

PART III

Coastal Structures



CHAPTER 95

DYNAMIC STABILITY OF ARMOR UNITS - A BEM APPROACH

Chimin Chian(1) and Franciscus Gerritsen(2)

ABSTRACT

A method is presented for predicting primary stability of armor units on a breakwater exposed to solitary waves. A 2-D flow model is developed based on a boundary element method for simulating run-up of a nonbreaking solitary wave on an impermeable, smooth slope. Since Laplace equation subject to exact boundary conditions is solved by the model, the 'mild slope' restriction in using a 1-D long wave model is eliminated. Schemes including inductive wave generation, Lagrangian shoreline motion and free surface regriding are proposed. The run-up flow model is then coupled with an armor stability model to predict a stability number for armor units as function of time and location on the slope. Aspects such as applicability of Morison-type approach and selection of lift coefficient are examined. Results of computed wave run-up and armor stability are compared with experimental data.

1. INTRODUCTION

A solitary wave, described as a disturbance of water surface that travels at a supercritical phase speed with a permanent and symmetrical form completely above the still water line, propagates from offshore region shoreward towards a coastal structure which has a sloping seaward face. After the leading tip of the wave reaches the slope, the wave deforms, runs up to a maximum height, and then runs down before propagating seaward. In its passage, the wave generates an unsteady flow field which exerts hydrodynamic forces on the exposed part of the structure. The subject of this study is the primary stability of the protective layer of rocks, or armor units, on the sea-facing slope of the structure under such flow conditions.

Kobayashi et al. (1986, 1987) studied the problem for oscillatory waves with a flow model based on a finite-difference method. A stability model was developed for armor units which predicted a stability number as a function of time and location on the slope

1) Graduate Student University of Hawaii, presently M.I.T., R.M. Parsons Laboratory, Cambridge, MA 02139.

2) Professor of Ocean Engineering, University of Hawaii, Honolulu, Hawaii.

Since a one-dimensional long wave equation was used, their model is applicable only to mild slopes. When the slope is relatively steep, a two-dimensional model is desirable for adequate description of the flow condition along the sloping bed. For this purpose, an approach based on the boundary element method is deemed appropriate if we may assume the slope is, to a good approximation, smooth.

Run-up of solitary waves on a smooth slope has been studied, among a few others by Kim et al. (1983) using a boundary element method. To broaden the range of problems being treated and to reflect the development in our study of nonlinear wave simulation, we present in this paper a model based on the potential theory and a boundary element method (BEM) for evaluating the stability of armor units exposed to solitary waves on relatively steep, impermeable, smooth slopes.

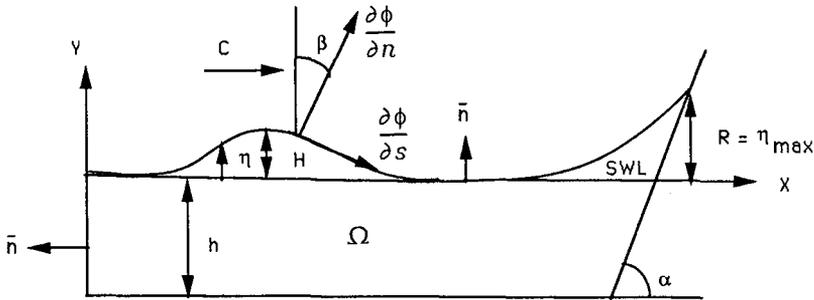


Fig.1 Definition sketch.

2.PROCEDURE

For a problem defined in Fig.1 where potential theory is assumed valid, the appropriate governing equation and boundary conditions are

$$\nabla^2 \phi = 0 \quad \text{in } \Omega, \tag{1}$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on solid bed}, \tag{2}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial \phi}{\partial x} \frac{\partial \eta}{\partial x} = \frac{\partial \phi}{\partial y} \quad \text{on } y = \eta, \tag{3}$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \left\{ \left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right\} + g \eta = 0 \quad \text{on } y = \eta. \tag{4}$$

Nondimensionalizing variables such that

$$(x^*, y^*, \eta^*) = \frac{(x, y, \eta)}{h}, \quad t^* = t \sqrt{\frac{g}{h}}, \quad \phi^* = \frac{\phi}{\sqrt{gh^3}}, \quad (u^*, v^*) = \frac{(u, v)}{\sqrt{gh}}$$

dropping the stars, and expressing the free surface boundary conditions in surface-fitting coordinates (Liu, 1978), we have

$$\nabla^2 \phi = 0 \quad \text{in } \Omega, \quad (5)$$

$$\frac{\partial \phi}{\partial n} = 0 \quad \text{on solid bed}, \quad (6)$$

$$\frac{\partial \eta}{\partial t} = \frac{1}{\cos \beta} \frac{\partial \phi}{\partial n} \quad \text{on } y = \eta, \quad (7)$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} \left\{ \left(\frac{\partial \phi}{\partial n} \right)^2 + \left(\frac{\partial \phi}{\partial s} \right)^2 \right\} + \eta = 0 \quad \text{on } y = \eta. \quad (8)$$

To specify the condition at the offshore (left) boundary, we differentiate between wave generation and run-up simulation. In wave generation, an inductive method is used (Chian, 1989) in which the wave field is generated by gradually introducing a known-a-priori wave form into the computational domain with other variables being solved stepwise in the process until the entire wave (cut at 0.1%H) is in the domain. For this procedure, the condition at the offshore boundary is

$$\frac{\partial \phi}{\partial n} = -u, \quad (9)$$

where u is the horizontal fluid particle velocity. In the ensuing run-up simulation, the generated wave field is taken as the initial condition and the offshore boundary is set open by using an Orlanski condition given by

$$\frac{\partial \bar{\phi}}{\partial t} + c \frac{\partial \bar{\phi}}{\partial n} = 0, \quad (10)$$

where c is the wave phase speed and overbars denote depth-averaged values, all being evaluated at each time step. The assumption underlying eq.(10) is that the shape of the outgoing wave keeps approximately unchanged.

At the shoreline, we write the dynamic free-surface boundary condition (8) as

$$\frac{d\phi}{dt} = \frac{1}{2} \left[\left(\frac{\partial \phi}{\partial n} \right)^2 + \left(\frac{\partial \phi}{\partial s} \right)^2 \right] - \eta \quad \text{on } y = \eta. \quad (11)$$

Assuming that the fluid element at the shoreline consists of the same fluid particles at all times, eq.(11) further reads

$$\frac{d\phi}{dt} = \frac{1}{2} u_s^2 - \eta \quad \text{on } y = \eta, \quad (12)$$

with u_s , the velocity of the fluid element at the shoreline, being evaluated using the boundary solutions of velocity potential along the slope. To match this scheme, Lagrangian nodes are used on the slope to follow fluid motion.

Applying Green's second identity to the boundary value problem governed by the Laplace equation, it may be shown that

$$\epsilon(\zeta)\phi(\zeta, t) = \int_{\Gamma} \left\{ G(\zeta, \omega, t) \frac{\partial \phi}{\partial n}(\omega, t) - \phi(\omega, t) \frac{\partial G}{\partial n}(\zeta, t) \right\} d\Gamma, \tag{13}$$

Where ζ, ω are field and source points, Γ is the boundary enclosing Ω , ϵ is the interior angle of the boundary at ζ , and G is a fundamental solution of the form

$$G(\zeta, \omega, z) = -1/r(z), \text{ with } r \text{ being the distance between } \zeta \text{ and } \omega.$$

Discretizing eq.(13) by dividing Γ into N linear elements, and differencing eqs. (7), (8), (10), (12) in time by a Crank-Nicholson implicit scheme, a set of simultaneous equations subject to boundary conditions may be obtained (Kim et al., 1983; Chian, 1989) and solved in time domain for ϕ and $\partial\phi/\partial n$ around Γ . Differentiating solutions of ϕ along the slope then yields a history of fluid particle velocities at various locations on the slope, which are used as input for the armor stability model as will be discussed later.

In implementing run-up simulation, a regridding scheme is used on the free surface at each time step, with which the size of projections on x of all elements are kept equal so as to avoid excessive elongation or shortening of certain elements and improve the accuracy of numerical differentiation along the free surface in evaluating $\frac{\partial\phi}{\partial s}$ and β . The scheme is given by

$$\begin{aligned} \left(\phi, \frac{\partial\phi}{\partial n}, \frac{\partial\phi}{\partial s} \right)_{i'} &= \xi_1^+ \left(\phi, \frac{\partial\phi}{\partial n}, \frac{\partial\phi}{\partial s} \right)_i + \xi_2^+ \left(\phi, \frac{\partial\phi}{\partial n}, \frac{\partial\phi}{\partial s} \right)_{i+1} && \text{(advancing wave)} \\ \left(\phi, \frac{\partial\phi}{\partial n}, \frac{\partial\phi}{\partial s} \right)_{i'} &= \xi_1^- \left(\phi, \frac{\partial\phi}{\partial n}, \frac{\partial\phi}{\partial s} \right)_i + \xi_2^- \left(\phi, \frac{\partial\phi}{\partial n}, \frac{\partial\phi}{\partial s} \right)_{i+1} && \text{(receding wave)} \end{aligned}$$

where

$$\begin{aligned} \xi_1^+ &= \frac{1}{2} \left(1 - \frac{dS - S_i/2}{S_i/2} \right), & \xi_2^+ &= \frac{1}{2} \left(1 + \frac{dS - S_i/2}{S_i/2} \right), \\ \xi_1^- &= \frac{1}{2} \left(1 - \frac{dS - S_{i-1}/2}{S_{i-1}/2} \right), & \xi_2^- &= \frac{1}{2} \left(1 + \frac{dS - S_{i-1}/2}{S_{i-1}/2} \right). \end{aligned}$$

In the above, S_i and S_{i-1} are the lengths of the i th and the $(i-1)$ th elements. dS is the difference in element length between two time steps. Primes for the subscripts denote the interpolated nodal points. Linear interpolation is chosen in this scheme for consistency with the linear elements used in this study.

To relate wave field with armor stability, Kobayashi et al. (1986) showed, based on analysis of forces acting on armor stones, that the commonly used stability number

could be dynamically represented by the one calculated from the threshold condition against sliding/rolling. Modified in this study, using the scaling relationships given previously, this number is given by

$$N_s = A \frac{H}{u_b^2} \left[\tan \phi \cos \alpha - \frac{u_b}{|u_b|} \left(B \frac{du_b}{dt} - \sin \alpha \right) \right], \quad (14)$$

$$A = \frac{2C_3^{2/3}}{C_2(C_D + C_L \tan \phi)}, \quad B = \frac{C_M}{s-1}, \quad s = \frac{\gamma_s}{\gamma_w},$$

in which u_b is the fluid velocity tangential to the slope, C_D, C_M, C_L are the drag, inertia and lift coefficients, respectively, and C_2, C_3 , , , are, respectively, area and volume coefficients of the stone, frictional angle of the stone, specific gravity of the stone and that of the water.

In this study, the same values for various coefficients in eq.(14) are adopted as in Kobayashi et al. (1986, 1987) except for C_L which is chosen to be 0.18. The value of 0.4 as used for some cases by Kobayashi et al. (1986, 1987) is close to the solution of Milne-Thomson (1960) for low Keulegan-Carpenter number flow and minor effect of wall proximity. These premises are apparently at odds with the present problem.

In coupling the BEM flow model with the stability model, special considerations are given to the fact that the Morison-type approach, on which eq.(14) is based, may be considered valid only when the depth of the flow is sufficiently large compared with the sizes of the stones. Since it is observed that a thin-water sheet trailing the down-running wave usually develops within a section near the shoreline, most obviously when the initial wave is relatively large, eq.(14) does not, and in fact should not be expected to, provide reasonable prediction of N_s . This difficulty is circumvented in this study by starting computing N_s from the location where the difference between the bed slope and the free surface slope becomes larger than 0.2, assuming this difference in slope increases monotonically within a certain distance from the shoreline at all times. This procedure, however, is found not necessary for waves of heights smaller than about 0.2 where no pronounced thin water sheet is observed.

3.RESULTS

Numerical experiments are run for solitary waves due to Boussinesq and Laitone on slopes steeper than 15° . The profile due to Boussinesq (1872) is given by

$$\eta = H \operatorname{sech}^2 \left\{ \left(\frac{3H}{4} \right)^{\frac{1}{2}} (x - ct) \right\}, \quad (15)$$

where

$$c = \sqrt{1 + H},$$

and the one due to Laitone (1960) is given by

$$\eta = H \operatorname{sech}^2[\sigma(x - ct)] \left\{ 1 - \frac{3}{4}H [1 - \operatorname{sech}^2\sigma(x - ct)] \right\} \tag{16}$$

where

$$c = 1 + \frac{1}{2}H - \frac{3}{20}H^2 + O(H^3),$$

$$\sigma = \sqrt{\frac{3}{4}H} \left(1 - \frac{5}{8}H \right) + O(H^{5/2}).$$

For eq.(14), we use $s = 2.71, C_2 = 0.9, C_3 = 0.66, C_D = 0.5, C_M = 1.5,$ and $C_L = 0.18.$

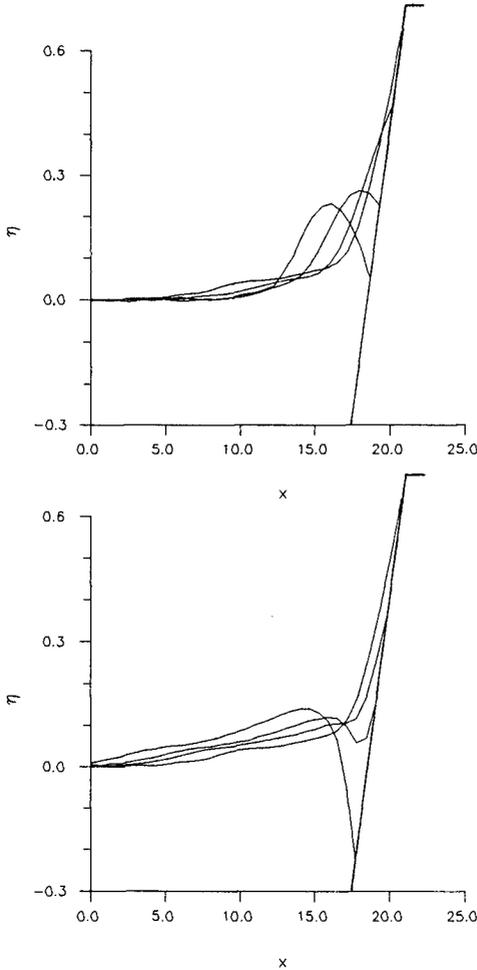


Fig.2 Sequences of free surface evolution during run-up (upper) and run-down (lower); $H=0.2, =15\%.$

Fig.2 shows typical sequences of the evolving free surface profiles during run-up and run-down of a wave of $H=0.2$ on a 15° slope. The deflected shape of the down-running wave with a thin tail near the shoreline when it approaches the still water line is clearly observed, which actually becomes more pronounced for higher waves.

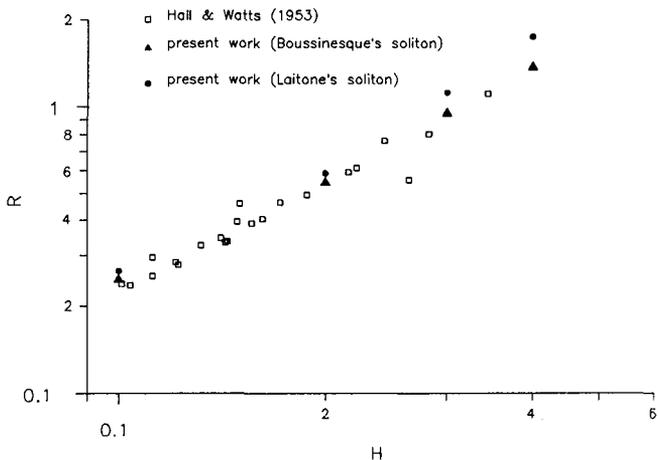


Fig.3 Maximum run-up of two different solitary waves on a 25° slope compared with experimental data by Hall and Watts (1953).

Fig.3 presents the predicted maximum run-up on a 25° slope as compared with experimental data by Hall and Watts (1953). Fairly favorable agreement is achieved for both types of waves. The reflected wave formed after run-down is found to be around $3/4$ of the initial wave height. This reflected wave is observed to pass through the offshore open boundary with less than 3% increase in height and a phase speed agreeing well with the analytical value corresponding to its height. For waves higher than 0.38, backwash breaking is found to occur near the end of run-down, when fatal numerical instability occurs.

Computed stability numbers are compared with the experimental results by Ahrens (1975) where oscillatory waves are used. The reason for using this work for comparison is that no experimental work for the solitary wave has ever been done in this subject area. To seek a common reference parameter for comparison, we use an equivalent Iribarren number given by

$$\xi = \frac{\tan \alpha}{\left(\frac{H}{L_e}\right)^{\frac{1}{2}}}, \tag{17}$$

where L_e is the equivalent wave length defined by the length of the wave truncated at $\eta = 0.1\% H$ on both ends. Figs.4 and 5 present computed N_s as a function of ξ on 1:2.5 and 1:3.5 slopes, respectively. The trends of stability variation over the concerned

range of surf conditions are predicted satisfactorily with locations of minimum stability found at $2.0 < \xi < 3.0$ for cases tested. The value of minimum N_s , however, is somewhat underestimated in the case of 1:2.5 slope, which seems to be related to uncertainties in identifying regimes of applicability for a Morison-type approach in the present problem as mentioned earlier. For a particular wave, the minimum stability is always found to occur in the vicinity of the still water shoreline position during backwash.

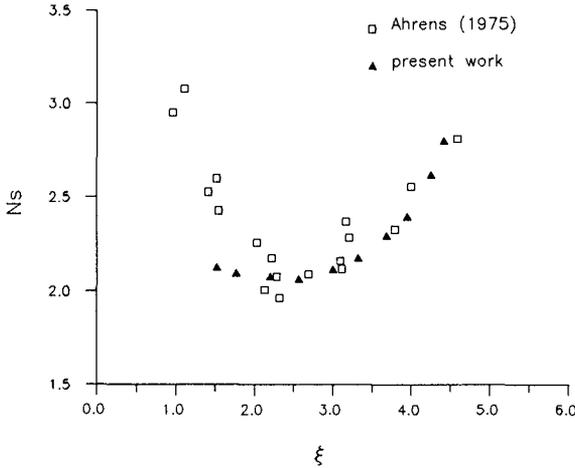


Fig.4 Comparison of computed stability number with experimental data by Ahrens (1975) on a 1:2.5 slope.

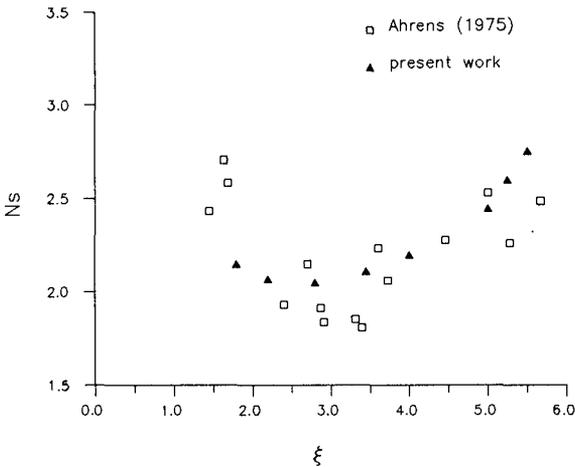


Fig.5 Comparison of computed stability number with experimental data by Ahrens (1975) on a 1:3.5 slope.

4. CONCLUSIONS

The stability of armor units on a breakwater slope exposed to solitary waves can be favorably modelled using the wave field predicted by the procedure of boundary element method as presented in this study. No 'mild slope' assumption is needed.

The proposed schemes including inductive wave generation, Lagrangian shoreline condition and free surface regridding combined yield satisfactory results in run-up simulation. The Orlanski-type radiation condition also exhibits fair performance.

The formulation of stability number based on the Morison equation does not yield realistic results unless the local water depth is sufficiently larger than the characteristic stone size. The simple empirical criterion used in this study to exclude the region where the Morison-type approach does not apply yields reasonable results. Further study is needed to better determine regimes of applicability of the Morison-type approach in the present problem.

A reasonable range of lift coefficients of the armor stones should be close to the lower limit of the range used by Kobayashi et al. (1987), i.e. around 0.18.

5. ACKNOWLEDGEMENTS

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