

CHAPTER 92

DECOMPOSITION AND INTERCEPTION OF LONG WAVES BY A SUBMERGED HORIZONTAL PLATE

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Abstract

The results of two-dimensional hydraulic experiments are presented on the decomposition of regular and irregular surface waves by a fixed, submerged horizontal plate with a finite length. Nonlinear interactions, containing the near-resonant interaction as well as the self- and mutual-interactions, among the waves affected by the submerged plate are found to induce the generation of higher harmonic components which propagate independently in the lee side, and to result in appreciable disintegration of wave form. This wave decomposition may produce considerable modification of the spectrum of a transmitted-wave train through the nonlinear transfer of energy from primary wave components to higher frequency components and consequently reduce its significant wave period. With the above mechanism, thus, the submerged horizontal plate may serve as an effective interception device against waves, especially of long wave length.

1 Introduction

There are three conceivable mechanisms by which a submerged horizontal plate intercepts incoming waves: (1) linear interaction between surface waves passing over the plate and an oscillatory fluid motion beneath the plate, resulted from the phase differences between them; (2) wave breaking on it and turbulence around it which result in decreasing transmitted-wave height through energy loss; (3) wave decomposition due to nonlinear effects taking place in the shallow water region over the plate, which would cause the frequency band of transmitted waves to be diversified, thereby reducing the impact of individual waves. The first two mechanisms and their wave attenuation effects, which numerous researchers have studied theoretically and experimentally (for example, Ijima et al, 1970; Siew & Hurley, 1977; Patarapanich, 1984 and Tabuchi et al, 1987), are well known. Although the observation of wave disintegration by a submerged object like a submerged breakwater has been reported (Horikawa, 1960; Jolas, 1960), little attention has been placed on the third mechanism in the light of wave interception effects, nor extensive study has been performed to understand its phenomenon.

The purpose of this study is to examine the phenomenon of wave decomposition accompanied with the generation of higher harmonics by a submerged horizontal plate

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and their propagation in the water region over the plate and in the transmitted-wave side in order to discover the conditions under which a significant energy transfer and shortening of wave length can take place. We also evaluate the effects of decomposing a wave into higher harmonic components on its statistical properties and attenuation.

2 Experimental Procedure and Analysis

A wave tank used is 29m long, 0.3m wide and 0.5m deep with the piston-type wave generator, which can generate both regular and irregular waves and absorb more than 90% of reflected waves. The tank ends in a very efficient wave absorber, the reflection coefficient of which is less than 10%. The water depth h is maintained constant at 0.38m. A horizontal plate was fixed at a certain depth h_s from the water surface. The length of the horizontal plate B was varied, and for a comparative purpose one plate was extended to the end of the tank, which plate was regarded to be infinitely long (hereafter referred to as the infinite plate).

The laboratory experiment has three phases:

- (1) The first phase involves regular wave tests where a monochromatic sinusoidal wave and a composite wave of two primary frequency components are used. To investigate the mechanism of wave decomposition by a submerged horizontal plate, we observed wave profile transformation in the regions over and past the plate of various lengths. Measurement of the wave profile was made with four capacitance wave gages at a number of stations along the tank, successive stations being 20cm apart. At each station the passage of the first 20 to 30 wave data was recorded, which were then converted at the sampling rate of 0.05sec. to digital data. The Fourier analysis was applied to the profile data to obtain the spatial distribution of harmonic component amplitudes.
- (2) The second phase is irregular wave tests where forced disintegration of random incident waves associated with real ocean waves is examined. The random waves generated in the experiments are based on the Bretschneider-Mitsuyasu spectrum. The range of significant wave periods $T_{1/3}$ used in the tests was 0.8 to 2.42 sec. ($h/L_{1/3} = 0.387$ to 0.085; $L_{1/3}$: the incident wave length associated with the significant wave period) and a significant wave height was 4.5cm ($H_{1/3}/h = 0.118$). Spectral densities for incident, transmitted and reflected waves were estimated from smoothing estimated periodgrams by the resolution technique (Goda and Suzuki, 1976). We also obtained statistical properties of the transmitted waves which may be modified by the nonlinear interactions.
- (3) The third phase deals with evaluation of the effects of wave disintegration on wave interception. The case where a vertically-slitted wall was placed behind a submerged horizontal plate was experimented and possible changes in wave transmission-reflection coefficients and wave force exerted on the permeable wall due to disintegration of incoming waves were studied.

3 Test Results and Discussions

3.1 Evolution of Wave Profiles

Figure 1 shows the evolution of wave profiles when a monochromatic wave of an infinitesimal amplitude ($H_I/L = 0.0066$) enters and travels over the infinite plate (Figure

1-(a)) and the finite plate of $B/h = 5.25$ (Figure 1-(b)). The wave profiles for two wave periods (T) are depicted from the data of every other station and each of the highest wave crest is aligned at zero on the abscissa for a comparative purpose. It is seen that waves propagating over the infinite plate will break down into multiple secondary peaks in their profiles, as observed by Multer and Galvin(1967) and Boczar-Karakiewicz(1972) in a wave tank of constant depth. Since the higher crest propagates faster than the smaller ones, the primary crest approaches the preceding secondary crest from behind. When the two crests encounter and interact each other, the large crest decreases in height while the smaller one increases, and the wave form is nearly identical with the initial wave form. Then they exchange roles in both amplitude and speed and start to separate each other. This event is repeated at a certain distance called the recurrence distance. The event clearly indicates that the well-known soliton effects occur in the shallow water region over the infinite plate.

Markedly different evolution of wave profiles is observed once waves passed over the finite plate and entered the original water depth(see Figure 1-(b)). One of the obvious features is that the wave profiles appear somewhat irregular so that it is difficult to follow a particular wave crest. Another is that as the wave train of a primary wave and secondary waves travels down the tank, some of the secondary waves grow as nearly great as the primary wave. A consequence of increase in the height of secondary waves would be apparent reduction in period of the wave train.

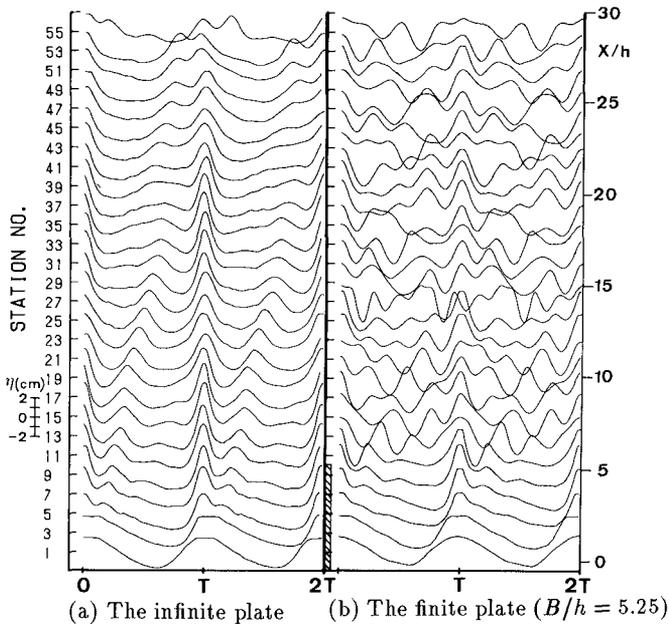


Figure 1 Wave profile transformation along the wave tank ($h/L = 0.100, H_I/L = 0.0066$).

3.2 Fourier Analysis

We study quantitatively wave profile data based on Fourier analysis. Applying this analysis to wave profile data yields the amplitude of any Fourier component, A_i ($i = 1, 2, 3, \dots$) associated with corresponding angular frequency σ_i at each measuring station. Figures 2 and 3 depict the spatial distribution of the amplitudes of the 1st, 2nd and 3rd harmonic components (A_1, A_2, A_3), normalized by the incident wave amplitude a_0 , for the case of the infinite and the finite plates, respectively. The heights of the two largest waves (primary and the largest secondary), Hm and Hs , normalized by the incident-wave height are also plotted versus distance from $x = 0$, the front end of the plate. The prominent difference between the results of the infinite and the finite plates lies in the variation of harmonic amplitudes in the region beyond the extent of the finite plate; the amplitude variation in the transmitted-wave side becomes nearly constant, independent of space, although small disturbances (or undulations) of spatial periodic manner is seen, while variation of the wave heights (Hm, Hs) is still spatially periodic. This implies that each of the harmonic component waves generated over the plate by nonlinear interactions should propagate independently and without mutual interactions in the water of the original depth with the phase speed corresponding to its own frequency. Thus the height of the primary wave would rise to the maximum when other harmonics are in phase with it; the height would decrease as phase is shifted.

Of interest here is the magnitude of harmonic component amplitudes in the lee side of the finite plate, because the increase in their amplitudes, comparable to the first harmonic one, indicates considerable transformation of surface wave form, which may consequently reduce an apparent wave period. As seen from the figures, the harmonic amplitudes are equivalent to those at the end of the plate, their magnitudes being related to the length of a submerged horizontal plate. It is then obvious that if the plate length is half of the recurrence distance, the amplitude of the first and third harmonics is minimum, while that of the second harmonic maximum. Similar results are discussed by Mei(1988) through the theory of nonlinear resonant interactions in water of constant depth.

3.3 Generation and Growth of Higher Harmonics

We examine factors affecting on the generation and growth of higher harmonics, which include fluid motion under the plate and water depth above the plate. In doing so, we introduce a parameter indicating the magnitude of the higher harmonics, namely, the square root of the ratio of the energy flux of harmonic components to the incident wave energy flux T_{p_i} ($i = 1$ through 3), referred to as the energy flux ratio. When $i = 1$, T_{p_1} is equivalent to a transmission coefficient of the first harmonic component.

The energy flux ratios versus relative water depth for the submerged horizontal plate and impermeable rectangular body are shown in Figure 4-(a) and Figure 4-(b), respectively. The solid line in the figure indicates the transmission coefficient obtained from the linearized computation of the boundary element method proposed by Ijima, Chou and Yoshida(1976). A distinct difference between the horizontal plate and rectangular body lies in the energy flux ratio of the first harmonic component, i.e., the transmission coefficient. The results of the horizontal plate show a strong tendency of selectively intercepting wave of a certain wave length, the measured value of T_{p_1} being

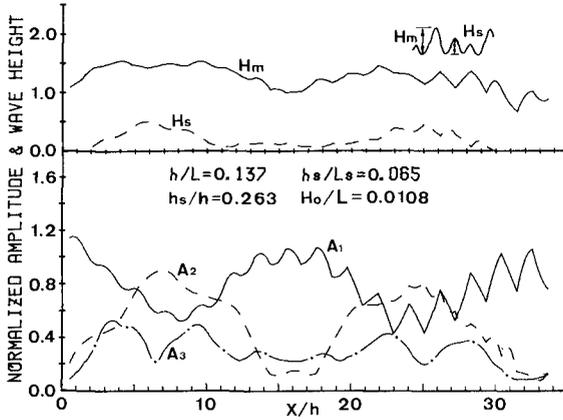


Figure 2 Spatial distribution of the amplitude of harmonic components for the infinite plate ($h/L = 0.100$).

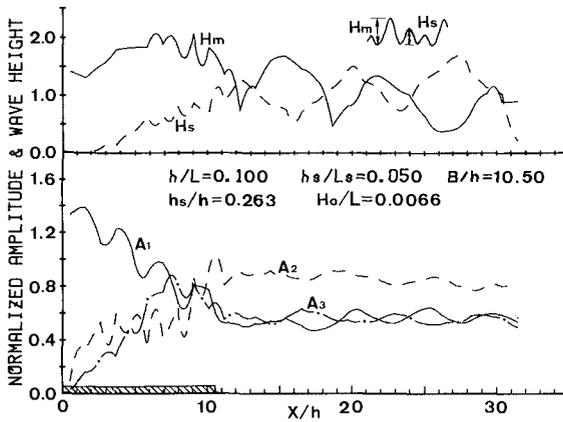


Figure 3 Spatial distribution of the amplitude of harmonic components for the finite plate ($B/L = 10.50$, $h/L = 0.100$).

less than 0.4 at $h/L = 0.15$. In contrast, the rectangular body has no such tendencies and the measured value of T_{p1} appears to be around 0.75 in the entire range of h/L . The energy flux ratio of higher harmonics for the both cases has similar trends showing the maximum values of about 0.5 at around $h/L = 0.225$ for T_{p2} , except in the lower range of relative water depth where the energy of the higher harmonics generated by the rectangular body is relatively greater. Since the amplitude of the first harmonic component in that range is quite small for the horizontal plate, however, the relative magnitude of the second harmonic amplitude to the first is much larger in the case of the horizontal plate than the impermeable rectangular body. These results lead to

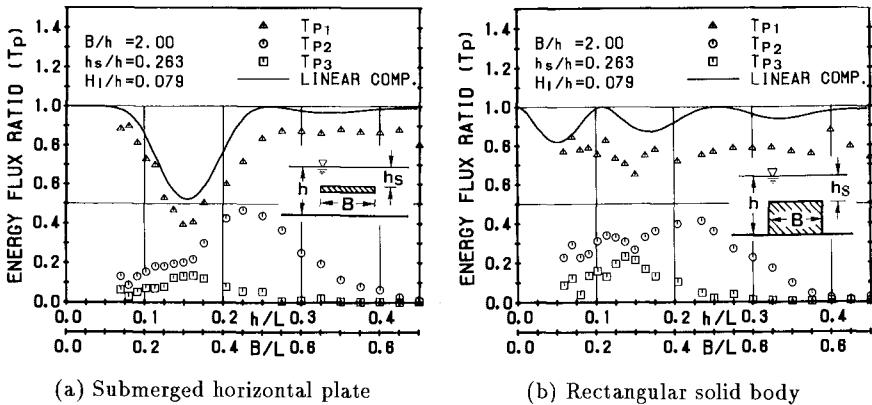


Figure 4 The magnitude of harmonic components in the transmitted waves.

conclusions that the fluid motion beneath the plate has little effect on the generation of higher harmonics and a submerged horizontal plate is more effective in reduction of wave period than a impermeable rectangular body.

The effects of a plate length B and water depth over the plate h_s on the generation and growth of higher harmonics are also studied. A brief summary is discussed here; detailed results can be seen elsewhere (Kojima and Ijima, 1989). Increase in the plate length results in heightening the maximum values of T_{p2} and T_{p3} as well as shifting these values to lower relative water depth. A horizontal plate of longer length will thus more effectively generate higher harmonics against waves of longer wave length. Decrease in the plate depth h_s results in augmenting the values of T_{p2} in the larger relative water depth and the maximum value of T_{p3} as well.

3.4 Decomposition of Two Primary Waves

Figure 5 shows an example of the normalized amplitude spectra of the incident waves of two primary frequencies $f_1 = 0.500_{Hz}$ and $f_2 = 0.667_{Hz}$ (upper) and those of the transmitted waves (lower). The amplitude spectra of the incident waves are measured at constant water depth without existence of a submerged plate and those of the transmitted waves are obtained from the data measured at the point $4.75h$ away from the rear end of the plate. It is obviously seen that many modes of harmonics are present in the transmitted waves, where the sum interaction components and the difference interaction components are generated other than the self-interaction modes, i.e., $2f_1, 2f_2, 3f_1$ and $3f_2$, the sum interactions being dominated. Secondary waves ($f_1 + f_2, f_2 - f_1$), the products of binary interactions between primary components, are evident together with tertiary waves ($2f_1 + f_2, f_1 + 2f_2$), the components arising from the interactions between primary and secondary wave; the sum interactions of these are appreciably larger than any of the components resulting from the self-interaction effects.

It has been shown in the theory of nonlinear resonant interactions by Phillips (1960)

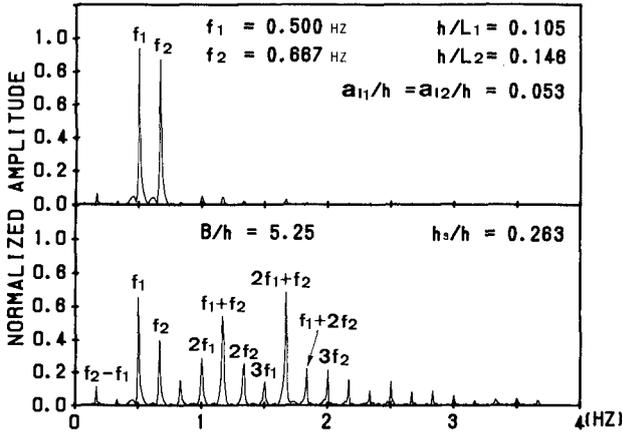


Figure 5 Amplitude of spectra of the incident wave of two primary components(upper) and the transmitted wave (lower).

and Longuet-Higgins(1962) that in the case of deep water the secondary interactions will not occur since the phase velocity of the secondary components is always different from the phase velocity of a free infinitesimal wave of the same wave number, whereas the tertiary interaction will occur. In the case of waves transmitting over the submerged plate where the ratio of water depth to wave length becomes considerably small, however, the near-resonant interaction(Bryant, 1973) would give rise to secondary waves due to the fact that the wave dispersion relation approaches asymptotically a straight line as non-dimensional wave number kh becomes small. The harmonic modes present in the transmitted waves may thus be generated by the nonlinear interactions comprised of the near-resonant interaction as well as the self- and mutual-interactions among the waves affected by the submerged plate.

The amplitude of the harmonic components together with the amplitude of the two primary components is plotted versus distance from $x = 0$ in Figure 6. As the waves travel over the plate, the amplitudes of secondary and tertiary waves increase with decrease in the amplitudes of the two primary components. Once the waves passed the plate and entered the water of the original depth, however, amplitude variations appear to be independent of space, similar to the results of a monochromatic incident wave. Thus, disintegrated waves of all the harmonics in the transmitted-wave side are most likely to propagate at phase speeds corresponding to their own frequencies.

In order to examine changes in the magnitude of secondary and tertiary amplitudes generated by nonlinear interactions with varying the frequency ratio of two primary waves f_2/f_1 , the energy flux ratio of each harmonic component of the transmitted waves to the incident wave is obtained and depicted in Figure 7. One of the two fundamental frequencies f_1 is kept constant at $f_1 = 0.400_{Hz}$ ($h/L_1 = 0.082$) for the upper figure and $f_1 = 0.667_{Hz}$ ($h/L_1 = 0.148$) for the lower figure, while the other frequency f_2 is varied. Comparison of the two figures reveals an interesting result that when the frequency of one of the two primary waves is low, the frequency range of the other wave

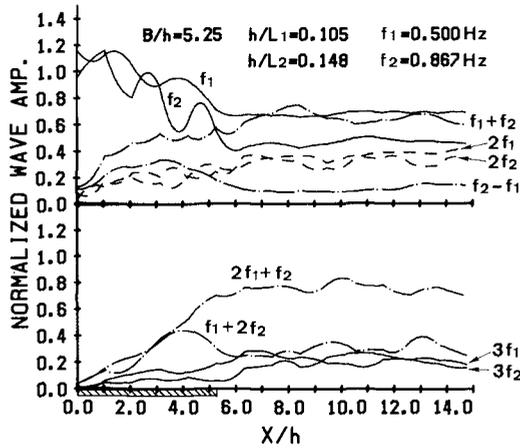


Figure 6 Spatial distribution of harmonic component amplitudes, measured along the tank, for two primary waves.

that induces nonlinear interactions tends to become wider and the frequency range appears to be shifted to the higher range. Another noticeable point is that although a monochromatic wave of high frequency, say $f = 1.314_{Hz}$ ($h/L = 0.425$), striking the submerged plate alone produces hardly any harmonics (see Figure 4), coexistence of a low frequency wave with it could give rise to secondary and tertiary components (see in Figure 7 the energy flux ratio at $f_2/f_1 = 3.285$ for $f_1 = 0.40_{Hz}$ and at $f_2/f_1 = 1.970$ for $f_1 = 0.667_{Hz}$). These results can be taken as evidence that the decomposition of an incident wave by the submerged horizontal plate through the mechanism of nonlinear interactions is much more excited when multiple wave components exist in the incident wave field rather than a single component. It is concluded therefore that a submerged horizontal plate can be considered as a device that effectively transfers the energy of incident waves to higher components of transmitted waves, thereby reducing the impact of primary wave components.

3.5 Decomposition of Random Waves

3.5.1 Transformation of frequency spectrum

Forced wave disintegration by a submerged horizontal plate is examined for irregular incident waves associated with real ocean waves with a sharp spectral peak around a significant wave period. Figure 8 shows the transformation of frequency spectral shape, along the tank, for the random incident wave train with a significant wave frequency $f_{1,3} = 0.625_{Hz}$ ($h/L_{1,3} = 0.105$) and wave height $H_{1,3}/h = 0.118$. It is evident that the spectral shape differs from station to station over the plate. At the front end of the plate ($x = 0.1B$) the spectral shape is similar to that of the incident-wave train, and as the waves propagate further down the tank over the plate, its shape is modified in such manner that some part of the wave energy around the significant wave frequency is transferred to the higher frequency components, thereby flattening the spectral shape. However, little variation of spectral shape is observed when the waves propagate the

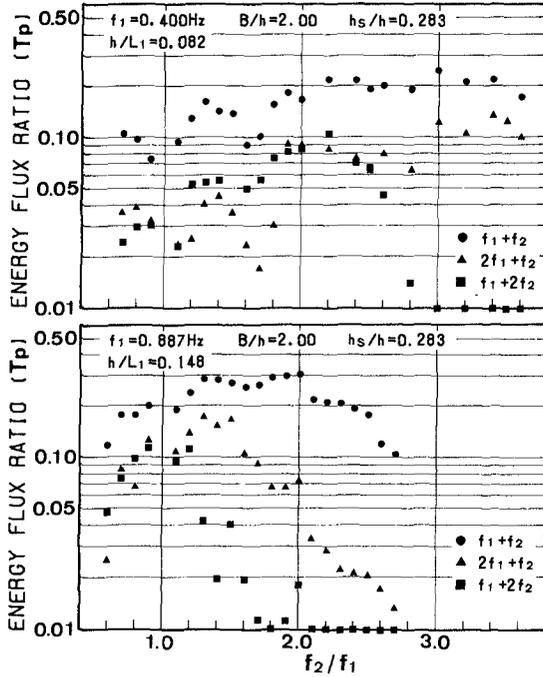


Figure 7 The magnitude of secondary and tertiary components for two primary waves.

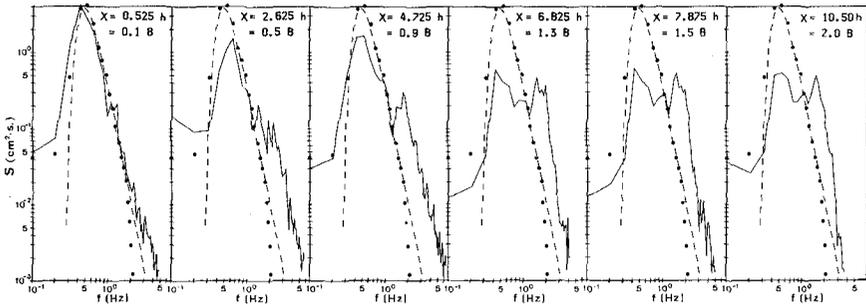


Figure 8 Transformation of the spectral shape of random waves measured along the tank ($h/L_{1/3} = 0.105, H_{1/3}/h = 0.118$); --- target spectrum, ●●●● incident wave, ——— transmitted wave.

water of the original depth after passing the plate ($x > 1.0B$), the spectral shape of which holds the presence of large energy in the high frequency range. This significant energy transfer reflects the effects of the nonlinear interactions taking place over the plate, as discussed in Section 3.4.

3.5.2 Statistical properties of decomposed transmitted waves

We investigate statistical properties of a transmitted-wave train. Wave profile data taken from four wave probes situated 20 to 40cm apart are used. To define individual waves, we utilize the zero-upcrossing method. The number of individual waves sampled is between 400 and 600. All of the statistical values obtained from four probes are averaged to determine one representative value.

One of the main aims for the present research is to confirm the evidence of reduction in wave period of an incoming wave train by a submerged horizontal plate. The significant wave period of a transmitted-wave train $T_{T_{1/3}}$ is compared with that of an incident-wave train $T_{I_{1/3}}$ and their ratios for the cases of $B/h = 2.00, 3.00$ and 5.25 are plotted, in Figure 9, versus relative water depth $h/L_{I_{1/3}}$. Considerable reduction in wave period is seen; the lowest value of $T_{T_{1/3}}/T_{I_{1/3}}$ is about 60% for $B/h = 2.00$ and 45% for $B/h = 3.00$ within the range of relative water depth examined. The lowest value of the wave period ratio tends to be shifted toward the smaller value of relative water depth with increase in the plate length and the ratio comes close to one at the large value of relative water depth (say $h/L_{I_{1/3}} = 0.387$), where the amplitude of higher harmonics may be so small that the form of transmitted-wave profiles would barely alter. Of interest is that when considerable reduction in wave period occurs, the wave height distribution somewhat deviates from the Rayleigh distribution, similar to the wave height distribution for the case of wave transmission over a protruding vertical breakwater, which waves also have shorter periods than those of incident waves, as discussed by Goda and Suzuki(1976).

Figure 10 shows the relationship between the spectrum and the characteristic wave height of a transmitted-wave train and parameters indicating nonlinearity of waves, in which plotted versus relative water depth are the ratio of significant wave height to the square root of the total wave energy (equivalent to the root-mean-square value of the surface elevation) $H_{T_{1/3}}/\sqrt{m_0}$, the skewness $\sqrt{\beta_1}$, and the kurtosis β_2 . The skewness and kurtosis, indicating the extent of the distortion of the statistical distribution of the instantaneous surface elevation from the normal, are expressed as

$$\sqrt{\beta_1} = \frac{1}{\eta_{rms}^3} \frac{1}{N} \sum_{i=1}^N (\eta_i - \bar{\eta})^3, \quad \beta_2 = \frac{1}{\eta_{rms}^4} \frac{1}{N} \sum_{i=1}^N (\eta_i - \bar{\eta})^4 \quad (1)$$

where η_{rms} and $\bar{\eta}$ are the root-mean-square value and the mean value of the surface elevation, respectively.

The theoretical value based on the normal distribution is $\sqrt{\beta_1} = 0$ and $\beta_2 = 3.0$, the latter value of which is drawn with the chain line in the figure. The proportionality constant between $H_{T_{1/3}}$ and $\sqrt{m_0}$ is also plotted at 3.8, with the broken line, the value of which is determined from many wave observation data throughout the world(Goda, 1985). Two distinct trends are obvious: the proportionality constant tends to decrease to a value less than 3.5 in the range of relative water depth between 0.085 and 0.160,

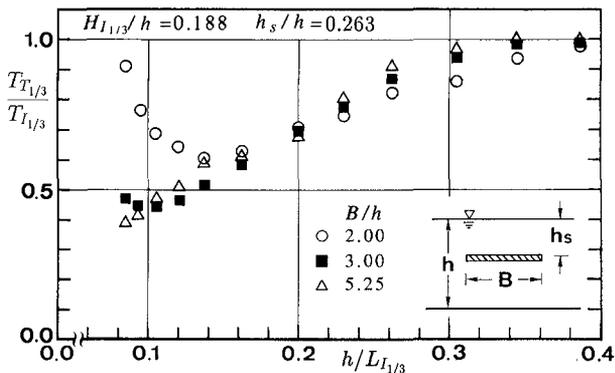


Figure 9 Change in period of transmitted waves for different lengths of the submerged horizontal plate.

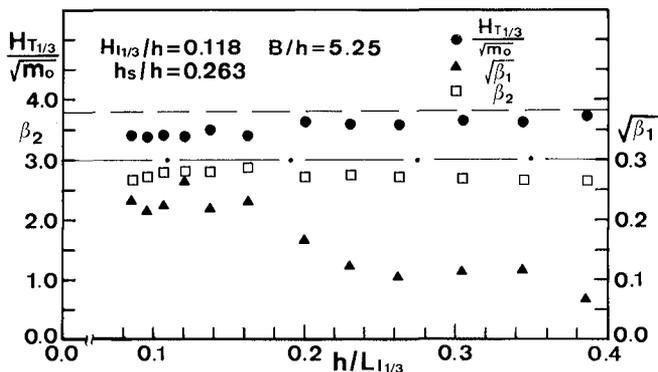


Figure 10 Relationship between the spectrum and the significant wave height and nonlinearity parameters for transmitted-wave trains.

where appreciable reduction in period of the transmitted-wave train occurs, as shown in Figure 9; in the same range the skewness value tends to become a larger value, being more than 0.2. With increase in relative water depth, both the values tend to approach the expected value of 3.8 for $H_{T_{1/3}} / \sqrt{m_0}$ and zero for $\sqrt{\beta_1}$. The value of the kurtosis β_2 has no clear tendencies with respect to relative water depth, but the value stays less than the theoretical value of 3.0 over the entire relative water depth examined, indicating the distribution shows lower peak than the corresponding normal distribution and long tails on both sides.

4 Wave Interception Effects

The disintegration of incoming waves by a submerged plate likely results in shortening of wave length in its shoreward side. We might thus anticipate that any structures located in the lee side of it would be exposed to less wave force and furthermore permeable structures which have a definite drawback in wave dissipation effectiveness against waves of long length would effectively intercept them once their lengths became shorter prior to striking the permeable structures. A schematical description of an experimental setup is shown in Figure 11, where a vertically-slitted wall with porosity $\varepsilon = 0.2$ and thickness $W/h = 0.171$ is placed in the lee side of the submerged horizontal plate. Figure 12 shows one of example of variation of transmission-reflection coefficients and non-dimensional wave force, F/F_s (F_s : standing wave force), acting on the permeable wall with the distance x_w between the plate center and the wall front along the wave channel. With the distance ratio x_w/h the values of a transmission coefficient K_T and wave force vary in a similar manner, while reflection coefficient values K_R vary in an opposite fashion, two peak locations of K_T and F/F_s being at about $x_w/L = 0.3$ and 0.85 where K_R indicates a minimum value.

Taking an average value of these fluctuations over the distance ratio and plotting the average value together with a vertical line indicating the maximum and minimum values of the fluctuations for K_T and F/F_s , with its top and bottom ends respectively, we have a representative value for wave transmission and force versus relative water depth in Figures 13 and 14. In these figures, the results for the case without a horizontal plate (Kojima, et al, 1988) are also plotted with a triangle mark for the measured and a solid line for the computed. It can be easily observed that the existence of the submerged horizontal plate considerably reduces both the wave transmission and force over the entire relative water depth tested. The experimental results for the case of the permeable wall plus the horizontal plate follow the same trend as the wave period changes shown in Figure 9; K_T and F/F_s tend to decrease with the relative water depth to a minimum value at about $h/L = 0.15$ and then increase. This trend can be taken as the evidence of the effects of wave disintegration on wave interception.

5 Conclusions

The major conclusions obtained are as follows:

- (1) Wave decomposition accompanied with the generation of higher harmonics by a submerged horizontal plate can be considered as a phenomenon that is induced by nonlinear interactions including the near-resonant interactions as well as the self- and mutual-interactions, which are much more excited when multiple wave components exist in the incident wave field. The non-spatial variation of harmonic component amplitudes in the lee side of the plate, different from the spatial variation above the plate, suggests that each component wave should propagate independently in that water region at its own phase speed without the self- and mutual-interactions.
- (2) By comparing the harmonic generation by the horizontal plate with that by the submerged rectangular body, the fluid motion beneath the plate was determined to have little effects on the wave decomposition.
- (3) A submerged horizontal plate induces a considerable modification in the ocean

wave spectrum, in such manner that the energy around significant wave frequency is transferred to higher frequency components due to the nonlinear interactions. This modification results in reducing significant wave period of the transmitted wave train by about 50%.

(4) When the significant wave period of a transmitted-wave train is reduced considerably compared with that of an incident wave train, interesting statistical properties of the transmitted-wave train are observed; namely, the wave height distribution somewhat deviates from the Rayleigh distribution, the ratio of the significant wave height to the root-mean-square value of surface elevation becomes about 3.3, and the skewness appears to be over 0.2.

(5) A submerged horizontal plate can serve as one of the effective wave interception devices against waves of long wave length, due to the fact that the energy of the long incident waves is disintegrated or dispersed into that of higher harmonic components

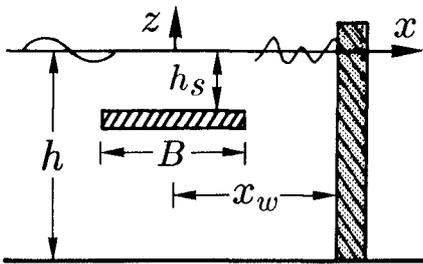


Figure 11 Schematic description of a submerged horizontal plate and vertical permeable wall.

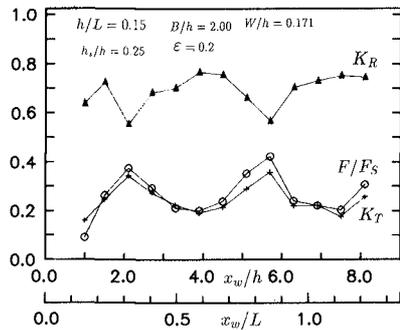


Figure 12 Variation of transmission-reflection coefficients and wave force with the distance ratio x_w/h .

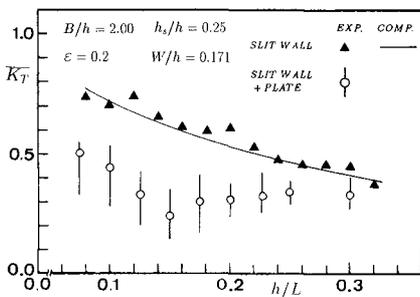


Figure 13 The average, maximum and minimum wave transmission coefficient versus relative water depth.

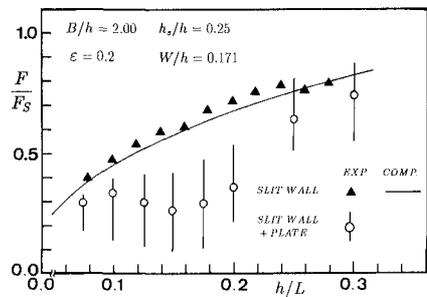


Figure 14 The average, maximum and minimum wave force versus relative water depth.

in the transmitted waves, thereby reducing the impact of the primary waves.

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