CHAPTER 162

Method for Prediction of Bar Formation and Migration

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Abstract

A computational model based on sediment conservation and transport equations is presented. The bottom shear stress associated with breaking waves is the primary forcing function for the model. A breaking wave model is used to calculate distributions of wave height and mean water level setup across the surf zone and the profile elevations are updated at the end of every time step for each grid across the profile. Examples of the model prediction using large wave tank data are presented. The results of the tests are encouraging for the prediction of bar formation and migration.

Introduction

A significant feature normally found in beach profiles immediately following a storm is the offshore bar, a shore parallel deposition with a scour trough on the landward side. This feature forms as a result of large wave heights, and short wave periods. Once a bar forms, it can dissipate substantial quantities of wave energy thereby reducing the energy reaching the shore and limiting erosion of the berm and dune. After storm water levels and wave conditions have returned to their normal state, the bar usually acts as a significant element in the recovery process of the beach profile. The sediment stored in the bar is available not only to be returned to the beach face during recovery, the bar also can dissipate wave energy during any subsequent storm event. An example of a bar which formed after a storm along the Florida coast in March 1989 and the recovery which occurred during the subsequent months is shown in Fig. 1.

Cross-shore erosion models, such as that developed by Kriebel and Dean (1985), do not typically allow the formation of bars in the profile. They are therefore only capable of predicting berm and dune erosion over time scales on the order of hours to at most days. More recently Larson and Kraus (1989) developed a numerical cross-shore sediment transport model, which is based on extensive correlations of wave, sediment, and profile characteristics. The model was calibrated using large wave tank data and it was compared with profiles predicted by the Kriebel and Dean model for various wave and water level conditions. Larson and Kraus found their model more realistically described the profile at the dune toe when no bar was present in the profile, and because it allowed the formation of bars, there was less erosion of the dune in a case in which

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Figure 1: Example of observed barred (March, 1989) and unbarred (August, 1989) beach profiles at Beverly Beach in Flagler County, Florida USA.

offshore bars were present. The size of the model predicted bars and troughs were underestimated and the problem of beach recovery was not included. This paper presents an explicit computational sediment transport model using bottom shear stress parameters to predict the formation and evolution of offshore bars in two-dimensional cross-shore sediment transport processes.

Approach

The development of the berm and offshore bar has been found to be associated with the dimensionless parameter suggested by Dean (1973) and later tested by Wright and Short (1984)

$$H_b/wT$$
 (1)

where H_b is the breaking wave height, w is the fall velocity of the sediment, and T is the wave period. Wright and Short (1984) found the value of Eq. (1) to be 4.0 ± 1.5 at Eastern Beach in Australia, where offshore bars were present. This dimensionless parameter is important for bar formation because the breaking wave acts to displace sediment shoreward of the breakpoint in the beach profile. Stive and Wind (1986) investigated the mean cross-shore flow in a twodimensional surf zone and developed an undertow model based on experimental and theoretical considerations. They found the result of the strong spatial decay after breaking was equivalent to a shear stress at the wave trough level, which caused a mean offshore flow in the water near the bottom. Svendsen and Hansen (1988) considered the problem of incorporation of cross-shore circulations into numerical models, which predict wave height and setup. One of the important forcing functions they investigated was the average bottom shear stress, τ_b . They concluded that computed values of τ_b could be included in comprehensive nearshore numerical models.

The approach used in the computational model presented herein to determine the location, volume, and mobility of offshore bars is based on the assumption that transport of sediment across beach profiles is related to the momentum fluxes due to waves. Breaking waves transfer momentum by exert-



Figure 2: Schematic of the moment due to the wave-related momentum applied to the center of gravity of the water column.

ing a force on the water column directed toward the shore. The momentum force is defined by

$$F = -\frac{\partial S_{yy}}{\partial y} \tag{2}$$

where y is the distance offshore from the shoreline and S_{yy} is the flux in the y-direction of the y-component of momentum due to waves. This momentum is not applied at the centroid of the water column. Instead, the moment of the applied momentum about the center of gravity of the water column, following Boreckci (1982), is

$$M = \frac{Eh}{4} (1 + \frac{H^2}{2h^2})$$
(3)

where H is the wave height, h is the water depth and $E = (1/8)\rho g H^2$ is the total energy per unit surface area in the wave as determined by linear wave theory. Figure 2 shows a sketch of the water column with the moment due to the applied momentum force. Balancing the moment with the average bottom shear stress leads to

$$\overline{\tau_b} = \frac{2}{h} \frac{\partial M}{\partial y} \tag{4}$$

which is the time-averaged seaward directed bottom shear stress due to the transfer of the wave related moment of momentum. If the applied shear stress is strong enough and the value of Eq. (1) is greater than approximately 4, then a bar would be expected to form in the profile where there was sediment convergence under the breaking wave. A trough would form in the scour region on the shoreward side of the bar, as a result of the turbulence generated by the breaking wave. Once the bar forms, if the wave conditions and water level remain the same, then the breakpoint can move offshore from the bar. This can result in growth and migration of the bar some distance offshore, until there is a balance of transport components, thereby resulting in equilibrium.

Dean (1987) observed that in addition to the tendency for sediment motion due to waves to be offshore, there must be a net "constructive" shoreward force on the bottom sediment. If not, there would not be an upward slope in the beach profiles in the landward direction. Therefore, the equilibrium profile is the result of a balance between landward forces and seaward forces, including gravity. The average bottom shear stress, which retards the motion of a fluid in unidirectional open channel flow is expressed in terms of a quadratic friction law

$$\overline{\tau_o} = \frac{pf}{8} \overline{|U||U} \tag{5}$$

where f is the Darcy-Weisbach friction factor and U is the velocity of the fluid, and the absolute value ensures the proper duration of the shear stress. It can be shown that whereas simple harmonic bottom velocities result in a zero average bottom shear stress, nonlinear waves cause a shoreward directed average stress. Based on stream function wave theory (Dean, 1974), $\overline{\tau_o}$ can be approximated as

$$\overline{\tau_o} = \frac{\rho f}{8} \frac{0.09 L_o}{d} \frac{H^2}{T^2}$$
(6)

where $L_o = gT^2/2\pi$ is the deep water wavelength.

Description of the Computational Model

The computational model used to predict the creation and migration of an offshore bar is based on a sediment continuity and a transport equation. The expression for the conservation of sediment over the profile is

$$\frac{\partial h}{\partial t} = \frac{\partial Q}{\partial y} \tag{7}$$

where h is the water depth, t is time, and Q is the offshore transport. The sediment transport equation used to calculate changes in depth contours is

$$Q = K_1 \overline{\tau_b} + K_2 \overline{\tau_o} + K_3 \frac{\partial h}{\partial y}$$
(8)

The forcing functions on the right hand side of this equation include contributions from τ_b , the mean bottom shear stress due to nonlinear waves, $\tau \overline{o}$, and the effect of gravity which is related to the bottom stress, $\partial h/\partial y$. The quantities K_1, K_2 , and K_3 are transport rate coefficients. The computational model is an "open-loop" explicit model, which uses finite

The computational model is an "open-loop" explicit model, which uses finite difference forms of Eqs. (7) and (8) to predict cross-shore sediment transport across a profile. The portion of the profile over which these equations are applied is represented as uniformly spaced offshore grids, although, if desired, smaller grids could be employed in areas of specific interest. Unlike many previous models, this representation allows the model to produce offshore bars, because it does not require monotonic depth increases offshore. The two-dimensional profile for the explicit model begins at the berm or dune and continues offshore to well beyond the maximum breaking depth. The distance offshore can be represented by the contour location, y_i , which is referenced to an arbitrary baseline located landward of the shoreline. To determine the depth at any contour, each elevation contour, h_i , must be considered along with the water level, η_i , which includes tide, storm surge, and wave setup and setdown effects. The total depth at each grid can be represented as $d_i = h_i + \eta_i$. The continuity equation (Eq. 7) in finite difference form using a spacecentered finite difference method, is expressed as

$$\Delta h_i = \frac{\Delta t}{\Delta y} (\overline{Q_{i+1}} - \overline{Q_i}) \tag{9}$$

where $\overline{Q_i}$ represents the time-averaged sediment flux. The terms in the transport equation (Eq. 8) can be considered individually. The most important term for the determination of the location, volume, and mobility of offshore bars is based on the momentum fluxes due to waves. Breaking waves transfer momentum by exerting a force directed toward the shore. By substituting Eq. (3) into Eq. (4), the average moment can be balanced with the average applied shear stress, yielding the following equation:

$$\tau_b = \left(\frac{E}{H}\frac{\partial H}{\partial y} + \frac{E}{2}\frac{\partial h}{\partial y}\right)\left(1 + \frac{H^2}{2h^2}\right) + \frac{E}{2h}\left(\frac{H}{h}\frac{\partial H}{\partial y} - \frac{H^2}{h^2}\frac{\partial h}{\partial y}\right) \tag{10}$$

which can be used to calculate time-averaged moment of momentum induced mean bottom stress acting on the sediment in the bed. expanding this equation and collecting terms leads to the following simplified form of the average bottom shear stress in finite difference form using the total depth, d_i :

$$(\overline{\tau_b})_i = \frac{\rho g}{8} \left[\frac{1}{2} \frac{(H_i)^2}{(d_i)^2} d_i \left(\frac{\partial h}{\partial y} \right)_i + H_i \left(\frac{\partial H}{\partial y} \right)_i \left(1 + \frac{(H_i)^2}{(d_i)^2} \right) \right]$$
(11)

Implementation of the Model

The explicit model requires input of a two-dimensional profile with constant offshore grid size, Δy . This input profile (see Fig. 3) must encompass the entire active region from onshore in the dune or berm region to offshore beyond the closure depth. Before transport computations can be made in any time step, the wave height and wave setup model developed by Dally (1980) is implemented. Using this breaking wave model, a realistic calculation of the average bottom shear stress parameters can be made in the explicit model. The instantaneous value of depth, $d_i = h_i + \eta_i$, is found by using the value of wave setup, η_i , determined from the model.

To demonstrate the effects of the average bottom shear stresses, $\overline{\tau_0}$ and $\overline{\tau_o}$, in bar formation and migration, examples of the variations in their values across the profile and in time are presented. The examples used here are from the large wave tank tests documented by Kraus and Larson (1988). It is stressed that in the following, the results are presented for the measured (NOT predicted) profiles. In Case 400 of these tests, the conditions included a breaking wave height of 2.3 m, a wave period of 5.6 s, a constant water level, and a sediment size of 0.22 mm ($w = 0.031 \text{ ms}^{-1}$). Figure 4 shows the values of H/wT across the initial profile (planar with slope 1:15), as well as the values of $\overline{\tau_0}$ and $\overline{\tau_o}$. The maximum value of H/wT (approximately 13, which is much greater than the threshold of 4), located at a distance 30 m offshore, corresponds to the location of the initial break point. The calculated value of $\overline{\tau_0}$ is very small offshore from the breaking wave, but at that point it increases sharply to 700 Nm⁻². The average shear stress due to nonlinear waves, which is always negative, has the greatest magnitude at the breakpoint of -3 Nm⁻². Toward the shore both values of average bottom shear stress gradually approach zero as the wave



right s. Model representation of beach prome showing depth and transport related to grid definitions; the cross-shore grid elements are of constant length, Δu .

height decreases across the swash zone. One hour later, a bar has formed and the break point has moved offshore to about 37 m, as shown in Fig. 5. The greatest magnitudes of both $\overline{\tau_b}$ and $\overline{\tau_o}$ have also moved offshore with the breaking wave location. After 10 hours Fig. 6 shows the Case 400 bar crest is located at 42 m and the break point is at 48 m, while the largest magnitudes of $\overline{\tau_b}$ and $\overline{\tau_o}$ are both at approximately 45 m.

In the implementation of the predictive model for bar formation and migration, the values of $\overline{\tau_b}$, $\overline{\tau_o}$, and $\partial h/\partial y$ are calculated at each grid point. Because the momentum induced shear stress is based on a local balance, it does not accurately reflect the spreading due to the breaking wave over some distance toward the shore from the breakpoint. In addition, the transport of suspended sediment offshore from the breakpoint must be considered in the computation. The model includes a weighting function which distributes the values of $\overline{\tau_b}$ over adjacent grid cells, with the weights for the onshore values being slightly greater than the offshore portion. The bottom shear stress is lagged using the following expression:

$$\overline{\tau_{bi}} = \sum_{i=-5}^{5} W_i \overline{\tau_{bi+j}}$$

Table 1: Lag Weights Used for Prediction of Bar Formation and Migration

i+j	Weight, W_j
i+5	0.01
i+4	0.03
i+3	0.04
i+2	0.05
i+1	0.06
i	0.07
i-1	0.09
i-2	0.10
i-3	0.16
i-4	0.32
i-5	0.07



Figure 4: Initial profile and the ratio H/wT for Case 400 shown in lower panel. The upper panel shows the calculated values of $\overline{\tau_b}$ (units = 100 * Nm⁻²) and $\overline{\tau_o}$ (units = Nm⁻²).





$$V_{i} = \Delta t [K_{1}(\overline{\tau_{b}})_{i} + (0.3(Q_{2p})_{i} + 0.7(Q_{2n})_{i}) + K_{3}(\partial h/\partial y)_{i}]$$
(12)

where Q_{2p} and Q_{2n} are described in the Appendix. When Eq. (12) is input into Eq. (9), the equation for the change of depth at any grid in the profile is

$$\Delta h_i = \frac{V_i - V_{i-1}}{\Delta y} \tag{13}$$

The portion of the profile over which calculations are made is from onshore at the upper limit of the setup to offshore at the index, imax - 1, where imax is the total number of grid points in the profile. The two boundary conditions on the model are 1) $V_{is} = 0.0$ where *is* is the instantaneous upper limit along the profile of the water level setup and 2) $V_{imax-1} = 0.0$. The elevation contours are updated each time step the explicit model is run, and in the next time step the new profile is used as input into the Dally (1980) model to calculate the new wave heights and setup.

Sensitivity Tests

Sensitivity tests were performed on the explicit model to determine which parameters caused significant changes in the prediction of the size and location of offshore bars. In addition, the stability of the model was investigated for various coefficients and different lags of the bottom shear stress parameter. A few examples of these sensitivity tests will be presented here. The first example was a test in which only the average bottom shear stress, $\overline{r_b}$, term was used in the transport equation, with the value $K_1 = 2.9 \times 10^{-6} \text{ m}^4(\text{Ns})^{-1}$ and the time step $\Delta t = 360 \text{ s}$. Figure 7 shows that the predicted profile is unstable after 10 times steps when only the parameter $\overline{r_b}$ is used in Eq. (13). By simply adding the slope term, $\partial h/\partial y$, to the transport equation with a value for $K_3 = 5.8 \times 10^{-4}$



Figure 7: The original, one hour observed, and the model predicted beach profiles after 10 time steps; the upper graph is for the model prediction with $\overline{\tau_0}$ only and in the lower graph both the $\overline{\tau_0}$ and $\partial h/\partial y$ terms are included.

 $m^{2}s^{-1}$, the model prediction after one hour shown in Fig. 7 now has a stable profile with the bar located accurately, even though its height is smaller than was observed in reality. Figure 8 shows the result of combining only the bottom shear stress due to nonlinear effects, $\overline{\tau_{o}}$, using $K_{2} = 5.8 \times 10^{-5} m^{4} (Ns)^{-1}$, and the slope term in the transport equation. After 100 time steps, the model produces a stable profile, with sand deposited between 0 m and 15 m offshore.

<u>Results</u>

The predictive offshore bar model was first calibrated using the large wave tank data in Case 400. The simulation of the changes in the bars over time periods longer than the sensitivity tests reported above required that different transport rate parameters be used. The coefficients were established at the values: $K_1 = 2.4 \times 10^{-6} \text{ m}^4(\text{Ns})^{-1}$, $K_2 = 8.4 \times 10^{-5} \text{ m}^4(\text{Ns})^{-1}$, and $K_3 =$ $4.2 \times 10^{-4} \text{ m}^2 \text{s}^{-1}$. Using the larger K_i values in these computations was found to be necessary to allow the forcing function, $\overline{\tau_o}$, which produces shoreward sediment motion in the model, to compensate for the increasing magnitude of the slope term, $\partial h/\partial y$, as the bar forms. The time steps were also reduced to $\Delta t = 180$ s, which is half the interval used in the sensitivity tests. This had the result of increasing the stability of the model, which requires very little



Figure 9: Case 400 profiles initially and one hour observed and predicted. computational time for the large wave tank beach profiles. The prediction for the first hour shown in Fig. 9 shows the location of the bar agrees very closely with the observed profile, but the size of the predicted bar and trough is nearly half of the actual one hour result. This is an indication of the reduction of the size of the transport rate coefficients. Figure 10 shows the model is simulating the 5 hour profile reasonably well if the size of the bar and its migration offshore are compared with the observed profile. After 300 time steps or 15 hours of simulation, the observed and predicted profiles in Fig. 11 continue to be in good agreement. By the end of 30 hours (Fig. 12), the agreement between the predicted and actual profiles remains quite good. Some of the small differences are likely due to wave reflections in the measured profile.

The model was also run for Case 401, which had nearly the same wave characteristics as Case 400 (H = 2.0 m, T = 5.6 s), but the sediment size in the large wave tank beach profile was 0.4 mm. The larger sediment size would be expected to result in steeper slopes in the offshore bar, especially on the seaward side of the bar, due to the greater fall velocities associated with sediment having a larger diameter. The profile shown in Fig. 13 after the first 20 time steps of the explicit model run shows very good agreement between the location of the predicted offshore bar when compared with the actual system. Even after five hours (see Fig. 14), the model has predicted correctly the rate of



Figure 10: Case 400 profiles initially and 5 hours observed and predicted.



Figure 11: Case 400 profiles initially and 15 hours observed and predicted.



Figure 12: Case 400 profiles initially and 30 hours observed and predicted.



right 14. One of the par offshore, but the predicted trough is slightly less deep than that of the observed profile. The model is obviously overestimating the amount of movement which occurs in the simulations for longer periods of run times (see Figs. 15 and 16). The effects of the coarser sediment need to be included in a realistic manner to slow the bar migration offshore in the computational model.

Conclusions

An "open-loop" explicit model can be used to determine the location of the formation and subsequent migration of bars. The calculation of average bottom shear stress values based on the use of the moment due to the waverelated momentum applied about the center of gravity of the water appears realistic. The model is based on sediment continuity and transport equations, and the latter equation is used by combining the mean bottom shear stress due to breaking waves, $\overline{\tau_b}$, the mean shear stress due to nonlinear waves, $\overline{\tau_o}$, as well as a slope term, $\partial h/\partial y$. The $\overline{\tau_b}$ term acts to direct sediment offshore from the breakpoint, while the $\overline{\tau_o}$ term produces mostly shoreward sediment motion. The slope term maintains stability in the transport model can be applied to beach profiles which have waves breaking over them in such a manner that scouring occurs shoreward of the breakpoint. The prediction of the trough created by the scouring process is accomplished by applying weighting lags to the $\overline{\tau_b}$ values. The coupling of the sediment continuity/transport model with the Dally (1980)



Figure 15: Case 401 profiles initially and 15 hours observed and predicted.



Figure 16: Case 401 profiles initially and 30 hours observed and predicted.

breaking wave model allows the formation and migration of the bar.

Appendix

The value of τ_o is decreased across the profile by including the seaward directed component of bottom shear stress due to nonlinear waves using the expression

$$(Q_{2p})_i = K_2[\{-(\overline{\tau_o})_i + \frac{2}{3}\omega\delta\sin\phi_i\} - \tau_{cr}]$$
(14)

if $\{\} \geq \tau_{cr}; (Q_{2p})_i = 0.0$ otherwise. The critical bottom shear stress is $\tau_{cr} = \frac{2}{3}\rho g(s-1)\delta sin\phi_{cr}$, where ϕ_{cr} is the critical slope angle (a function of the sediment diameter) and s is the ratio of mass density of sediment to the mass density

of water. The immersed specific weight of the sediment is ω , δ is the sediment diameter, and ϕ_i is the profile slope. The component of the shear stress directed toward the shore is

$$(Q_{2n})_i = K_2[\{3(\overline{\tau_o})_i + \frac{2}{3}\omega\delta\sin\phi_i\} + \tau_{cr}]$$
(15)

if $| \{ \} | \geq \tau_{cr}; (Q_{2n})_i = 0.0$ otherwise.

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