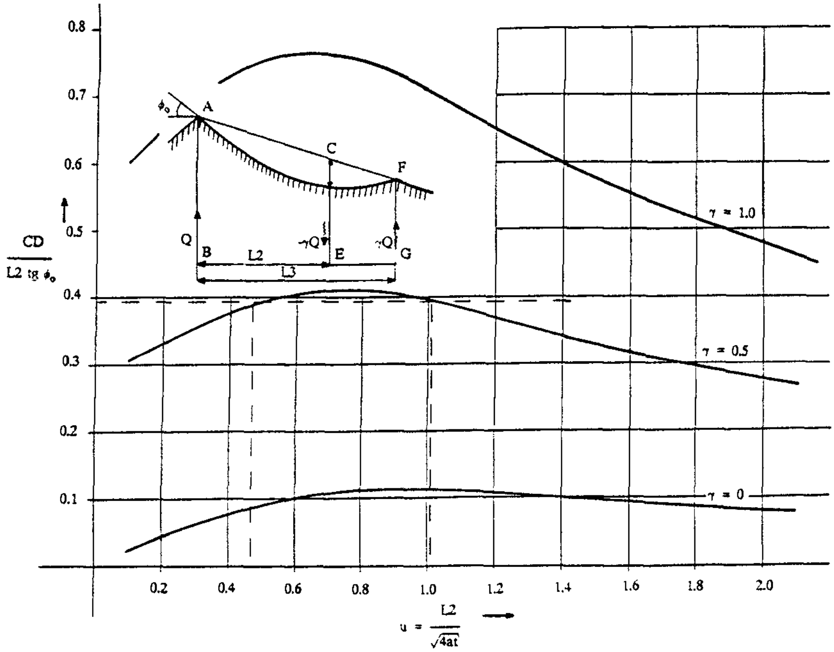

Figure 4: Characteristic dimension CD, expressed in terms of $\sqrt{4at}$.

The distance CD in Figure 4 can be expressed in geometrical dimensions of the delta as follows:

$$CD + DE = AB - (AB - FG)\, L2/L3 \qquad 4.4.1$$

In terms of x, y and t, equation 4.4.1 can be expressed as:

$$CD + y(L2,t) = y(o,t) - \{\, y(o,t) - y(L3,t) \,\}\, L2/L3 \qquad 4.4.2$$

Taking the angle near the mouth of the Kelantan as the reference angle, the following expression of CD in terms of u22 can be obtained:

$$\frac{CD}{L2 \operatorname{tg} \phi_o} = \frac{1}{L2 \operatorname{tg} \phi_o} \{\, y(o,t) - y(L2,t) - (y(o,t) - y(L3,t))\, L2/L3 \,\} \qquad 4.4.3$$

where the displacement of the delta y(x,t) is described by 4.3.7.

The functional relation between CD, u22 and γ, as represented by equation 4.4.3, is shown in Figure 4.

The following geometrical parameters have been selected for the Kelantan delta: CD = 5,900m, L2 = 36,100m, L3 = 50,100m and ϕ_o = 22,5°. The value of parameter CD/(L2 tg ϕ_0) equals 0,395. According to Figure 4 both u = 0,47 and u = 1,07 satisfy 4.4.3 for γ = 0,5. It is interesting to note that there are two values of u (u_1 = 0,47 and u_2 = 1,07) which satisfy CD = 5,900m. This implies that both at time t_1 and t_2 after the start of the build up of the delta, that CD/(L_2 tg_o) = 0,395. For a constant value of a and L, it follows from the definition of u that:

$$\frac{t_2}{t_1} + \left(\frac{u_1}{u_2}\right)^2 \qquad 4.4.4$$

If the present shoreline is the result of a continuous process of t_1 = 6000 years, that at t_2 = 1150 years after the start of the build up of the delta CD reached also the present value. This is an example of many valuedness in the analysis of shoreline evolution.
In the next paragraph the selected values of γ and $\surd$4at will be validated.

4.5 Validation of the values of $\surd$4at and γ.
The shoreline

A fair reconstruction of the shoreline by means of influence functions is shown in Figure 3. The following values of the parameters have been used: $\surd$4at = 75,790m, γ = 0,5, ϕ_o = 22.5°.

The base line

On basis of a LANDSAT image of the Kelantan delta the alignment of the various series of beach ridges could be reconstructed. In Figure 3 a tentative line is drawn to indicate the delta growth over the last 6000 years. The distance between baseline and the mouth of the Kelantan is roughly 18,000m. A theoretical estimate of the position of the baseline, using 3.2.1 and realising that x = 0 or F(u) = 1 at the origin of the river, than 3.2.1 leads to a value of y = 17,711m. The fact that the effect of the rivers 2 and 3 on the width of the delta near the Kelantan is small, is due to the fact that the location of both rivers is of the same order of magnitude as the length scale $\surd$4at in the diffusion equation, and that both rivers are of equal magnitude, but of opposite sign.

Sediment yield of the river.

The natural geological rate of erosion in the Kelantan delta is in the order of $100m^3/km^2/yr$ from the totally undisturbed catchment area. The yearly sediment yield of the Kelantan would be in the order of 1,300,000 m^3/yr. In the last century, due to human activities, this figure has increased to 1,700,000 m^3/yr. Only the bedload will contribute to the growth of delta. From measurements the bedload appeared to be 50% of the total load, leading to a sediment yield of the Kelantan in the order of 700.000 m^3/yr to 800,000 m^3/yr.

A numerical estimate of the sediment yield Qt of the Kelantan can be obtained from the continuity equation. Near the mouth of the river this formula leads to:

$$Qt = 2St = 2at\ h\ tg\ \phi \qquad 4.5.2$$

Assuming an accretion depth of 4 m. to 5 m., this leads to a total sediment yield of $4,8 * 10^9\ m^3$ to $5,9 * 10^9\ m^3$. According to the geological study the present delta formation has continued over the last 6000 years, leading to an annualy sediment yield of the Kelantan of 790.000 m^3/yr to $991.000 m^3/yr$.

The conclusion is that on all three aspects: the shape of the shoreline, the position of the baseline and the total sediment the theoretical approximation and the selected parameters yield a reasonable approximation.

5. Summary and remarks

The approach presented in this paper has been tested in various coastal morphological projects, were wave drive longshore sedimenttransport played a dominant role. In the analysis the morphological project is placed within a larger framework, allowing for various disciplines to contribute. The experience is that application of the approach is rather straightforward, highlighting the important aspects and conflicting sources of data and theory for a specific site.

The use of influence functions requires a clear perception of the large scale morphology. The set up of the equations such as shown in paragraph 4.3. and 4.4 is rather straight forward. Personal Computers are particularly suitable for the construction of the dimensionless graphs as shown in Figures 2 and 4. Reconstruction of the present coastline using the obtained values of $\sqrt{4at}$ requires a similar approach as for the construction of the dimensionless graphs. For the verification of the longterm morphology, information can be used from geological and geophysical sources. For calibration of recent developments particularly use is made of the rate of accretion in recent decades.

The framework of analysis allows for highlighting

conflicting data sources and the resulting effects on the large scale morphology if either of the data sources would be true. Equally important however is that influence functions can be used to form a test for the basic hypothesis that the longshore transportproces related to ϕ_{br} is the dominant proces for the coast under consideration. If this is not the case a different approach than the one-line theory should be devised. An example is the offshore transport term S_y.
However the concept of influence functions remains applicable for those beaches where offshore transport can be represented as : S_y = constant or S_y = constant fraction of the longshore transportgradient $\partial S/\partial x$. The effect on the shape of this offshoretransport on the present shoreline in that case is nil. Obviously the effect can be noted in the large scale delta evolution and in the total volume of longshore transport.

In some morphological projects it appeared that long term trends in beach evolution differed considerably from recent trends in beach evolution. In that case the values of the parameters for the recent evolution were used for prediction of future changes. Within the realm of the projects it was not possible to find the cause for the changing trends on different time scales. It may be that if experience from more sites are combined a coherent picture may emerge, for instance the effects of a rise of mean sea level.

Acknowledgement
The theory of influence functions has been developed by the author in the period 1976 - 1987. In this period the author was employed as an project engineer of Delft Hydraulics. The theoretical development was initiated by the investigations for the Gioia Tauro Port project. The Kelantan Minor Port project, carried out by mr. W. Tilmans led to a further elaboration and illustration of the theory. The author acknowledges the permission of collegues, Delft Hydraulics and the clients of the Gioia Tauro Port project and the kelantan Minor Port project to allow publication of the results.

Notation

a	diffusion coefficiënt
A	constant
C	characteristic dimension of a beach plan
c_o	deep water wave celerity
F	influence function for shoreline displacement
G	influence function for shoreline rotation
h	depth of accretion
H_o	significant deep water wave height
H	influence function for rate of accretion of the shoreline
L, L_2, L_3	distance between two rivers

Q	sediment yield of the river
S	sediment transport rate
t	time
u	$x/\sqrt{4at}$; parameter
u1	idem $L/\sqrt{4at}$
u2	idem $\lvert x - L2 \rvert/\sqrt{4at}$
u3	idem $\lvert x - L3 \rvert/\sqrt{4at}$
u22	idem $L2/\sqrt{4at}$
x	co-ordinate axis
y	co-ordinate axis
γ	ratio of the sediment yield of two rivers
η	reduction factor of the sediment yield of the river
ϕ_b, ϕ_o	angle between deep water wave crest respectively the breakerline and the positive x-axis
subscripts	
b	breakerline, y
x,y	parallel to x,y-axis
pet., mes.	Petrace, Mesima
1,2,3	refering to river 1,2 or 3
0	deep waterwave characteristicts

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