CHAPTER 128

Scale Effect of Wave Force on Armor Units

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ABSTRACT

This paper describes the scale effect in experiments on the stability of armor units from the point of view of the wave forces. The relationship between the drag coefficient C_D and the stability coefficient K_D of Hudson formula is theoretically derived to be that $K_D \propto C_D^{-3}$ on the condition that the inertia force is negligible. Three kinds experiments were performed by using various sizes of Tetrapods ranging from 16 g to 6800 g according to the Froude law for scaling, 1) to measure wave force on an armor unit placed in an armor layer of a breakwater, 2) to determine the drag and inertia coefficients in wave fields, and 3) to determine the drag coefficient in a steady flow. It is found that the wave force in the small-scale experiments is relatively larger than that in the large-scale experiments. As the wave height increases, the drag force becomes predominant in comparison with the inertia force. It is concluded that the scale effect of the wave force on armor units is mainly due to the change of the relative drag force number as a function of Reynolds number.

1. INTRODUCTION

The weight of armor units in breakwaters and sea walls are usually determined by laboratory experiments or stability formulae. Hudson formula (1959) is prevalently used as a stability formula for its simplicity. In order to apply the experimental results to prototype designs, the scale effect of the experiments on armor units should be taken into account. Thomsen et al.

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(1972) and Shimada et al. (1986) investigated the scale effect of the stability of armor units by using the large-scale models. Both researches showed that small-scale models were less stable than large-scale models.

In other words, the experimental coefficient in Hudson formula, K_D, at a certain constant damage rate depends on a model scale. As a result, small-scale experiments give conservative stability criteria. The weights of armor units are overestimated when the results are directly applied to prototype designs by the Froude law for scaling. On the other hand, van der Meer (1988) showed that the stability of an armor layer of rock was not influenced by Reynolds number ranging from 4×10^4 up to 7×10^5.

It was reported that the transmission and reflection coefficients in small-scale tests become smaller than those in large-scale tests (Johnson et al., 1966; Shuto and Hashimoto, 1970; Delmonte, 1972; Wilson and Cross, 1972; Shimada et al., 1986). It means that the energy dissipation in small-scale tests is relatively larger than that in large-scale tests. It also implies that the drag force in small tests is relatively larger than that in large-scale tests. As long as tests are performed by using the Froude law the similarity of the viscosity is neglected.

Although the experimental results of the scale effects were presented in the previous papers and the quantitative evaluation was described, the mechanism of the scale effects has not adequatly explained so far.

For the purpose, we focus our attention on the wave forces acting on a single armor unit. This paper firstly aims to theoretically interpret the scale effects of the stability of armor units. Secondly, three kinds of experiments were performed to show that relative wave force depends on the model scales and to approve the theoretically presented relationship between the parameters relevant to the scale effect.

2. THEORETICAL CONSIDERATION

Hudson formula (1959) was derived from the balance between the wave force and the resisting force. The process of the derivation of Hudson formula was reviewed here from the point of view of the scale effect of the wave force. Fig. 1 shows the definition sketch. A wave force is given by Morison equation as shown by eq.(1).

$$F = \frac{1}{2} \rho C_D A u \mid u \mid + \rho C_M V \dot{u} \tag{1}$$

where ρ is the fluid density, C_D, C_M the drag and inertia coefficients, A and V the projected area and volume of an armor unit and u, $\dot{u}$ the velocity and acceleration of water particle.

The resisting force is expressed by eq.(2).

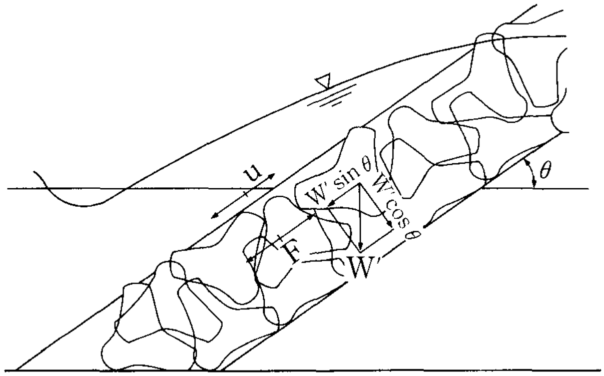


Fig. 1 Definition sketch

$$F_r = \pm fW'\cos\theta - W'\sin\theta \tag{2}$$

where f is the friction coefficient, W' the buoyant weight of the armor unit and θ the angle of the breakwater slope. The signs of "+" and "–" are applied for wave run-down and wave run-up, respectively. For incipient instability of armor units, the wave force is equal to the resisting force. After letting $F=F_r$, the following equation was obtained to describe the relationship between the weight of an armor unit, W, and other parameters.

$$W = \frac{k_a^3 \gamma_r C_D^3 (u|u|)^3}{8k_v^2 ((\pm f\cos\theta - \sin\theta)(S_r - 1)g - C_M \dot{u})^3} \tag{3}$$

where k_a, k_v are the shape coefficients defined as $A=k_a q^2$, $V=k_v q^3$ and $q=(W/\gamma_r k_v)^{1/3}$ as defined in Hudson (1959).

Hudson formula was derived on some assumptions as follows.

1) The inertia force is neglected.
2) The velocity u is substituted by that of the linear long wave theory, $u=\alpha\sqrt{gh}$, where α is a proportional coefficient.
3) Wave height is assumed as $H=\gamma h$, where γ is a function of the wave steepness. Together with assumption 2), the velocity is given by $u=\alpha\sqrt{gH/\gamma}$.

These assumptions are also introduced into eq.(1). Eq.(1) becomes eq.(2) as follows.

$$W = \frac{k_a^3 \alpha^6 \gamma_r C_D^3 H^3}{8 k_v^2 \gamma^3 ((\pm f\cos\theta - \sin\theta)(S_r-1) g)^3} \tag{4}$$

In order to investigate the mechanism of the scale effect, the parameters which are functions of Reynolds number should be kept in the formula.

On the other hand, Hudson formula is expressed by eq.(5).

$$W = \frac{\gamma_r H^3}{K_D \cot\theta \ (S_r - 1)^3} \tag{5}$$

where K_D is the experimentally determined coefficient which is a function of various parameters as mentioned in Hudson (1959).

Hudson (1959) assumed that the resisting force was equal to the buoyant weight of the armor units because an armor unit is isolated for inciepient instability. The difference of the term of θ is cot θ instead of $\pm$ fcos θ $-$ sin θ which is equivalent to the term in Iribarren's formula. Hudson introduced the slope of the breakwater θ by the stability number $N_s = (K_D \cot\theta)^{1/3}$.

The rest of the differences are the coefficients which are not related to the scale effects. Comparison between eq.(4) and eq.(5) gives the following relationship between K_D and C_D.

$$K_D \propto C_D^{-3} \tag{6}$$

So far, for the design a value of K_D of a specific kind of armor unit has been considered as a constant at a fixed damage rate. Eq.(6), however, implies that K_D depends on the model scale because the drag coefficient C_D is a function of Reynolds number. It is necessary to ascertain experimentally the relationship between K_D and C_D given by eq.(6). It is also important to check the assumptions which were introduced to the derivation of eq.(4) and Hudson formula. From among them, assumption 1) which is related to the scale effect will be investigated in section 3.2.

3. EXPERIMENTS AND RESULTS

In previous researches the experiments of the stability of armor units were done in the ways of counting the numbers of armor units which rolled down an armor layer of the breakwater and measuring the damaged area due to wave attack. The experimental results might be subjected to the handling of the armor units in the invividual experiments. However, it is more accurate to measure wave vorces directly than to investigate the damage rate.

3.1 Wave Force on Armor Unit

The first series of experiments was performed in order to measure wave forces acting on an armor unit in an armor layer. Wave force were obtained by using load-cell type wave force gages for various sizes of Tetrapods. Fig. 2

shows the experimental setup which was constructed in a wave flume 51 m long, 0.9 m wide and 1.2 m deep. Wave forces were measured by using the strain gages transdusers which transform displacement of armor unit to wave force. The armor unit installed to the wave force gage was protected not to have contact with armor units surrounding it. By the present measuring system, wave force acting on a cylinder which connected the armor unit and the force gage was also included in the data. In order to reduce the wave force acting on the cylinder, as shown by Picture 1 in section 3.2, a pair of wave force gages were used. One force gage was used for the armor unit and the cylinder and the other for the cylinder. Table 1 shows the experimental conditions. According to the Froude law for scaling, the geometric parameters of breakwater model and water depth etc. were determined. Wave heights were chosen by the capacity of the wave generator.

Table 1 Experimental conditions (wave force)

CASE	Tetrapod		breakwater	period	wave height
	weight W (g)	height b (cm)	depth h(cm)	T (s)	H (cm)
F4	2200	15.0	68.2	2.32	16.9~33.3
F3	570	9.6	43.6	1.85	11.6~27.7
F2	120	5.8	26.4	1.44	6.9~17.6
F1	60	4.6	20.9	1.27	5.7~14.3

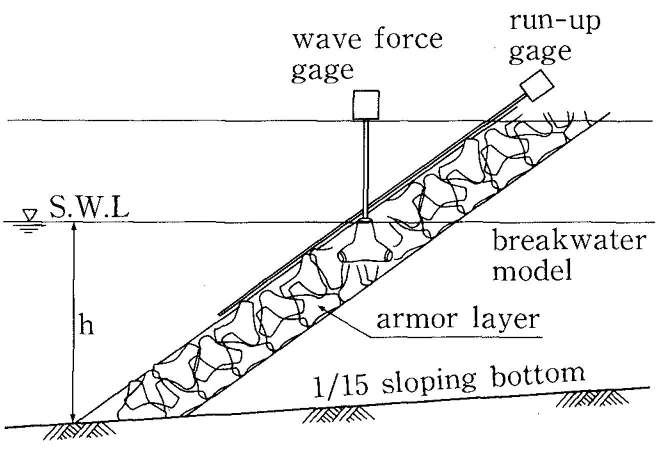


Fig. 2 Experimental setup to measure wave force acting on armor unit in breakdater

Fig. 3 shows an example of time histories of wave run-up, ξ, the wave force F_b in the direction along the breakwater slope and the uplift force F_u. It is observed that the uplift is negative at the moment when a wave hits the armor unit. It means that wave force works in the direction into the breakwater body.

Fig. 4 shows the relationship between the relative wave force F_b/W' and the deep-water wave steepness H_o/L_o for four different sizes of model scales. As this figure shows, the relative wave force increases as model scale becomes small. It means that an armor unit in small-scale experiments becomes unstable under ralatively smaller wave action that large-scale experiments.

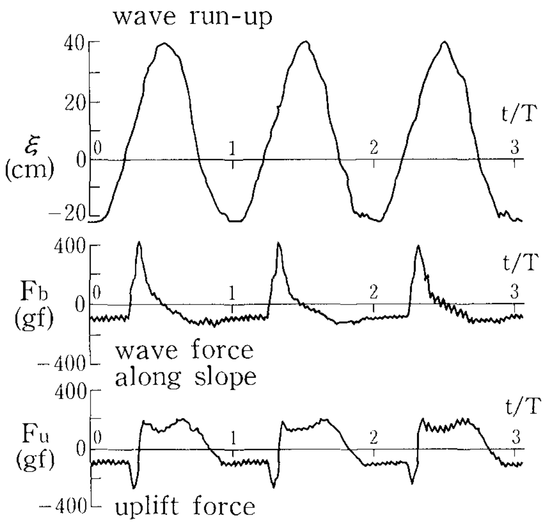


Fig. 3 Time histories of wave forces and wave run-up

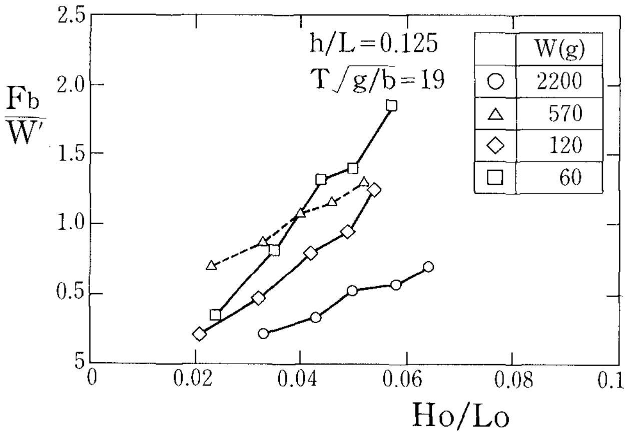


Fig. 4. Relative wave force depending on model scale

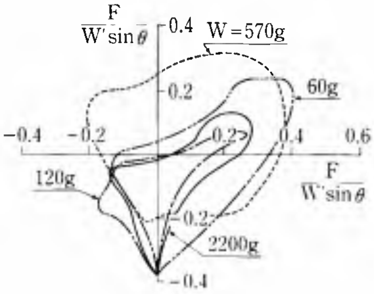


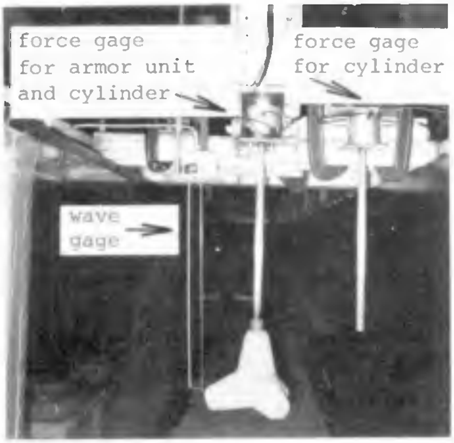
Fig. 5 Hodograph of wave forces

The result of W=570 g shows a different tendency. The armor unit of W=570 g was set at a different level from the others. Fig.5 shows the hodograph of the wave force. The pattern of W=570 g shows acutely different change from the others. When a wave runs down, the wave force is equal to the weight of a part of the armor unit which is under the still water level. The values for W=60, 120, 2200 g were about -0.4 but -0.2 for W=570g.

From the above investigation, it is confirmed that the wave force in smallscale experiments is relatively large compared to that in large-scale experiments.

3.2 Drag and Inertia Forces

The second series of experiments was performed in order to check assumption 1), that is, to measure the magnitude of the drag and inertia forces



Picture 1 Experimental setup to determine drag and inertia coefficients

by using five different sizes of Tetrapods. Picture 1 shows the experimental setup. Surface displacement was also measured by using a capacitance-type wave gage. The drag and inertia coeffcients were determined by Fourier analysis of the measured wave forces, the estimated wave velocities and accelerations based on Morison equation (Sarpkaya, 1976). Wave velocities and accelerations at the position of Tetrapod were estimated by the fifth order solution of Stokes wave theory (Isobe et al., 1978). The projected area of the armor unit in Picture 1 is $0.59 \times b^2$. Table 2 shows the experimental conditions. The scale of armor units and wave period were determined by the Froude law.

Table 2 Experimental conditions (drag and inertia coefficients)

CASE	Tetrapod		uniform	period	wave height
	weight W (g)	height b (cm)	depth h(cm)	T (s)	H (cm)
C5	6800	22.0	92.5	2.87	6.2 ~ 31.8
C4	2200	15.0	90.8	2.36	7.0 ~ 34.3
C3	570	9.6	89.9	1.87	7.3 ~ 35.9
C2	120	5.8	57.3	1.45	4.2 ~ 21.4
C1	60	4.6	49.7	1.28	4.4 ~ 20.9

Fig. 6 shows the comparison between the measured and calculated results of the surface displacement and of the wave forces. At top, the measured surface displacements were in good agreement with the calculated ones. The measured wave force is also in good agreement with the calculated total wave force which consists of the drag and inertia forces as shown by Morison equation.

Fig. 7 shows the ratio of the drag force F_D to the inertia force F_I. Reynolds number Re is defined as $Re = u_c b/\nu$, where u_c is the velocity under the wave crest. As Reynolds number increases, the ratio F_D/F_I increases for each scale model. Since a wave height is large at the incipient instability of armor units, the drag force becomes predominant compared to the inertia force.

Fig. 8 shows the inertia and drag coefficients as functions of Reynolds number and KC number where KC is defined as $KC = u_c T/b$. Since KC number is a ratio of the drag force to the inertia force, as shown in Fig. 7(a), as KC decreases, C_M increases. As Re increases, C_M increases at a certain KC number; on the other hand, C_D decreases. At a constant KC number, Froude number is also constant. It is necessary to concentrate on the changes of C_M and C_D at a constant KC in order to investigate the scale effect of wave force. C_M and C_D are non-dimensional inertia and drag forces, respectively. At a certain constant Forude number, C_M and C_D change depending on Reynolds number.

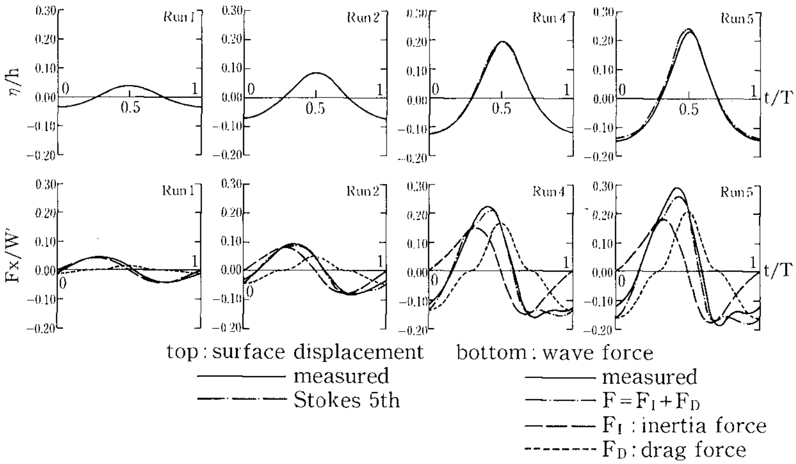


Fig. 6 Surface displacement and wave forces

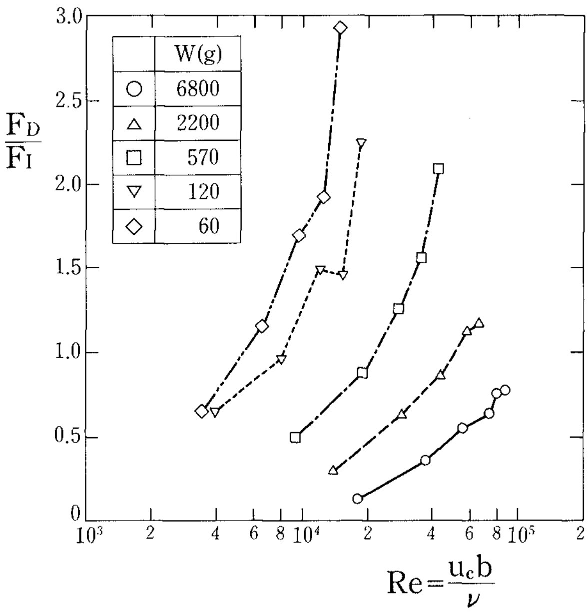
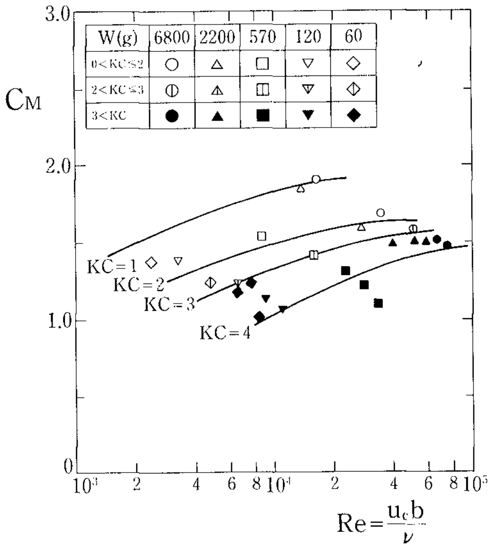
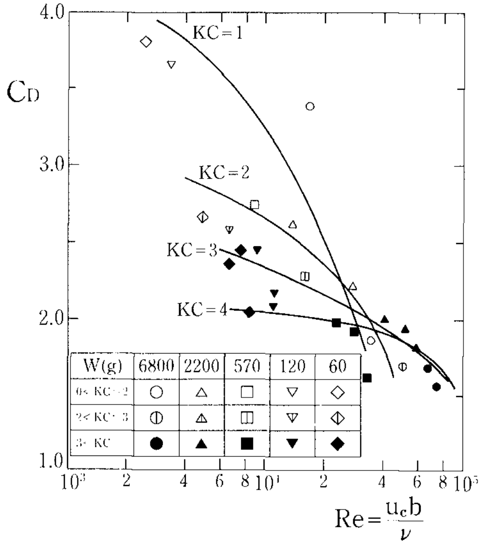


Fig. 7 Ratio of drag force to inertia force



(a) Inertia coefficient C_M



(b) Drag coefficient C_D

Fig. 8 Inertia and drag coefficients depending on model scale

The ratio of the gravity to the inertia force is constant but that of the inertia force to the viscous force changes. The similarity of the dynamics is not kept constant. The inertia force is proportinal to wave acceleration and the drag force proportional to the velocity squared. As a result, as wave height increases, the drag force becomes predominant to the inertia force as shown in Fig. 7.

3.3 Scale Effects of Drag Force and Stability

The drag coefficients of various sizes of Tetrapods in a steady flow were obtained by measuring the fall velocities in still water. The weight of Tetrapod ranged from 16 g to 6800 g. A 4-m deep and 2-m by 2-m section water tank as shown in Fig. 9 and a set of video recording system were used to measure fall velocities. Fall velocities were obtained by measuring the falling time for the distance of 1.0 m at three levels (z = -2.5, -3.0, -3.5 m). Measurements were repeated from five to ten times for each size of Tetrapod.

By this series of experiments, the determined drag coefficients were obtained on the condition that no inertia force worked and the relative drag force was a constant. It means that the drag force is equal to the buoyant force ($F_D/W'=1.0$) for each size of Tetrapod. If the drag coefficient, which is the relative drag force, is constant, there will be no scale effect due to the viscosity.

Fig. 10 shows vertical changes of the fall velocities of Tetrapods. It was observed that there was no significant change vertically. It is confirmed that the fall velocities were obtained on the condition that no inertia force worked. Maximum fall velocity is up to about 2 m/s at W=6800 g on the ex-

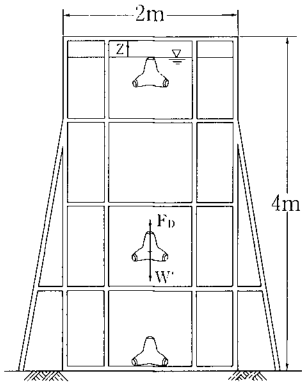


Fig. 9 Water tank to measure drag coefficient in steady flow

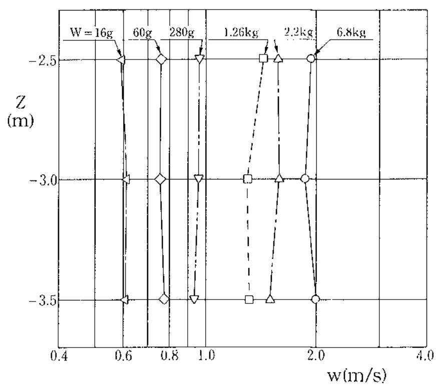


Fig. 10 Fall velocity of Tetrapod

perimental conditions. Vertically averaged values of the fall velocities in Fig. 10 were used in the following analysis. The drag coefficient C_D was calculated with the formula $C_D= 2(S_r-1)gV/(w^2 A)$, where w is the fall velocity and the projecred area $A=0.65Xb^2$ was chosen as an averaged value.

Fig. 11(a) shows the normalized drag coefficient C_{Dm}/C_{Dp} depending on Reynolds number, R_b, where C_{Dp} is the prototype drag coefficient and is assumed as 0.6 which is the value of the drag coefficient at $R_b=10^8$. Reynolds number R_b is defined as $R_b=wb/\nu$. As Reynolds number decreases, C_{Dm}/C_{Dp} increases. It means the relative drag force increases, as a model scale decreases.

The drag coefficient is dependent on conditions of the environment of the armor units. When armor units in an armor layer are attacked by waves, an upper half side of the armor units is exposed to waves. It seems that an armor unit is isolated for incipient instability. It is possible to substitute the drag coefficient of a single armor unit for that in an armor layer in order to discuss the scale effect of the wave force.

Fig. 11(b) shows the scale effect of the stability of armor units (Shimada et al., 1986). This result agrees with that by Thomsen et al. (1972). Although definitions of Reynolds number for R_b and R_N are not exactly same, values of R_b and R_N for a same size of Tetrapod is almost equal. As Reynolds numbers increase from 10^4 to 10^8, both C_{Dm}/C_{Dp} in Fig. 11(a) and $(K_{Dp}/K_{Dm})^{1/3}$ in Fig. 11(b) reduce roughly to a half. As presented in section 3.2, at large wave height, the drag force becomes predominant compared to the inertia force. However, the inertia force works under wave action. The slight difference between Fig. 11(a) and Fig. 11(B) may be due to the inertia force.

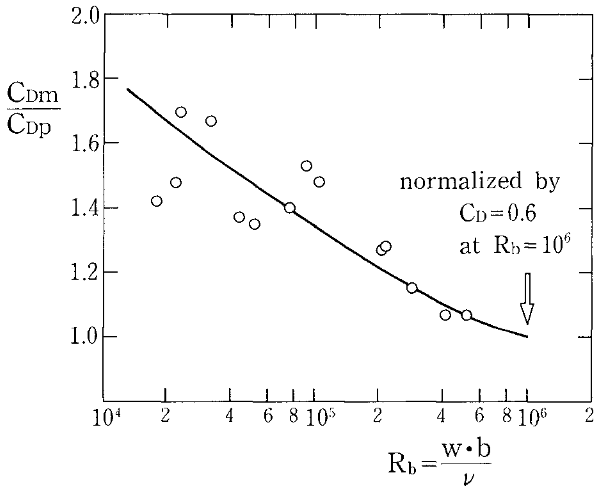
Although the resisting force was not measured in the present study, the result of the scale effect of the stability on the armor units in Fig. 11(b) include both the wave force and the resisting force. The resisting force may be not significantly contribute to the scale effect because an armor unit is isolated when it is unstable.

From the comparison of Fig. 11(a) and (b), eq.(6) which presents that K_D is inversely proportinal to C_D cubed, is approved by the experiments.

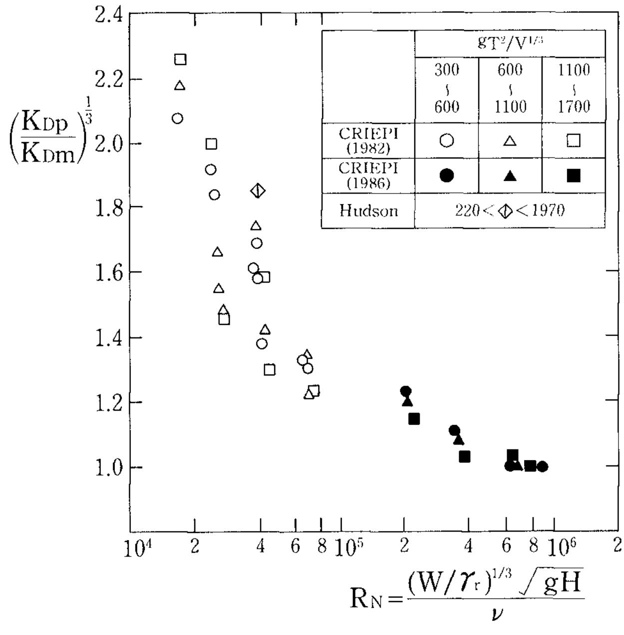
4. CONCLUSION

This paper presented the interpretation of the scale effect of the stability of armor units from the point of view of the wave forces. It is concluded from the results of the theoretical consideration and of the experiments that:

1) The relationship between the drag coefficient and the stability coefficient K_D of Hudson formula is theoretically derived to be that K_D is inversely proportional to the drag coefficient cubed on the condition that the inertia force is negligble. That relationship was approved by the experiments.
2) Wave force acting on an armor unit in small-scale model tests is relativty large compared with that in the large-scale model tests.



(a) Scale effect of drag force of armor unit



(b) Scale effect of stability of armor units

Fig. 11 Scale effect in experiments on armor units

3) For incipient instability of armor units, the drag force is predominant to the inertia force.
4) The scale effects of the wave force and of the stability of armor units can be interpreted as the scale-dependent change of the relative drag force because of the neglect of the similarity of the viscosity.

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