CHAPTER 231

Estuary Geometry as a Function of Tidal Range

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Abstract

The tidal characteristics, and consequently the morphology, of an estuary can be altered substantially by engineering works. A method is suggested for estimating the magnitude and the time-scale of the resulting morphological changes. This method is then applied to the Welsh River Usk which will be affected by the construction of a planned tidal power barrage across the nearby Severn Estuary.

Introduction

Engineering works in an estuary, such as dredging, training walls, reclamation, storm surge barriers and tidal power barrages, can cause a marked change in the upstream tidal characteristics. This, in turn, can lead to changes in the geometry of the estuary and any sub-estuaries, provided that there are sources of mobile sediment. As these alterations have an impact on such factors as navigation and land drainage, there is a need to predict their magnitude and time-scale.

A method of predicting the new equilibrium shape of a disturbed estuary system containing fine sediment and the time needed to attain this shape is outlined herein. The proposed method is then applied to a real situation, namely the River Usk which is located in South Wales and is a tributary of the River Severn. The latter is the site of a planned tidal power barrage which will reduce the existing 12m spring tidal range to just over 4m.

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Method

Two possible methods of determining the new geometry of a disturbed estuary are the detailed approach and the regime approach. The first method involves the use of two-dimensional, or even three-dimensional, numerical models of tidal motion and sediment transport. The second method, on the other hand, involves a simple one-dimensional tidal model in combination with empirical expressions which relate the flow field to the estuary geometry. The advantage of the detailed approach is the high level of accuracy but this is offset by appreciable running costs; the opposite holds true for the regime approach. Because of its simplicity and ease of operation, the regime approach is the basis for the proposed method.

The first empirical expression of the regime approach relates the estuary cross-sectional area to the tidal prism. If it is assumed that the tidal discharge at a given cross-section varies sinusoidally with time, then

$$P = \int_0^{T/2} Q_{MAX} \sin(2\pi t/T_T)\, dt$$

$$= Q_{MAX} T_T/\pi \qquad \ldots.(1)$$

where P = tidal prism (m^3); Q_{MAX} = maximum discharge during a tidal period (m^3/s); t = time (s); T_T = tidal period (s). In addition,

$$Q_{MAX} = A_{MWL} U_{MAX} \qquad \ldots.(2)$$

where A_{MWL} = cross-sectional area below mean water level (m^2); U_{MAX} = maximum cross-sectionally-averaged flow velocity during a tidal period (m/s). Hence, combining Eqs. 1 and 2 yields

$$A_{MWL} = (\pi/(U_{MAX} T_T))\, P \qquad \ldots.(3)$$

However, field studies (Bruun, 1967) have established that U_{MAX} is approximately constant along the length of a stable estuary, in which case Eq. 3 reduces to

$$A_{MWL} = a.P \qquad \ldots.(4)$$

where a is a constant. In practice, work carried out by O'Brien (1931), and later modified by Jarrett (1976), indicated that for a stable estuary

$$A_{MWL} = a(P_s)^b \qquad \ldots.(5)$$

where b is a constant; P_s = spring tidal prism (m^3).

The second empirical expression relates the centreline depth at mean water level of a cross-section, H_{MWL}, to the spring tidal prism. In a study of North American inlets, Vincent and Corson (1981) found that H_{MWL} is directly related to A_{MWL}. Hence, from this information and from Eq. 5, it follows that

$$H_{MWL} = H_{MWL}(P_s) \qquad \ldots.(6)$$

Also in a study of North American inlets, Bruun (1978) concluded that the mean water level width of a cross-section, W_{MWL}, is a function of H_{MWL}. This function, in combination with Eq. 6, therefore, yields the third empirical expression

$$W_{MWL} = W_{MWL}(P_s) \qquad \ldots.(7)$$

It can also be shown that, under idealised flow conditions, W_{MWL} varies exponentially with distance downstream of the head of the estuary. Details are given in Appendix 1.

The fourth and final empirical expression concerns the shape of a cross-section, an idealised version of which is shown in Fig. 1. The cross-sectional shape is described by the relationship

$$W/W_{MWL} = (H/H_{MWL})^S \qquad \ldots.(8)$$

where W = cross-sectional width (m) at any elevation H(m); S = side slope at the mean water level. It is argued that S is a function of the local sediment type which, in turn, is a function of the local flow conditions as represented by P_s. This line of reasoning leads to

$$S = S(P_s) \qquad \ldots.(9)$$

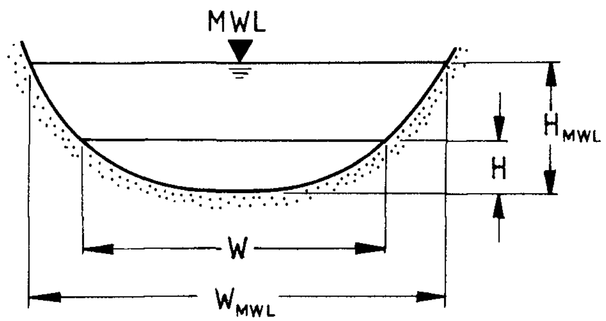

Figure 1. Cross-Sectional Shape

The determination of the new estuary geometry is carried out using Eqs. 5 to 9 in conjunction with a one-dimensional tidal model. The sequence of operations is shown in Fig. 2. First of all, the tidal model is run for the original estuary geometry and the original hydrodynamic boundary conditions, the aim being to establish the exact form of the empirical relationships (Eqs. 5, 6, 7 and 9). The resulting P_s - A_{MWL} relationship (Eq. 5) is then compared with the Jarrett version of this curve in order to check that the estuary was originally in a state of equilibrium. Assuming this to be so, the tidal model is then run again but this time for the original estuary geometry and the new hydrodynamic boundary conditions. This yields a new set of P_s values for various locations along the length of the estuary. Eqs. 5 to 9 are then used to give a first estimate for the new estuary geometry. The latter half of the above cycle of operations is then repeated until the P_s - A_{MWL} relationship converges to the version associated with a stable estuary.

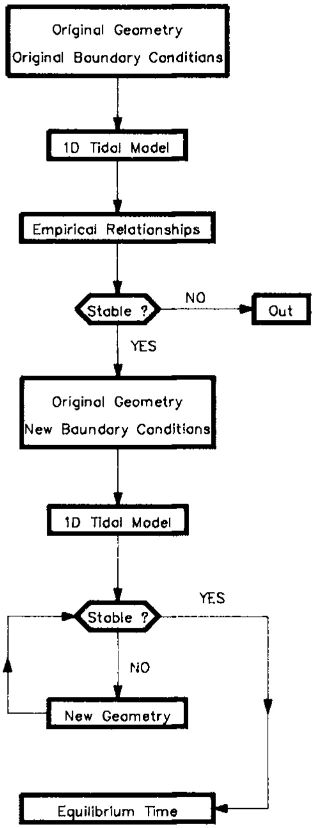

Figure 2. Sequence of Operations

The method used to compute the time taken by a disturbed estuary to attain a new state of equilibrium requires, firstly, that a relationship be established between mean sediment concentration and tidal range. This can only be done using available field data and makes it possible to derive the quantity of sediment carried into the estuary by any flood tide occurring during the lunar tidal cycle. Then, if an assumption is made regarding the fraction of the suspended sediment load deposited per tidal period, the rate of change in the estuary capacity immediately after the estuary is disturbed can be computed. Next, it is assumed that the rate of change of the estuary capacity varies exponentially with time until nominal equilibrium has been attained; a typical value for the latter is 95 per cent, say, of the capacity change. Finally, knowing the total quantity of siltation (from the difference between the original and the new estuary geometries), integration of the area under the curve describing the rate of change of the estuary capacity with time yields a solution for the total time required to reach equilibrium. Full details are given in Appendix 2.

Application

The proposed method of establishing the new equilibrium shape of a disturbed estuary was applied to the River Usk. This river, the location of which is shown in Fig. 3, discharges into the Severn Estuary and it is planned that a tidal power barrage be built across the latter. The River Usk is approximately 120 km long, of which the lower 27 km are tidal, and it has a mean spring tidal range at its mouth of just over 12 m. After construction of the barrage, the mean spring tidal range will be reduced to 4.3 m.

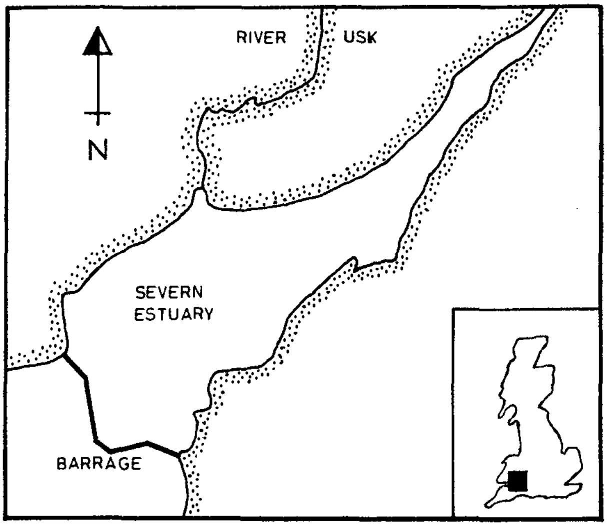

Figure 3. Location of the River Usk

The new estuary geometry was determined as described in Fig. 2. Following the running of a one-dimensional tidal model for the pre-barrage layout and hydrodynamic boundary conditions, the resulting relationships between P_s and A_{MWL}, H_{MWL}, W_{MWL} and S are as shown in Figs. 4 to 7 respectively. As can be seen, there is a strong correlation between the spring tidal prism and the first three variables but the situation is less clear for the fourth variable, S.

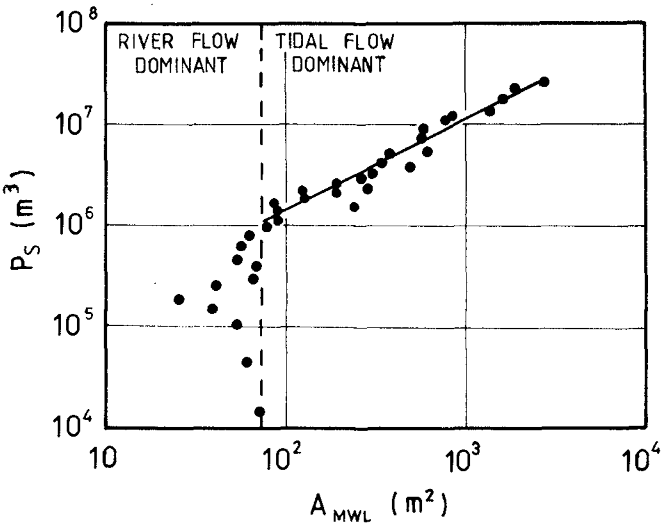

Figure 4. P_s versus A_{MWL}

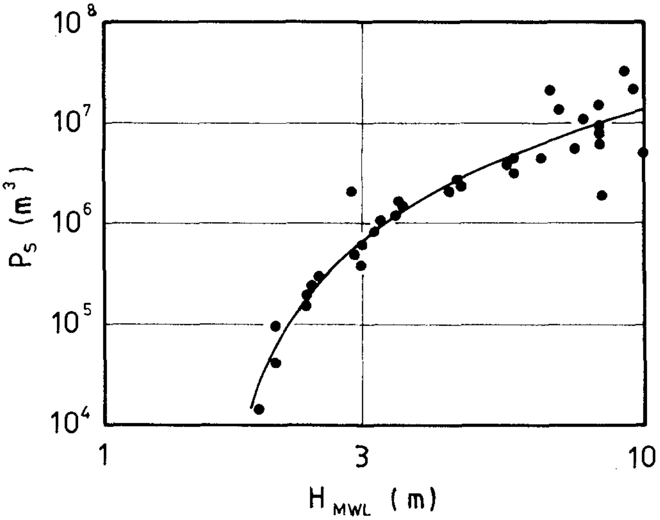

Figure 5. P_s versus H_{MWL}

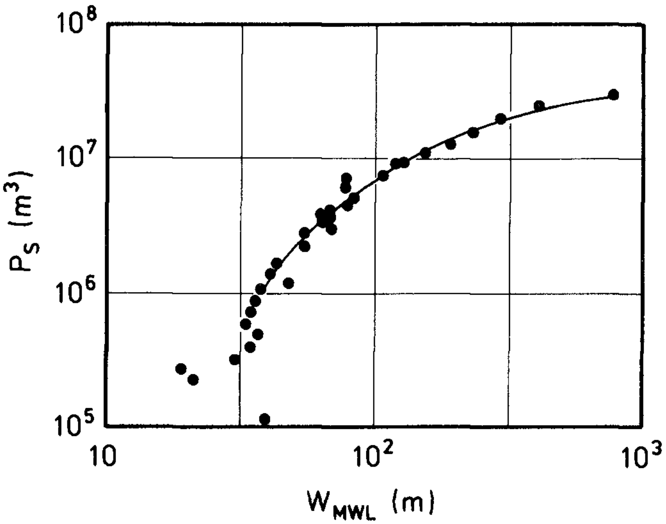

Figure 6. P_s versus W_{MWL}

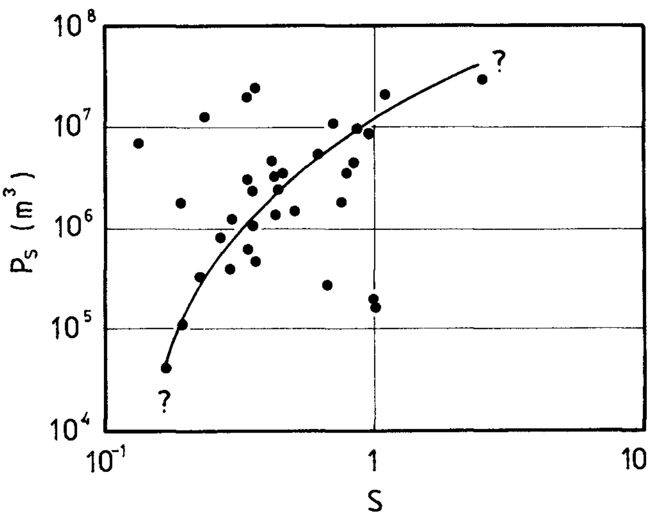

Figure 7. P_s versus S

The stability of the existing layout was then checked by comparing the form of the P_s - A_{MWL} curve with that obtained by Jarrett (1976) for stable estuaries. This comparison is contained in Fig. 8 which shows a close measure of agreement between the two curves. It was concluded, therefore, that the existing layout of the River Usk is in a state of equilibrium.

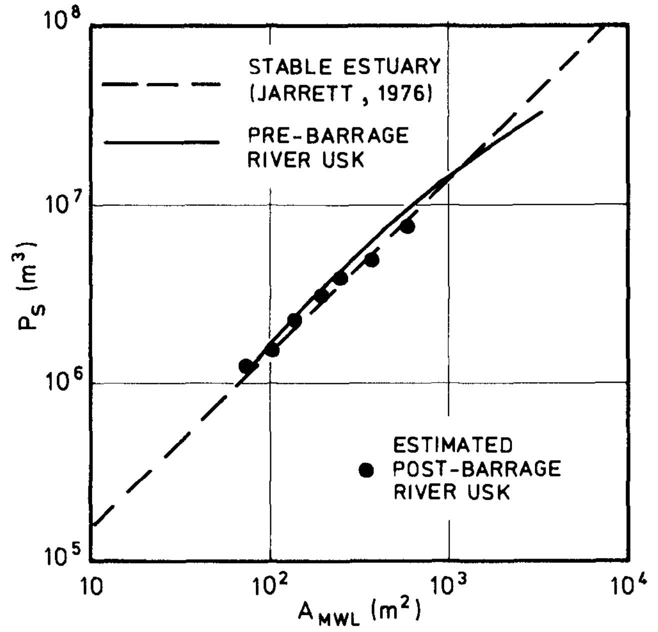

Figure 8. P_s versus A_{MWL}

The one-dimensional tidal model was then run again, this time for the pre-barrage estuary geometry but the post-barrage hydraulic boundary conditions. This yielded a first estimate for the new estuary geometry which, following one more circuit of the tidal model computational loop (see Fig. 2), resulted in a P_s -A_{MWL} curve which closely matched that of Jarrett; the comparison is included in Fig. 8. The model output produced at this point, therefore, was considered to be an acceptable estimate for the final equilibrium geometry of the post-barrage estuary. An example of the pre-barrage and post-barrage shapes of a typical cross-section is shown in Fig. 9.

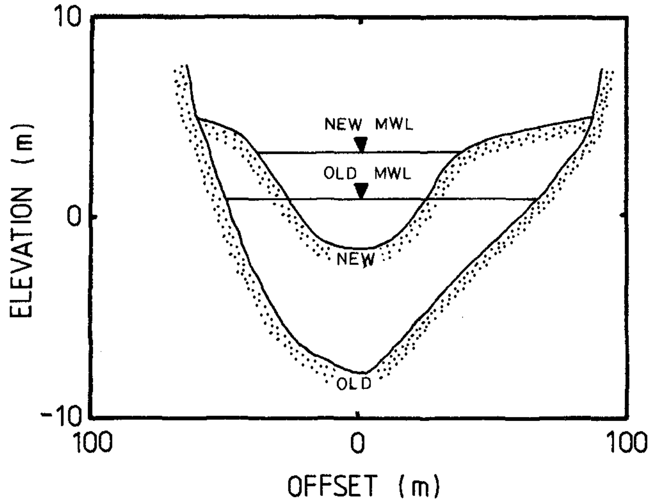

Figure 9. Typical Old and New Cross-Sections

The time required to attain a new state of equilibrium was computed as described in Appendix 2. Values adopted for the relevant independent variables were as follows:

F_E = 0.95 $\quad$ R_N = 2.8 m $\quad$ ρ_D = 1100 kg/m^3

P_N = 8 10^6 m^3 $\quad$ R_s = 4.3 m

P_s = 12 10^6 m^3 $\quad$ V_{TOT} = 6.3 10^6 m^3

where F_E = fraction of equilibrium attained; P_N = neap tidal prism at the estuary mouth (m^3); R_N and R_s = neap and spring tidal ranges at the estuary mouth (m) respectively; V_{TOT} = total estuary capacity change (m^3); ρ_D = dry density of the bed material (kg/m^3). The results of this exercise are set out in Fig. 10, the estuary-mean spring tidal concentration, C_s, and the fraction of the sediment load deposited, F_D, being treated as variables. If best estimates of 0.5 g/ℓ and 0.10 are adopted for C_s and F_D respectively, then the computed equilibrium time, T_E is approximately 180 years.

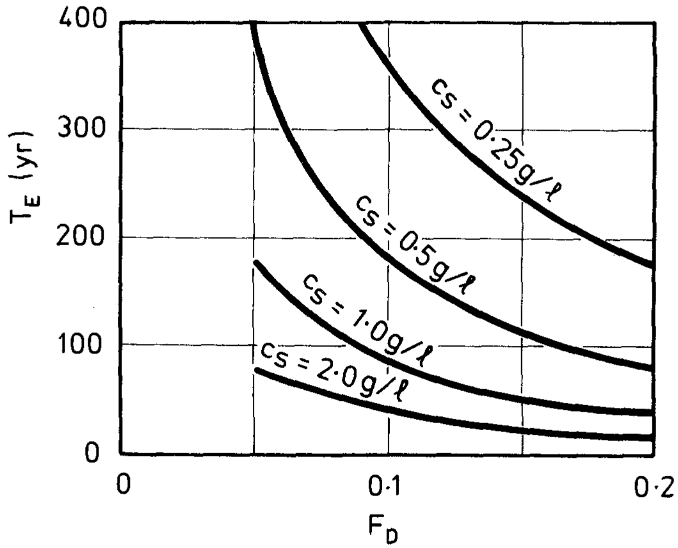

Figure 10. T_E versus F_D

Conclusions

The suggested method for estimating the new morphology and the equilibrium time of a disturbed estuary has been applied to a real situation. The indications are that the suggested method can be operated without undue difficulty and that it yields meaningful results. It is necessary, however, that the method be checked against a situation for which both pre-disturbance and post-disturbance information is available.

Acknowledgements

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Symbols

a,b = constants;
A = cross-sectional area (m^2);
C = estuary-mean concentration (g/ℓ);
F = fraction;
H = depth (m);
P = tidal prism (m^3);
Q = discharge (m^3/s);
R = tidal range (m);
S = side slope;
t = time (s);
T = period (s);
U = flow velocity (m/s);
V = volume (m^3);
W = width (m);
ρ = density (kg/m^3).

Subscripts:

D - dry;
E - equilibrium;
MAX - maximum;
MWL - mean water level;
N - neap;
S - spring;
T - tidal;
TOT - total.

Appendix 1 : Variation of W_{MWL} along the Length of the Estuary

Conservation of fluid mass, assuming a sinusoidal variation of tidal discharge with time, requires that

$$\partial Q_{MAX}/\partial x + W_{MWL}(\partial \eta/\partial t)_{MWL} = 0 \qquad \ldots.(1.1)$$

where x = distance downstream of the head of the estuary (m); η = elevation of the water surface above some arbitrary datum (m); the other symbols are as defined in the text. However,

$$Q_{MAX} = A_{MWL} V_{MAX} \qquad ....(1.2)$$

where

$$A_{MWL} \approx H_{MWL} W_{MWL}/2 \qquad ....(1.3)$$

Hence, combining the above equations gives

$$\partial(H_{MWL} W_{MWL} V_{MAX})/\partial x + 2 W_{MWL}(\partial\eta/\partial t)_{MWL} = 0 \qquad ....(1.4)$$

If it is also assumed that H_{MWL}, V_{MAX} and $(\partial\eta/\partial t)_{MWL}$ are independent of x, then integrating the above equation with respect to x yields

$$W_{MWL} = (W_{MWL})_o \exp(\alpha x) \qquad ....(1.5)$$

where $(W_{MWL})_o$ = mean water level width of a cross-section at the head of the estuary and α is a constant. Eq. 1.5 is the well known exponential shape of an 'ideal' estuary (McDowell and O'Connor, 1977).

Appendix 2 : Derivation of the Time to attain Equilibrium

Assume, for simplicity, that there is a linear relationship between suspended sediment concentration and tidal range, so that

$$C = \beta R \qquad ....(2.1)$$

where C = estuary-mean concentration (g/ℓ); R = tidal range (m); β is a constant. Then, because R varies sinusoidally with time,

$$C = \beta((R_N + R_s)/2 + (R_s - R_N) \sin(2\pi t/T_L)/2) \qquad ....(2.2)$$

where R_N and R_s are the neap and spring tidal ranges, respectively, at the estuary mouth (m); t = time (s); T_L = lunar period (s). In addition,

$$P = (P_N + P_s)/2 + (P_s - P_N) \sin(2\pi t/T_L)/2 \qquad ....(2.3)$$

where P, P_N and P_s are the typical, neap and spring tidal prisms, respectively, at the estuary mouth (m^3). Furthermore, by definition,

$$M = (1/T_L) \int_0^{T_L} C\, P\, dt \qquad ....(2.4)$$

where M = mass of sediment entering the estuary per tidal period averaged over a lunar cycle (kg). Hence, combining Eqs. 2.1 to 2.4 yields

$$M = \beta((P_N + P_s)(R_N + R_s)/4 + (P_s - P_N)(R_s - R_N)/8) \qquad ....(2.5)$$

Next, the initial change in the estuary capacity per tidal period averaged over a lunar cycle, V_o, can be described by

$$V_o = D/\rho_D \qquad \text{....(2.6)}$$

where D = deposition per tidal period averaged over a lunar cycle (kg); ρ_D = dry bed density (kg/m^3). However,

$$D = F_D . M \qquad \text{....(2.7)}$$

where F_D = fraction of sediment load deposited and

$$\rho_D = \rho_s(\rho_B - \rho_F)/(\rho_s - \rho_F) \qquad \text{....(2.8)}$$

where ρ_B = bulk bed density (kg/m^3); ρ_F = fluid density (kg/m^3); ρ_s = sediment density (kg/m^3). Hence, combining Eqs. 2.5 to 2.8 results in an expression for V_o containing variables the values of which are known or can be estimated.

Assume that the variation of the capacity change with time is of the form

$$N = \exp(\gamma(1 - V/V_o)) - 1 \qquad \text{....(2.9)}$$

where N = number of tidal periods; V = typical value of V_o (m^3); γ is a constant. However, at equilibrium, Eq. 2.9 becomes

$$N_E = \exp(\gamma(1 - V_E/V_o)) - 1 \qquad \text{....(2.10)}$$

where N_E and V_E are the equilibrium values of N and V, respectively. In addition, it is assumed that

$$V_E = (1 - F_E)\, V_o \qquad \text{....(2.11)}$$

where F_E = nominal fraction of equilibrium attained. Combining Eqs. 2.9 to 2.11 then gives

$$V = V_o(1 - F_E\ \ell n(1 + N)/\ell n(1 + N_E)) \qquad \text{....(2.12)}$$

However, by definition,

$$V_{TOT} = \int_0^{N_E} V\, dN \qquad \text{....(2.13)}$$

where V_{TOT} = total change in the estuary capacity (m^3). Hence, from Eqs. 2.12 and 2.13

$$V_{TOT} = V_o\ F_E\ N_E/\ell n(1 + N_E) - V_o(F_E - N_E + F_E\ N_E) \qquad \text{....(2.14)}$$

Eq. 2.14 is solved by trial and error for N_E, F_E being known or estimated, V_o being obtained from Eqs. 2.5 to 2.8 and V_{TOT} being derived from the known change in the estuary geometry.