CHAPTER 89

NUMERICAL 3-D CURRENT MODELLING OF STRATIFIED SEAS

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Abstract

A detailed knowledge of the velocity field is often needed, e.g. for environmental impact assessment and the presence of a continuously stratification often necessitate the use of three-dimensional models. The present paper describes such a three-dimensional numerical model. Its ability to model situations involving stratification is illustrated through two classical hydraulic problems and a practical application. The problems of boundary data in the three dimensions are discussed and a possible way of overcoming these problems is devised.

1. INTRODUCTION

The coastal sea will in many cases be stratified due to variations in temperature or salinity, and even very small differences in density may have a decisive influence upon the properties of the flow considered. During the last decade two dimensional layered models integrated over the layers have been developed to simulate stratified flow. These models typically describe two layers, and have been successfully applied in situations where a pronounced stratification exists. However, in practice the density varies often continuously and the mixing of the water can only be adequately described from a knowledge about the three-dimensional flow field. This has led to the development of three-dimensional numerical current

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models capable of calculating the three-dimensional velocity field and density differences in a continuously stratified sea.

In the present paper a three-dimensional numerical hydrodynamic and advection-diffusion flow model is described. The model takes into account density variations, bathymetry and external forcing from, for example, wind and hydrographic boundary conditions. The model is suitable for applications where a detailed description of the flow is necessary, such as water exchange in estuaries, and for environmental impact assessment.

An important aspect of three-dimensional modelling is the need for accurate boundary data that must be known a priori to the simulations. Often these are, however, not known or can only be poorly estimated in practice. Inconsistent boundary data will inherently lead to numerical instability in the model and this problem is more evident in the presence of density variations. A possible solution to this problem has been devised in the present paper.

2. DESCRIPTION OF A THREE-DIMENSIONAL MODEL

The mathematical model is based upon the equations for conservation of mass and momentum (the Reynolds-averaged Navier-Stokes equations) in three dimensions together with an advection-diffusion equation for the scalar quantities salt and temperature,

$$\frac{D\vec{u}}{Dt} + 2\,\Omega \times \vec{u} = -\frac{1}{\rho}\nabla P - \vec{g} + \nabla\{\nu_e \nabla\vec{u}\} + \vec{F}_{ext} \quad (1)$$

$$\frac{D\rho}{Dt} + \rho\nabla\vec{u} = \nabla\{D_m \nabla\rho\} \quad (2)$$

$$\frac{D}{Dt}\{\rho S\} = \nabla\{D_s \nabla S\} \quad (3)$$

$$\frac{D}{Dt}\{\rho T\} = \nabla\{D_T \nabla T\} \quad (4)$$

where

$\vec{u}$ is the velocity vector,
P is the fluid pressure,
ρ is the density,
ν_e is the effective viscosity,
$\vec{F}_{ext}$ is an external forcing such as wind shear,
D_m is the dispersion coefficient due to fluctuations in density,
Ω is the Coriolis tensor,
$\vec{g}$ is the gravitational vector,
S is the salinity,
T is the temperature,
D_s and D_T are dispersion coefficients for salt and temperature, respectively, and
t is time.

The conservation equations for mass and momentum are discretized with a second order accuracy into a finite difference formulation imposed on a space-staggered rectangular grid. The weak coupling between the fluid pressure and the three velocity components is solved by the artificial compressibility method which first was proposed by Chorin (1967).

The fractional step technique and a special handling of the convective terms allow for a non-iterative ADI (Alternating Directions Implicit) method to be applied for advancing the solution in time.

Simultaneously, the advection-diffusion equation is solved by an explicit quadratic upwind method, the so-called QUICKEST scheme proposed by Leonard (1979) and extended to two and three dimensions by Justesen et al. (1989).

3. MODELLING OF CLASSICAL HYDRAULIC PROBLEMS

Stratified seas are often characterized by migrating fronts of water with different density. The classical example of such flow is often referred to as the lock-exchange flow, as it is observed when a sluice gate separating saline and fresh water is opened. A numerical simulation of this test accentuates the properties of the model with respect to maintaining the density difference at the fronts and thereby the properties to simulate the flow correctly.

Consider a 4 km, 20 m deep channel. Initially the water is at rest, with the less dense water to the

left (salinity 20 ppt.) and the more dense to the rigth (salinity 32 ppt.), separated by a vertical wall. When the wall is suddenly removed, the two water bodies will start to move. A gridsize of $\Delta x = 100$ m and $\Delta z = 1$ m has been chosen. The time step is set equal to 2.5 sec. There is no wall friction applied and a constant eddy viscosity equal to 0.001 m/s^2 is used.

Fig. 3.1 shows the development in time of the flow. The very unsteady and complicated flow field at the two density fronts are clearly revealed. Due to the free surface, the flow is not symmetric.

Assuming a balance between the potential and kinetic energy the propagation speed, U_o of the density front can be expressed as

$$U_o = 0.5 \sqrt{\Delta g H_o} \qquad (5)$$

where H_o is the water depth and $\Delta = \Delta\rho/\rho$.

The factor 0.5 in Eq. (5) refers to symmetrical flow and to non-viscid flow. In the presence of a free surface, laboratory experiments yield a deviation of the factor 0.5 in Eq. (5). According to Simpson (1987) the factor becomes 0.465 and 0.59 for the underflow and overflow, respectively.

A comparison between the model estimates and the empirical estimates has been listed in Table 1.

	Emp. Expression	Num. Model
Underflow	0.62	0.62
Overflow	0.66	0.78

TABLE 1 Propagation Speed (m/s) of Density Fronts Based on an Empirical Expression and the Present Model.

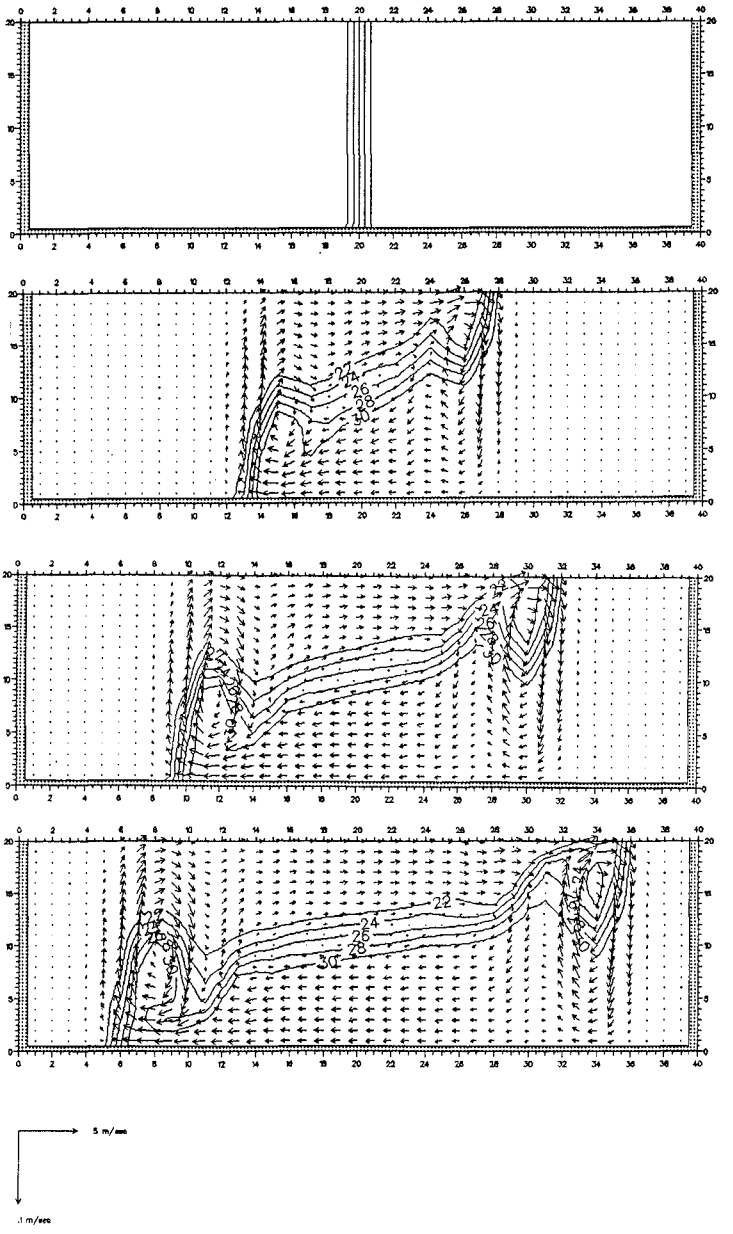

Fig. 3.1 Development of lock-exchange flow. Notice the characteristic head of the fronts which is also seen in laboratory experiments (cf. Simpson (1987)).

As seen from Table 1 there is an excellent agreement regarding the underflow whereas the model tends to overestimate the overflow compared to the empirical expression.

Another classical hydraulic problem in stratified seas is the wind-driven circulation and tilting of the interface. Consider again the 4 km and 20 m deep open channel; but with a stable, horizontal stratification at 10 m depth. The external force is a wind shear stress τ_ω equal to 0.58 Pa. The wind causes a set-up in the wind direction and a set-down in the opposite. This tilting of the surface induces additional water movements, resulting in a reversed tilting of the interface, and a wind induced, clockwise rotating current in the upper layer. In the lower

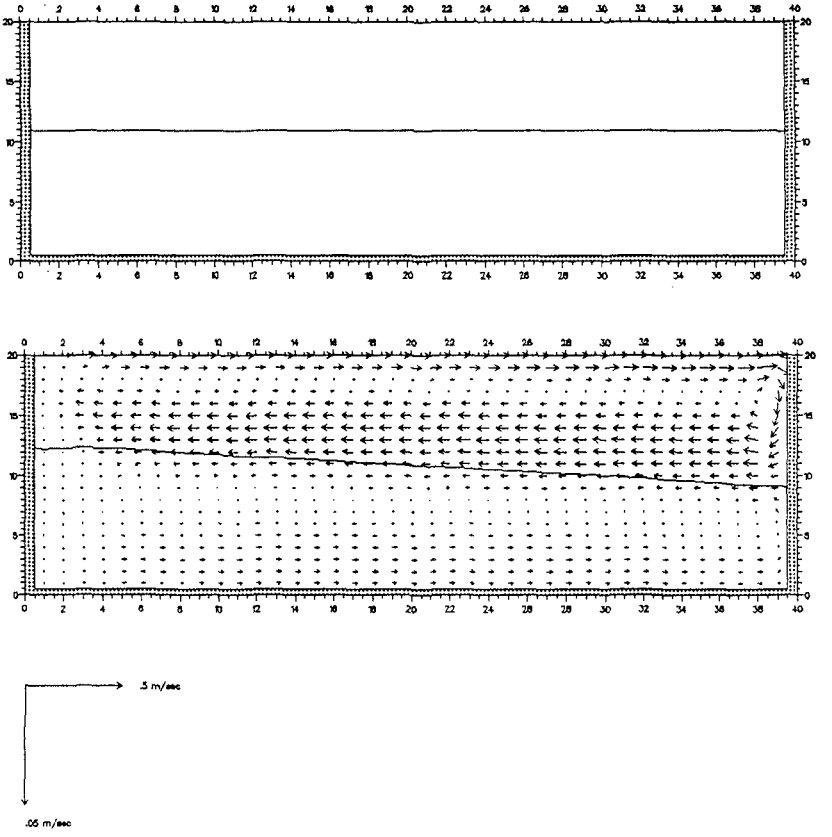

Fig. 3.2 Wind driven circulation. Initial and steady state conditions.

layer a weaker, counter clockwise rotating current is generated. The steady state situation has along with the initial conditions been shown in Fig. 3.2.Assuming a static equilibrium between the wind set-up and the wind shear stress, the set-up can be determined from

$$\Delta\eta = \frac{\tau_\omega L}{\rho g h} \qquad (6)$$

where L is the length of the channel and h the mean depth of the upper layer. The reversed tilting of the interface $\Delta\eta_i$ can be determined from

$$\Delta\eta_i = - \Delta\eta \; / \; \Delta \qquad (7)$$

assuming the lower layer is at rest. Inserting the actual values gives $\Delta\eta$ = 30 cm and $\Delta\eta_i$ = 3.18 m. These values are to be compared with the model results $\Delta\eta$ = 2,6 cm and $\Delta\eta_i$ = 2,76 m. With the given approximation this is a reasonable agreement.

4. APPLICATION IN THE GREAT BELT AREA

The three-dimensional model briefly described in Section 2 has been set up for the Great Belt which connects the Baltic Sea with the Kattegat (see Fig. 4.1).

The flow in the Great Belt may be very unsteady due to the meteorological changes and the water masses are often stratified with less dense water from the Baltic Sea above the more dense water from the North Sea. In connection with the building of a link across the Great Belt between Funen and Zealand a large field measurement programme has been carried out, providing an excellent basis for comparing model results with field data.

On basis of the available field data a five days period in November 1987 has been selected for simulation. The three-dimensional model has been set up with a horizontal grid spacing of 500 m and with 4 m in the vertical. The time step has been set to 10 min. The relatively high velocities that are observed and the length scales present in the Great Belt imply a significant influence of the Coriolis force. Thus, the effect of Coriolis has been included in the model, also. For simplicity, a bed friction based on a

quadratic law with a constant bed friction coefficient has been applied.

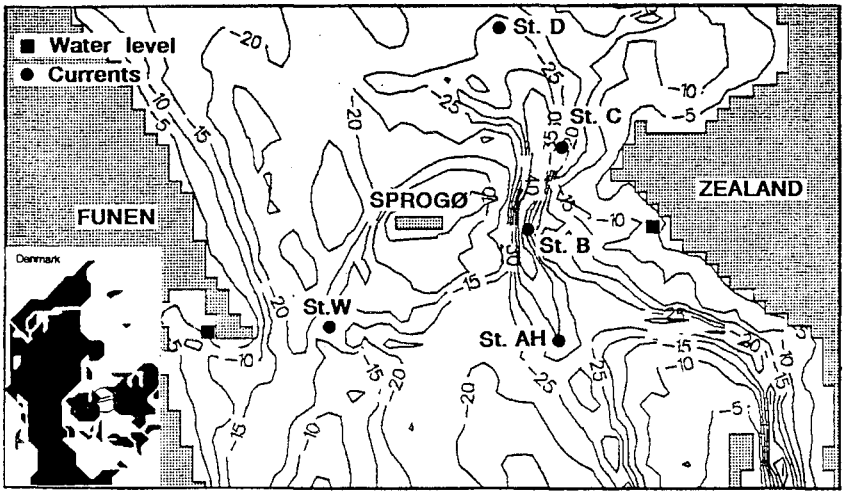

Fig. 4.1 Bathymetry of the Great Belt area with the locations of the measuring stations. Horizontal grid spacing is 500 m.

To describe the turbulence a Smagorinsky formulation of the eddy viscosity (see e.g. Smagorinsky (1963); Aupoix et al. (1982)) has been used. A simple damping function dependent on the gradient Richardsons number has been introduced to account for the reduction of the shear stress across a density interface,

$$\nu_e = \frac{\nu_e}{1+\psi Ri} \quad , \quad Ri = - \frac{g \frac{\partial \rho}{\partial z}}{\rho \left(\frac{\partial \vec{u}}{\partial z} \right)^2} \qquad (8)$$

where Ri is the gradient Richardsons number and ψ is a calibration factor.

The boundary conditions for the three-dimensional model have in the present study been extracted from a two-layered model assuming hydrostatic pressure and uniform velocity and density profiles for each layer

at the boundaries. Although this approximation is relatively rude, the model will within few grid points from the boundaries adjust itself to a non-uniform velocity distribution. This technique is known to be reliable and will prevent any inaccuracy and contradiction in the prescription of the boundary data.

A total of five stations with between two and four current devices on each string have been deployed during the simulation period. Also, two water level stations near Funen and Zealand, respectively, have been available. The approximate locations of these stations have been shown in Fig. 4.1.

The simulation period begins at 00:00 November 1, and ends at 12:00 November 5, 1987 of which the first 12 hours are a spin-up period. Initially, the density interface is positioned horizontally at 16 m depth.

In Fig. 4.2 a time series of the measured and calculated water levels at the two water level stations have been shown. It is seen that the water level is dominated by the semi-diurnal tide and that the Coriolis effect causes a difference of approximately 10 cm across the Belt under conditions with southward flow. A persisting southernly wind direction throughout the simulation period prevents a northward flow in the upper water masses and hence, neutralizes the Coriolis effect under ebb-tide conditions. Due to the limited model area the water level is strongly influenced by the boundary data and evidently - as seen in Fig. 4.2 - there must be a good agreement between the measured and calculated water levels.

In Fig. 4.3 an example of the horizontal velocities at the surface, at 16 m depth and 24 m depth, have been shown. At this particular time a strong southward flow is present in the upper water masses yielding at a Coriolis tilting at the surface (cf. Fig. 4.2). The reversed tilting of the density interface approximately positioned at -16 m causes a westward flow of dense water through a narrow trench south of the small island Sprogø in the middle of the Great Belt. This feature is also observed in many of the measurements, taken from a research vessel, carried out in parallel to the fixed stations measuring programme.

Typical examples of time series of measured and calculated current speeds have been given in Fig. 4.4.

The discrepancies between the measured and calculated speeds are more pronounced compared with the water levels. Naturally this is to be expected, due to the complexity of the physical (and numerical) velocity fields.

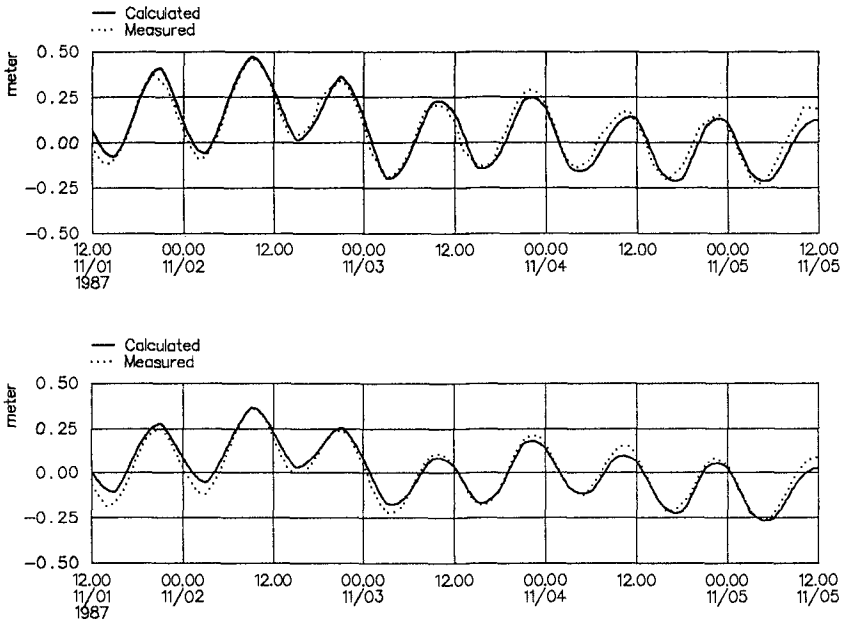

Fig. 4.2 Time series of measured and calculated water levels in Slipshavn (upper) and Korsør (lower)

As a qualitative assessment of the model performance the 12 hourly peak values of the current speeds have been extracted from the time series of each current meter along with the corresponding model data. In Fig. 4.5 the calculated vs measured peak values have been shown for all stations and all levels.

Station AH, B and C are deployed on the hillside of a deep, narrow trench (see Fig. 4.1) and thus, provide difficulties in the calibration due to an often inadequate spatial resolution. Comparing these three stations it is seen that there is a reasonable

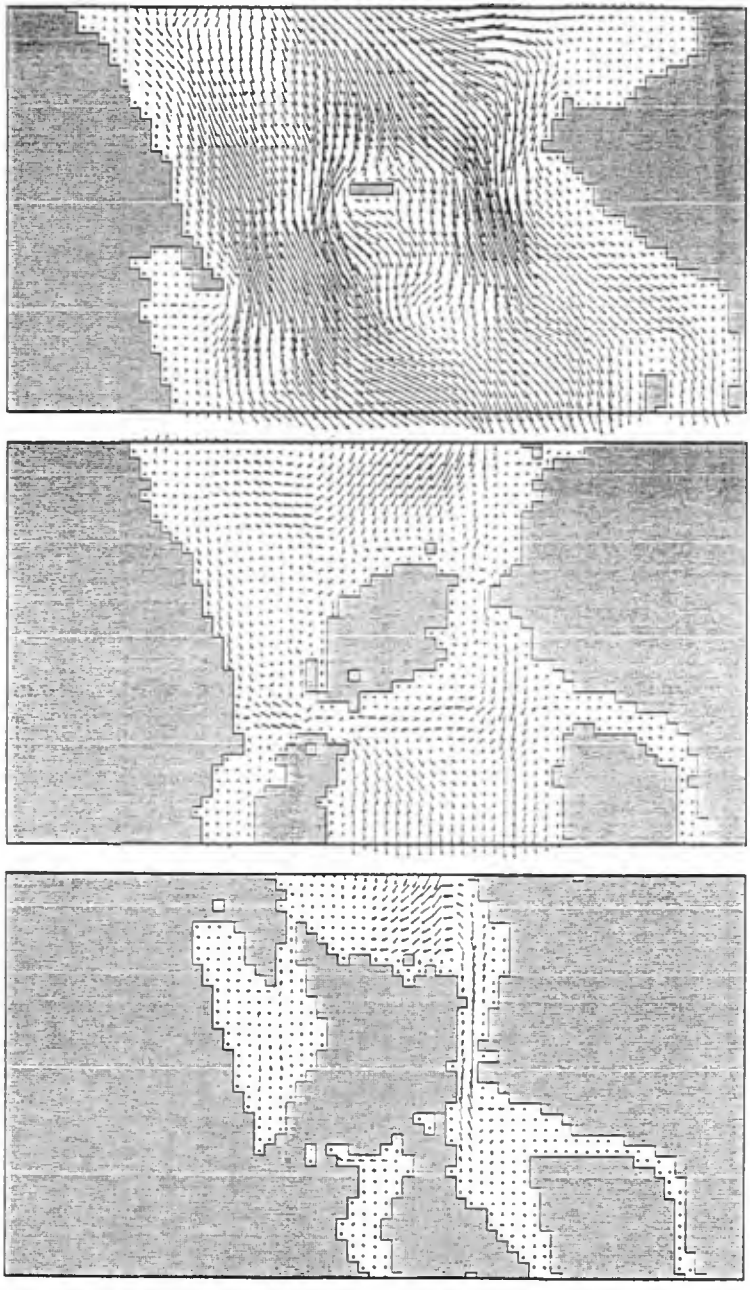

Fig. 4.3 Calculated horizontal velocities at the surface (A), -16 m (B) and -24 m (C) on November 4, 1987 at 12:00.

agreement for the upper levels except for station AH which seems to overpredict the current speed generally. At station B the model tends to underpredict the peak values at the lower levels which probably is due to a discrepancy in the actual and modelled density interface and the associated damping of the shear stress.

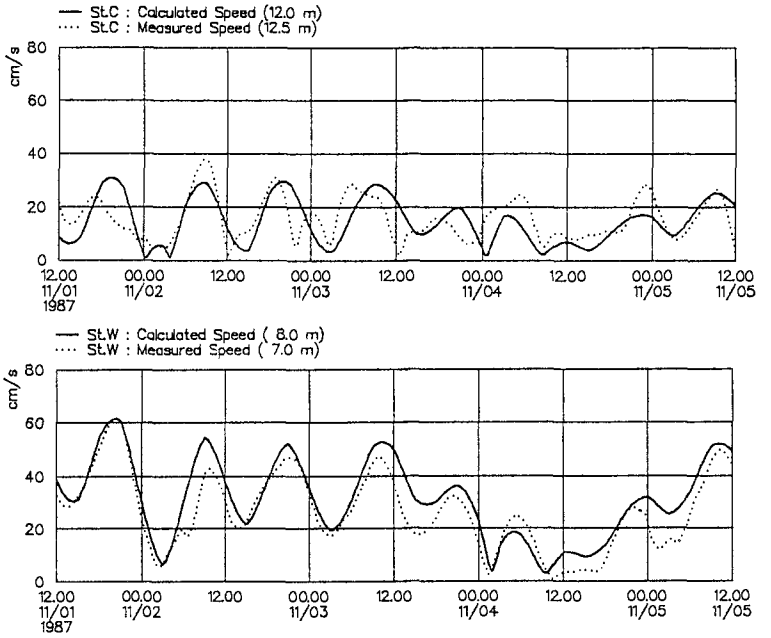

Fig. 4.4 Time series of measured and calculated current speed at Station C (upper) and Station W (lower)

The lower levels at station D show a fairly good agreement whereas the upper levels tend to overpredict. As station D is positioned close to the boundary the prescribed uniform velocity profile is a likely explanation for this behaviour. In nature the highest velocities are observed at the surface and decreasing downwards under the present meteorological and hydrographical conditions. At station W a good agreement is found at the upper level whereas the model overpredicts the peak values at the lower level. Similar to station B the likely explanation for this behaviour is the density interface.

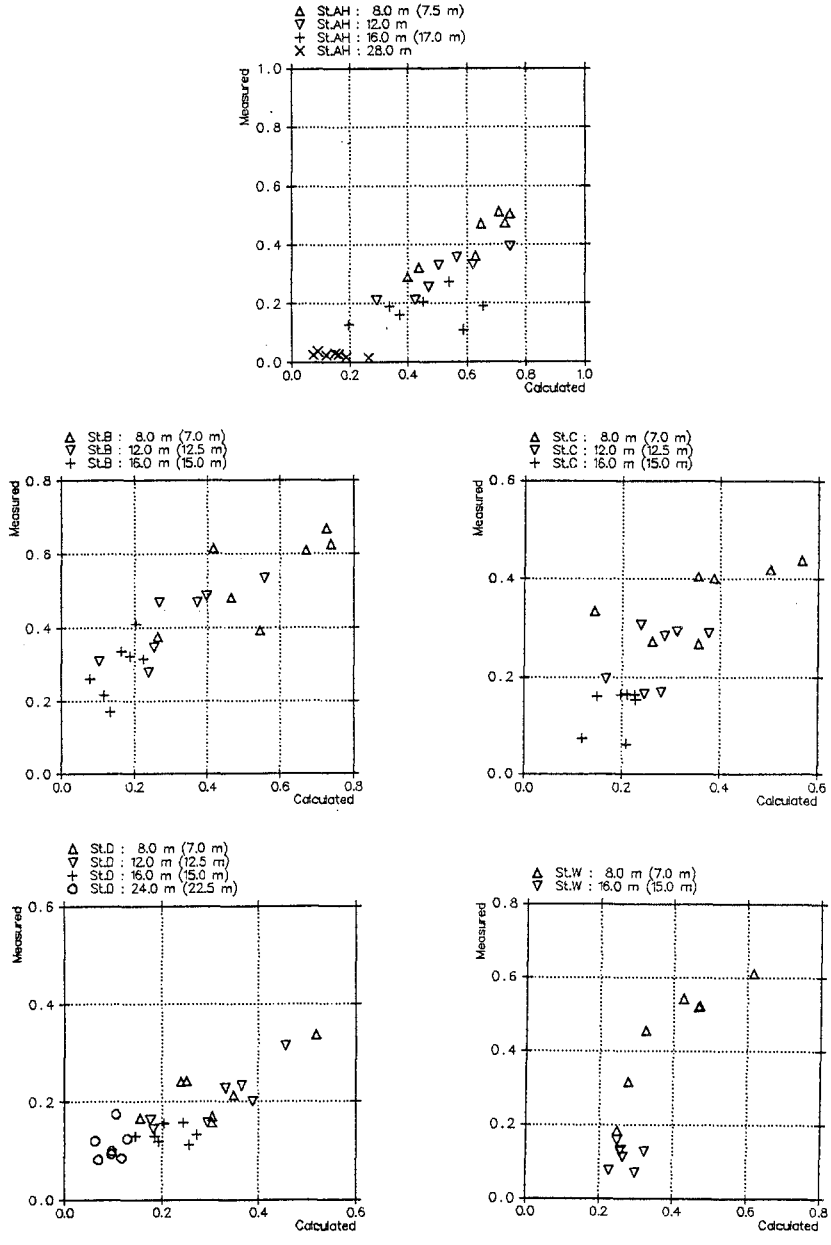

Fig. 4.5 Calculated vs measured 12 hourly peak values of current speeds (m/s) for all stations and all levels

The overall assessment of the model performance with respect to current modelling in stratified seas illustrates the importance of consistent boundary data and the ability of accurate modelling of stratification.

Although the present technique for establishing the boundary data for the three-dimensional model is not generally applicable and, furthermore, is time-consuming, it is a possible way out of the boundary data dilemma. On the other hand, this technique of utilizing results from layered models implies that only the area of interest needs to be modelled in detail and hence, often implies reduced computational costs. Alternatively, the three-dimensional model can be set up for so large an area that the open boundaries are better prescribed, e.g. so far away from the area of interest that the water masses can be regarded as homogeneous.

The problems associated with the boundaries may limit the practical applications of three-dimensional models seriously. Thus, further attention to these problems and especially those regarding the stratification is considered essential.

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