CHAPTER 80

MODELING OF WAVE TRANSFORMATION ON SUBMERGED BREAKWATER

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ABSTRACT

A mathematical model of wave transformation over a submerged permeable breakwater is developed on the basis of the equations for waves on a porous layer which are newly derived under the mild-slope assumption. The model equation is given as a two-dimensional elliptic equation analogous to the mild-slope equation on an impermeable bed. The model of wave breaking on a submerged permeable breakwater is proposed on the basis of the modified mild-slope equation. The validity of the model is confirmed through comparison with the experiments for a trapezoidal breakwater and with strict solution for a rectangular breakwater.

I. INTRODUCTION

A submerged breakwater has been shown as an effective wave control structure with less environmental impacts. Prediction of the wave transformation over a submerged permeable breakwater and the resultant effects on wave action is important in planning and designing structures for the coastal protection. So far, investigations have mostly been carried out on the basis of laboratory and field experiments such as Dattari *et al.* (1978). Though some theoretical studies were made by Liu (1973) and others, a simple and general predictive model has not been established yet.

In this study, a model equation is derived based on the mild slope assumption to analyze the transformation of waves over a submerged permeable breakwater. The resultant equation is given as a two-dimensional elliptic equation and is analogous to the standard mild-slope equation for an impermeable bed. A mathematical model based on the present mild-slope equation is used to compute the wave transformation for a general bottom configuration. The applicability of the model is demonstrated through comparison with laboratory data obtained from model experiments in a wave flume. The validity of the model is also examined by comparing with the strict solution for a rectangular submerged breakwater.

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Fig.1 Definition sketch; waves propagating over a horizontal porous layer.

II. MODELING OF WAVE TRANSFORMATION

The mathematical derivation of the basic equation for the transformation of nonbreaking waves over an inclined porous layer is given in Rojanakamthorn et al. (1989). In the following, the derivation is briefly reviewed and the model of breaking wave transformation is formulated.

2.1 Wave Transformation over an Inclined Porous Layer

The analytical approach starts with the formulation of the wave transformation over a horizontal porous layer. A definition sketch is shown in Fig.1 in which the depth of water, h_1 , and the thickness of the porous layer, $h_p = h_0 - h_1$, are constant. The governing equations describing the motion of an incompressible fluid inside the porous medium under the assumption of irrotational flow can be shown by Eqs.(1) and (2) in terms of the seepage velocity potential ϕ_s and pressure p_s .

$$\nabla^2 \phi_s = 0 \tag{1}$$

$$C_{\rm r}\frac{\partial\phi_{\rm s}}{\partial t} + \frac{1}{\rho}(p_{\rm s} + \gamma z) + f_{\rm p}\sigma\phi_{\rm s} = 0$$
⁽²⁾

where C_r is the inertia coefficient, ρ the mass density of water, γ the unit weight of water, f_p the linearized friction factor, σ the angular frequency of the periodic wave motion, $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ the gradient operator, and $\partial/\partial t$ the partial differential operator with respect to time. The friction factor f_p is evaluated from the Lorentz' condition of equivalent work. The relationship can be expressed as

$$f_{\rm p} = \frac{\int_{V} \int_{0}^{T} \left(\frac{\ell^2 \nu}{K_{\rm p}} |\vec{u}_{\rm s}|^2 + \frac{\epsilon^3 C_{\rm f}}{\sqrt{K_{\rm p}}} |\vec{u}_{\rm s}|^3 \right) dt \, dV}{\sigma \int_{V} \int_{0}^{T} \epsilon |\vec{u}_{\rm s}|^2 dt \, dV}$$
(3)

in which \vec{u}_s is the seepage velocity vector, ϵ the porosity of the medium, ν the kinematic viscosity, K_p the permeability, C_f the turbulent friction coefficient, V the volume under consideration, and T the wave period.

Equations (1) and (2) represent the Laplace equation and the unsteady Bernoulli equation for a seepage flow, respectively. The equations for the flow outside the porous medium can be obtained by substituting the coefficient C_r by unity and the friction

factor f_p by zero. The governing equations yield a potential flow problem. An analytical solution is derived for a monochromatic progressive wave train. The solutions for the velocity potentials were obtained as follows.

$$\phi = \frac{ig\eta}{\sigma} \frac{\epsilon \sinh(kh_{\rm p})\exp\{k(z+h_1)\} - \delta \cosh\{k(z+h_1)\}}{\epsilon \sinh(kh_{\rm p})\exp(kh_1) - \delta \cosh(kh_1)}$$
(4)

$$\phi_{s} = \frac{ig\eta}{\sigma} \frac{\cosh\{k(z+h_{0})\}}{\epsilon \sinh(kh_{p})\exp(kh_{1}) - \delta \cosh(kh_{1})}$$
(5)

where

$$\eta = a \cdot \exp\{i(\sigma t - kx)\}\tag{6}$$

$$\delta = \epsilon \sinh(kh_{\rm p}) - (C_{\rm r} - if_{\rm p})\cosh(kh_{\rm p}) \tag{7}$$

in which a is the amplitude of the incident wave prescribed at x = 0, g the gravitational acceleration, and $k = k_r - ik_i$ the complex wave number which is determined from the following relationship.

$$\sigma^{2} = gk \frac{\epsilon \exp(kh_{1})\sinh(kh_{p}) - \delta\sinh(kh_{1})}{\epsilon \exp(kh_{1})\sinh(kh_{p}) - \delta\cosh(kh_{1})}$$
(8)

It is noted that without a porous layer the above results reduce to the following linear wave solution (9) and the dispersion relation (10).

$$\phi = \frac{ig\eta}{\sigma} \frac{\cosh\{k(z+h_0)\}}{\cosh(kh_0)} \tag{9}$$

$$\sigma^2 = gk \tanh(kh_0) \tag{10}$$

A mild-slope equation is then derived for an inclined porous layer in a general bottom configuration. Referring to Fig.2 in which waves propagate over a submerged permeable breakwater, the governing equations derived from the equations of an incompressible fluid are expressed in terms of the amplitudes of velocity potentials as

$$\nabla_{\mathbf{h}}^{2}\Phi + \frac{\partial^{2}\Phi}{\partial z^{2}} = 0 \quad , \qquad \nabla_{\mathbf{h}}^{2}\Phi_{\mathbf{s}} + \frac{\partial^{2}\Phi_{\mathbf{s}}}{\partial z^{2}} = 0 \tag{11}, (12)$$

where $\nabla_{\rm h} = (\partial/\partial x, \partial/\partial y)$ is the horizontal gradient operator, and



Fig.2 Definition sketch; waves propagating over a submerged permeable breakwater.

$$\Phi = (ig\hat{\eta}/\sigma)G(z) \quad , \qquad \Phi_{s} = (ig\hat{\eta}/\sigma)H(z) \tag{13}, (14)$$

in which $\hat{\eta}$ is the complex amplitude of the water surface displacement, and

$$G(z) = \frac{\epsilon \sinh(kh_{\rm p})\exp\{k(z+h_1)\} - \delta \cosh\{k(z+h_1)\}}{\epsilon \sinh(kh_{\rm p})\exp(kh_1) - \delta \cosh(kh_1)}$$
(15)

$$H(z) = \frac{\cosh\{k(z+h_0)\}}{\epsilon \sinh(kh_p)\exp(kh_1) - \delta \cosh(kh_1)}$$
(16)

Equations (13) to (16) are assumed to be applicable to the slowly varying depth condition by using the local values of k, h_0 , h_1 and h_p . The mild-slope equation is derived by the vertical integration of Eqs.(11) and (12) after multiplying by appropriate functions.

$$\int_{-h_1}^0 \left[G\left(\nabla_h^2 \Phi + \frac{\partial^2 \Phi}{\partial z^2}\right) \right] dz + \int_{-h_0}^{-h_1} \left[\epsilon (C_r - if_p) H\left(\nabla_h^2 \Phi_s + \frac{\partial^2 \Phi_s}{\partial z^2}\right) \right] dz = 0 \quad (17)$$

By utilizing the relevant boundary conditions and the solutions of Φ and Φ_s with the form of Eqs.(13) to (16) are invoked and the variations of depth are taken into consideration, Eq.(17) is thus expressed as

$$\nabla_{h} \int_{-h_{1}}^{0} G^{2} \nabla_{h} \hat{\eta} dz + \int_{-h_{1}}^{0} k^{2} G^{2} \hat{\eta} dz$$

$$+ \epsilon (C_{r} - if_{p}) \left[\nabla_{h} \int_{-h_{0}}^{-h_{1}} H^{2} \nabla_{h} \hat{\eta} dz + \int_{-h_{0}}^{-h_{1}} k^{2} H^{2} \hat{\eta} dz \right]$$

$$= - \left[\int_{-h_{1}}^{0} G \nabla^{2} G \hat{\eta} dz + G \nabla_{h} G \cdot \nabla_{h} h_{1} \hat{\eta} \Big|_{-h_{1}}$$

$$+ \epsilon (C_{r} - if_{p}) \left\{ \int_{-h_{0}}^{-h_{1}} H \nabla_{h}^{2} H \hat{\eta} dz + H \nabla_{h} H \cdot \nabla_{h} h_{1} \hat{\eta} \Big|_{-h_{1}}$$

$$+ H \nabla_{h} H \cdot \nabla_{h} h_{0} \hat{\eta} \Big|_{-h_{0}} \right\} \right]$$

$$(18)$$

The assumption of slowly varying depth, *i.e.*, the mild slope assumption is imposed. The contributions of the terms on the right-hand side of Eq.(18) which are the second order of the bottom slope are considered negligible. The following elliptic equation is finally obtained.

$$\nabla_{\mathbf{h}}(\alpha \nabla_{\mathbf{h}} \hat{\eta}) + k^2 \alpha \hat{\eta} = 0 \tag{19}$$

where

$$\alpha = \alpha_1 + \epsilon (C_r - if_p)\alpha_2 \tag{20}$$

$$\alpha_1 = \beta_1^2 h_1 [(\beta_2^2/2kh_1)\{1 - \exp(-2kh_1)\} - (\beta_3^2/2kh_1)\{1 - \exp(2kh_1)\} - 2\beta_2\beta_3]$$
(21)

$$\alpha_2 = \frac{1}{2}\beta_1^2 h_{\rm p} \left\{ 1 + \frac{\sinh(2kh_{\rm p})}{2kh_{\rm p}} \right\}$$
(22)

$$\beta_1 = [\epsilon \exp(kh_1)\sinh(kh_p) - \delta \cosh(kh_1)]^{-1}$$
(23)

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$$\beta_2 = \epsilon \exp(kh_1) \sinh(kh_p) - (1/2)\delta \exp(kh_1)$$
(24)

$$\beta_3 = (1/2)\delta \exp(-kh_1) \tag{25}$$

Equation (19) is the mild-slope equation describing wave transformation on a mildly sloping porous layer and is an extension of the standard mild-slope equation derived by Berkhoff (1972) for an impermeable bed.

2.2 Breaking Wave Transformation on a Submerged Breakwater

(1) Modified mild-slope equation on porous layer

A submerged breakwater usually has a principal function of premature breaking of high waves to diminish the heights of waves transmitted to the shore. To develope a general predictive model for the breaking wave transformation, Eq.(19) is modified by incorporating an energy dissipation term as

$$\nabla_{\mathbf{h}}(\alpha \nabla_{\mathbf{h}} \hat{\eta}) + (k^2 \alpha - i \sigma \alpha f_{\mathrm{D}}) \hat{\eta} = 0$$
⁽²⁶⁾

where f_D is an energy dissipation function. The energy dissipation function f_D due to wave breaking will be derived on the basis of energy equation.

(2) Energy dissipation function of breaking wave

For a uniform porous layer and water depth condition, the value of α is constant and independent of horizontal coordinates. Equation (26) then reduces to

$$\nabla_{\mathbf{h}}^2 \hat{\eta} + (k^2 - i\sigma f_{\mathrm{D}})\hat{\eta} = 0 \tag{27}$$

The complex amplitude of the water surface displacement is now defined in terms of wave amplitude a as

$$\hat{\eta} = a \cdot \exp(-i\chi) \tag{28}$$

where χ is the phase angle.

Substituting Eq.(28) into (27) and grouping the real and imaginary terms, we obtain

$$\nabla_{\mathbf{h}}\chi \cdot \nabla_{\mathbf{h}}\chi = k_{\mathbf{r}}^{2}\{1 - (k_{\mathbf{i}}/k_{\mathbf{r}})^{2}\} + (1/a)\nabla_{\mathbf{h}}^{2}a$$
(29)

$$\nabla_{\mathbf{h}}(a^2 \nabla_{\mathbf{h}} \chi) = -(2k_{\mathrm{r}}k_{\mathrm{i}} + \sigma f_{\mathrm{D}})a^2 \tag{30}$$

The contribution from the last term in Eq.(29) may be considered negligible and the ratio k_i/k_r may be small, and hence Eq.(29) is approximated as

$$\nabla_{\mathbf{h}}\chi = (k_{\mathbf{r}}\cos\varphi, k_{\mathbf{r}}\sin\varphi) \tag{31}$$

where φ is the wave angle. If a unidirectional wave is considered, Eq.(30) reduces to

$$\frac{\partial}{\partial n}(a^2) = -(2k_{\rm i} + \frac{\sigma f_{\rm D}}{k_{\rm r}})a^2 \tag{32}$$

where n represents the coordinate in the direction of wave propagation.

Equation (32) is an energy equation of waves propagating over a horizontal porous layer in which the wave energy is dissipated due to porosity and wave breaking. The

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standard mild-slope equation describing the transformation of breaking waves on an impermeable bed is shown by Eq.(33).

$$\nabla_{\mathbf{h}}(cc_{\mathbf{g}}\nabla_{\mathbf{h}}\hat{\eta}) + (k^{2}cc_{\mathbf{g}} - i\sigma f_{\mathbf{D}}')\hat{\eta} = 0$$
(33)

where $f'_{\rm D}$ is the energy dissipation function, c the wave celerity, and $c_{\rm g}$ the group velocity. The energy dissipation function $f'_{\rm D}$ due to wave breaking was proposed by taking into account the processes of wave decay and recovery (Watanabe and Dibajnia, 1988).

$$f'_{\rm D} = \alpha_{\rm D} \tan \zeta' \sqrt{\frac{{\rm g}}{h}} \sqrt{\frac{\vartheta' - \vartheta'_{\rm r}}{\vartheta'_{\rm s} - \vartheta'_{\rm r}}}$$
(34)

where

 $\vartheta' = (|\hat{\eta}|/h), \quad \vartheta'_{\rm s} = 0.4(0.57 + 5.3\tan\zeta'), \quad \vartheta'_{\rm r} = 0.4(|\hat{\eta}|/h)_{\rm b}$ (35), (36), (37)

in the above equations α_D equals 2.5, $\tan \zeta'$ the bottom slope at the breaking point, and the subscript b denotes the value at the breaking point. The parameters ϑ'_s and ϑ'_r are expressed according to the results of experiment.

Under the same condition of uniform water depth, an energy equation of breaking wave transformation is derived in a similar fashion. The equation is expressed as

$$\frac{\partial}{\partial n}(a^2) = -\frac{f_{\rm D}'}{c_{\rm g}}a^2 \tag{38}$$

Supposing that the mechanism of energy loss due to wave breaking on a submerged permeable breakwater is similar to those on an impermeable bed, we obtain the following expression through comparison of Eqs. (32) and (38).

$$f_{\rm D} = (k_{\rm r}/\sigma c_{\rm g})f_{\rm D}^{\prime} \tag{39}$$

Equation (39) gives an energy dissipation function due to wave breaking on an inclined porous layer. Here, an effective depth is introduced to take into account the porosity of the structure.

$$h_1' = h_1 + \epsilon h_p \tag{40}$$

The energy dissipation function due to wave breaking on a submerged permeable breakwater is finally expressed as

$$f_{\rm D} = \frac{k_{\rm r}}{\sigma c_{\rm g}} \cdot \alpha_{\rm D} \tan \zeta \sqrt{\frac{{\rm g}}{h_1'}} \sqrt{\frac{\vartheta - \vartheta_{\rm r}}{\vartheta_{\rm s} - \vartheta_{\rm r}}}$$
(41)

where $\tan \zeta$ is the equivalent bottom slope at the breaking point which is defined as a mean slope in the distance $5(h'_1)_b$ offshoreward from the breaking point, and

$$\vartheta = (|\hat{\eta}|/h_1'), \quad \vartheta_s = 0.4(0.57 + 5.3 \tan \zeta), \quad \vartheta_r = 0.4(|\hat{\eta}|/h_1')_b$$
(42), (43), (44)



Fig.3 Breaking wave condition on submerged permeable breakwater.



Fig.4 Comparison of breaking wave condition.

(3) Breaking criterion

Wave breaking is usually detected from the condition which is determined by breaking indices. The information of wave breaking on a submerged permeable breakwater is scarce. Therefore, the wave breaking phenomena were preliminary investigated through a series of laboratory experiments.

The model experiments of submerged permeable breakwater were conducted with two sizes of gravel on a uniformly sloping bed and a horizontal bottom. The porosity of gravel were 0.39 and 0.44, respectively. On the basis of the experimental results, an empirical formula for the condition of wave breaking on a submerged permeable breakwater is proposed.

$$H_{\rm b}/L_0 = 0.127 \tanh\{k_0(h_1)_{\rm b}\}\tag{45}$$

where H_b is the wave height at the breaking point, L_0 the deep water wavelength, $k_0 = 2\pi/L_0$ the deep water wave number, and $(h_1)_b$ the depth of breakwater surface at the breaking point. The agreement of Eq.(45) with the experimental results is shown in Fig.3. The breaking wave condition in Eq.(45) has a similar form as that presented by Miche (1951).

$$H_{\rm b}/L_{\rm b} = 0.142 \tanh\{k_{\rm b}(h_1)_{\rm b}\}\tag{46}$$

in which $L_{\rm b}$ is the wavelength at the breaking point, and $k_{\rm b} = 2\pi/L_{\rm b}$ the breaking wave number. Equation (45) is plotted in comparison with Eq.(46) in Fig.4. It is shown that both equations give almost the same results for breaking wave height. However, the definition of the breaking wavelength and wave number for a submerged permeable breakwater is implicit. For the sake of the present application, Eq.(45) will be adopted as an index to detect the wave breaking.

2.3 Numerical Computation

A method of numerical computation is utilized for the computation of wave transformation in general breakwater configurations. A detailed description is also given in Rojanakamthorn *et al.* (1989). The governing equation is discretized by using a finite difference scheme. In this paper, a two-dimensional problem as in the wave-flume experiments is analyzed to examine the validity of the present model. Then, Eq.(26) reduces to

$$\frac{d}{dx}\left(\alpha\frac{d\hat{\eta}}{dx}\right) + (k^2\alpha - i\sigma\alpha f_{\rm D})\hat{\eta} = 0$$
(47)

The study area is divided into grids in the x-direction and a finite difference form of Eq.(47) is formulated. The computation is performed with the following boundary conditions. At the offshore boundary, an open boundary condition is imposed in which the water surface displacement is composed of incident and reflected wave components. Under the assumption of linear superposition, the complex amplitude of the water surface displacement is expressed as

$$\hat{\eta} = a_{\mathbf{i}} \exp(-ik_0 x) + a_{\mathbf{r}} \exp(ik_0 x) \tag{48}$$

where a_i and a_r represent the amplitudes of incident and reflected waves, respectively, and k_0 the complex wave number at the offshore boundary.

At the shoreline boundary, a non-reflective condition is imposed by considering that the wave energy is dissipated due to breaking on the shore. Therefore, the water surface displacement is the resultant of the transmitted waves.

$$\hat{\eta} = a_1 \exp(-ik_{\rm sh}x) \tag{49}$$

where a_t is the amplitude of transmitted waves, and k_{sh} the wave number in the shoreline region.

The solutions are obtained through an iteration. An initial value of the friction factor f_p is first assumed with zero energy dissipation due to wave breaking, $f_D = 0$. With a given incident wave condition, the complex wave number k at every grid is calculated from Eq.(8) via an iteration technique. The coefficient α is next evaluated from Eqs.(20) to (25) and then coefficients of the amplitudes $\hat{\eta}$ at every grid are determined. This results in a set of linear equations which can be solved by using the method of Gauss elimination. Wave breaking on the breakwater is then examined by Eq.(45) and the energy dissipation due to wave breaking is included from Eqs.(41) to (44). The calculation is iterated until the convergent results are obtained. The friction factor f_p is next evaluated from Eq.(3). The computed value of f_p is compared with the assumed one and the computation is iterated when required.



Fig.5 Definition sketch; waves propagating over a rectangular submerged breakwater.

For irregular waves, a technique of individual wave analysis is utilized. The advantage of this technique has been clarified in the analysis of wave breaking. In the computation, individual waves are defined from an irregular wave train through zerodowncrossing method and the transformation of each wave component is calculated.

2.4 Rectangular Submerged Breakwater

The validity of the mild-slope equation is examined by comparing with an analytical solution for a rectangular submerged permeable breakwater. A breakwater of finite width B is considered as shown in Fig.5. A monochromatic incident wave encounters the breakwater face at x = 0. Then, some part of the wave energy is reflected while the other part is transmitted through the breakwater. The flow field is separated into three regions. The solution in each region is shown in terms of unknown amplitudes as follows.

Region I (x < 0)

$$\phi_{\rm I} = \{a_{\rm i} \exp(-ikx) + a_{\rm r} \exp(ikx)\} \frac{\cosh\{k(z+h_0)\}}{\cosh(kh_0)} \exp(i\sigma t) + \sum_{m=1}^{\infty} a_{\rm rm} \exp(k_m x) \frac{\cos\{k_m(z+h_0)\}}{\cos(k_m h_0)} \exp(i\sigma t)$$
(50)

Region II (0 < x < B)

$$\phi_{\text{II}} = \sum_{n=0}^{\infty} [a_{1n} \exp(-ik_n x) + a_{2n} \exp\{ik_n(x-B)\}] G_n(z) \exp(i\sigma t)(-h_1 < z < 0)(51)$$

$$\phi_{\text{sII}} = \sum_{n=0}^{\infty} [a_{1n} \exp(-ik_n x) + a_{2n} \exp\{ik_n(x-B)\}] H_n(z) \exp(i\sigma t)(-h_0 < z < -h_1)(52)$$

Region III (x > B)

$$\phi_{\text{III}} = a_{i} \exp\{-ik(x-B)\} \frac{\cosh\{k(z+h_{0})\}}{\cosh\{kh_{0}\}} \exp(i\sigma t) + \sum_{m=1}^{\infty} a_{im} \exp\{-k_{m}(x-B)\} \frac{\cos\{k_{m}(z+h_{0})\}}{\cos(k_{m}h_{0})} \exp(i\sigma t)$$
(53)

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In the above equations, a_{rm} and a_{tm} represent the amplitudes of evanescent modes of reflected and transmitted waves, a_{1n} and a_{2n} the amplitudes of progressive and evanescent modes of the transmitted waves through the front face and reflected waves from the rear face of the breakwater, and

$$G_n(z) = \frac{\epsilon \sinh(k_n h_p) \exp\{k_n(z+h_1)\} - \delta_n \cosh\{k_n(z+h_1)\}}{\epsilon \sinh(k_n h_p) \exp(k_n h_1) - \delta_n \cosh(k_n h_1)}$$
(54)

$$H_n(z) = \frac{\cosh\{k_n(z+h_0)\}}{\epsilon \sinh(k_n h_p) \exp(k_n h_1) - \delta_n \cosh(k_n h_1)}$$
(55)

in which

$$\delta_n = \epsilon \sinh(k_n h_p) - (C_r - i f_p) \cosh(k_n h_p)$$
(56)

The wave number k is determined from Eq.(10) while k_m and k_n can be determined from Eqs.(57) and (58).

$$\sigma^2 = -\mathbf{g}k_m \tan(k_m h_0) \tag{57}$$

$$\sigma^{2} = gk_{n} \frac{\epsilon \exp(k_{n}h_{1})\sinh(k_{n}h_{p}) - \delta_{n}\sinh(k_{n}h_{1})}{\epsilon \exp(k_{n}h_{1})\sinh(k_{n}h_{p}) - \delta_{n}\cosh(k_{n}h_{1})}$$
(58)

In Eqs.(50) to (53), unknown parameters are a_r , a_{rm} , a_t , a_{tm} , a_{1n} and a_{2n} . These unknowns are solved with the boundary conditions, namely the continuity of pressure and mass flux at the interfaces. The orthogonality of the eigenfunctions is utilized. After mathematical manipulations, the amplitudes a_{1n} and a_{2n} can be solved from the following equations.

$$\sum_{n=0}^{\infty} \left[a_{1n} \left\{ \left(1 + \frac{k_n}{k} \right) \lambda_{10n} + \left(C_r - if_p + \epsilon \frac{k_n}{k} \right) \lambda_{20n} \right\} + a_{2n} \exp(-ik_n B) \left\{ \left(1 - \frac{k_n}{k} \right) \lambda_{10n} + \left(C_r - if_p - \epsilon \frac{k_n}{k} \right) \lambda_{20n} \right\} \right]$$
$$= \frac{2a_i}{\cosh(kh_0)} \left\{ \frac{h_0}{2} + \frac{\sinh(2kh_0)}{4k} \right\}$$
(59)

$$\sum_{n=0}^{\infty} \left[a_{1n} \left\{ \left(1 + \frac{ik_n}{k_m} \right) \lambda_{3mn} + \left(C_r - if_p + \epsilon \frac{ik_n}{k_m} \right) \lambda_{4mn} \right\} \right. \\ \left. + a_{2n} \exp(-ik_n B) \left\{ \left(1 - \frac{ik_n}{k_m} \right) \lambda_{3mn} \right. \\ \left. + \left(C_r - if_p - \epsilon \frac{ik_n}{k_m} \right) \lambda_{4mn} \right\} \right] = 0 \quad (m = 1, 2, 3 \cdots)$$

$$(60)$$

$$\sum_{n=0}^{\infty} \left[a_{1n} \exp(-ik_n B) \left\{ \left(1 - \frac{k_n}{k} \right) \lambda_{10n} + \left(C_r - if_p - \epsilon \frac{k_n}{k} \right) \lambda_{20n} \right\} + a_{2n} \left\{ \left(1 + \frac{k_n}{k} \right) \lambda_{10n} + \left(C_r - if_p + \epsilon \frac{k_n}{k} \right) \lambda_{20n} \right\} \right] = 0$$
(61)

$$\sum_{n=0}^{\infty} \left[a_{1n} \exp(-ik_n B) \left\{ \left(1 - \frac{ik_n}{k_m} \right) \lambda_{3mn} + \left(C_r - if_p - \epsilon \frac{ik_n}{k_m} \right) \lambda_{4mn} \right\} + a_{2n} \left\{ \left(1 + \frac{ik_n}{k_m} \right) \lambda_{3mn} + \left(C_r - if_p + \epsilon \frac{ik_n}{k_m} \right) \lambda_{4mn} \right\} \right] = 0 \quad (m = 1, 2, 3 \cdots) (62)$$

where

$$\lambda_{10n} = \int_{-h_1}^0 G_n(z) \cosh\{k(z+h_0)\} dz, \quad \lambda_{20n} = \int_{-h_0}^{-h_1} H_n(z) \cosh\{k(z+h_0)\} dz \quad (63)$$

$$\lambda_{3mn} = \int_{-h_1}^0 G_n(z) \cos\{k_m(z+h_0)\} dz, \quad \lambda_{4mn} = \int_{-h_0}^{-h_1} H_n(z) \cos\{k_m(z+h_0)\} dz \quad (64)$$

The amplitudes of reflected and transmitted waves a_r and a_t are then calculated from Eqs.(65) and (66).

$$a_{\rm r} = \frac{1}{\frac{2}{\cosh(kh_0)} \left\{ \frac{h_0}{2} + \frac{\sinh(2kh_0)}{4k} \right\}} \sum_{n=0}^{\infty} \left[a_{1n} \left\{ \left(1 - \frac{ik_n}{k} \right) \lambda_{10n} + \left(C_{\rm r} - if_{\rm p} - \epsilon \frac{k_n}{k} \right) \lambda_{20n} \right\} + a_{2n} \exp(-ik_n B) \left\{ \left(1 + \frac{k_n}{k} \right) \lambda_{10n} + \left(C_{\rm r} - if_{\rm p} + \epsilon \frac{k_n}{k} \right) \lambda_{20n} \right\} \right]$$
(65)
$$a_{\rm t} = \frac{1}{\frac{1}{\cosh(kh_0)} \left\{ \frac{h_0}{2} + \frac{\sinh(2kh_0)}{4k} \right\}} \sum_{n=0}^{\infty} \{ a_{1n} \exp(-ik_n B) + a_{2n} \} \{ \lambda_{10n} + (C_{\rm r} - if_{\rm p}) \lambda_{20n} \}$$
(66)

The reflection and transmission coefficients, K_r and K_t , are by definition equal to the absolute value of ratio of the complex wave amplitudes.

$$K_{\rm r} = |a_{\rm r}/a_{\rm i}|, \quad K_{\rm t} = |a_{\rm t}/a_{\rm i}| \tag{67}$$

Equations (59) to (67) are the strict analytical solution for a simple rectangular submerged permeable breakwater.



Fig.6 Model experiment of submerged breakwater.

III. VERIFICATION OF THE WAVE MODEL

The validity of the present model is examined by comparing the results from the numerical computation with experimental data obtained in a laboratory experiment and a strict solution for a rectangular submerged breakwater.

The experiments on the wave transformation over a submerged permeable breakwater were carried out in a laboratory wave flume as shown in Fig.6. The model breakwater composed of gravel with average diameter of 2.5 cm was placed on a 1/20 uniformly sloping bed. The offshore slope of the breakwater was 1/3. Tests were conducted for various wave conditions of regular and irregular waves with different values of the breakwater crown width. The water surface displacement was measured along the center line of the test section from offshore to onshore region. Sample of the experimental data for the distributions of root mean square value of water surface fluctuation are presented in Fig.7. In the figures, H_i and T represent the incident wave height and period for regular waves while $H_{1/3}$ and $T_{1/3}$ represent the significant wave height and period for irregular waves. The symbols X_t and X_c indicate the locations of the toe and crown of the breakwater, and X_{sh} the location of the shoreline. It is found that the wave amplitude considerably decreases over the breakwater which indicates a significant loss of the wave energy.



Fig.7 Distribution of root mean square value of water surface fluctuation.

The mathematical model based on the present mild-slope equation is applied to the computation of wave transformation over a submerged breakwater. Some important parameters are evaluated from given properties of the model breakwater; the porosity is 0.39. Values of the permeability K_p and turbulent friction coefficient C_f are interpolated from the results of laboratory tests under steady flow conditions which were presented by Sollitt and Cross (1972). In the present computation, the values of $K_p = 3.77 \cdot 10^{-7}$ m² and $C_f = 0.332$ are adopted. The inertia coefficient C_r is approximately regarded as unity after Sollitt and Cross (1972). The computation is performed by the iteration procedure. Results of the numerical computation are also shown in Fig.7 in comparison with the experimental data. The agreement between the model computation and experiment is generally good although the computation gives a transmitted wave component slightly smaller than the measured one. The computation also shows a slight overestimation of reflected wave due to the energy dissipation.

The present model is applied to a rectangular submerged breakwater and the reflection and transmission coefficients are compared with the strict solutions. A breakwater of width B = 3 m with the same physical properties as mentioned is considered. The amplitudes of reflected and transmitted waves are obtained from the potential in the offshore and onshore region of the breakwater. Then, the reflection and transmission coefficients are calculated. The computation was performed for various conditions. Sample results of the comparison are shown in Fig.8 as a function of relative water depth, h_0/L_0 . The agreement is fairly good in spite of the assumption of mild slope used in the derivation.



Fig.8 Reflection and transmission coefficients of rectangular submerged breakwater.

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IV. CONCLUDING REMARKS

A mathematical model for the computation of wave transformation over a submerged permeable breakwater has been proposed on the basis of newly-derived mildslope equation. The model equation is simple and helpful in understanding the physical behavior of the phenomenon. Its application was described by means of a numerical computation and the validity of the model was confirmed through comparisons with the experimental data and with a strict solution for a rectangular submerged breakwater. The present model is applicable for a wide range of breakwater configurations. However, the extension of the present model for a three-dimensional problem is essential to make the model more practical and useful for the prediction of wave field around the submerged breakwater.

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