CHAPTER 70

BREAKING AND REFLECTION OF A STEEP SOLITARY WAVE CAUSED BY A SUBMERGED OBSTACLE

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ABSTRACT

A fully nonlinear potential flow theory is solved numerically but almost exactiy for a soritary wave passing over a submerged obstacle by using BIM. Based on the numerical solutions, the refrection characteristics and the breaker type and criterion are made clear for a solitary wave up to breaking caused by a step.

1. INTRODUCTION

A sound knowledge on the transformation including the breaking of steep coastal waves over a submerged obstacle is important for planning and designing submerged coastal structures. However,few knowledge is obtained on the breaking caused by a submerged obstacle including a discontinuity in depth.

It has often been remarked that waves on beaches resemble solitary waves. In fact, steep coastal waves are demonstrated to be representable as a random train of solitons(Tsuchiya & Yasuda, 1986). Hence, it may be better to consider each wave crest as a solitary wave and investigate its transformation, rather than to examine directly that of steep coastal waves.

A computational model to be used here is based on BIM and has already developed by authors (1989).It can describe almost exactly the transformation up to overturning of the solitary wave propagating over a bed

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containing a submerged obstacle.

This study aims to make clear the breaker criterion and reflection characteristics of steep solitary waves passing over a step by using the computational model(Yasuda et al., 1989) which can describe almost exactly the transformation up to overturning of the solitary wave.

2. COMPUTATIONAL MODEL

In this study, the solitary wave given by the exact steady solution of fully nonlinear potential-flow theory is supposed over the planar of left side from the obstacle, and the still water is supposed over the obstacle and its right side planar in a two-dimensional domain. Further, the Cauchy integral theorem is introduced to solve Laplace's equation under the condition on the rigid boundary. The updating of the free surface profile and the velocity potential is based on the second-order Taylor expansion in a mixed Eulerian-Lagrangian formulation as well as Dold & Peregrine(1986).

3. VALIDITY OF THE MODEL

The first check of the time stepping accuracy is provided by examining the growth of the error energy defined by

$$E(t) = \frac{1}{2}\int_{-\infty}^{\infty} [\eta(x,t) - Y(x-ct)]^2 \, dx \, , \tag{1}$$

where Y is the surface displacement of the steady solitary wave mentioned above, c its propagation speed and η the water surface profile given by the present numerical solution. A test is made on the solitary wave in still water up to the dimensionless time $t\sqrt{g/h}=12$. Here, h is the undisturbed water depth and g the acceleration of gravity. While the wave has propagated on the distance of 15h, E(t) remains less than 3×10^{-5} in value. (Note, for comparison, that for the wave under consideration $(1/2)\int Y^2 dx=0.42$.)

The second check is provided by examining the validity of the model against carefully controlled experiments in a wave tank(1m×1m×54m). The desired steep solitary wave is generated by using a servo-controlled, hydraulically activated wave maker. The free surface displacement is measured with the capacitance-type wave gauges installed at three locations, P.1, P.2 and P.3 which are placed at the front of the obstacle, just behind of it and at the breaking point of the transmitted solitary wave, respectively. The breaking point is defined as the onset of the formation of a jet or bubble plume. The location of P.3(breaking point) is decided for each incident solitary wave by using a high speed video camera and observing the presence of the jet or bubble plume.

Figure 1 shows an example of the computed shape of the free surface at some evolution times. The initial wave-height H/h of the incident solitary wave is 0.4, and the relative height R/h and length B/h of the submerged breakwater model are 4/7 and 24/7, respectively. It is found that while the transmitted

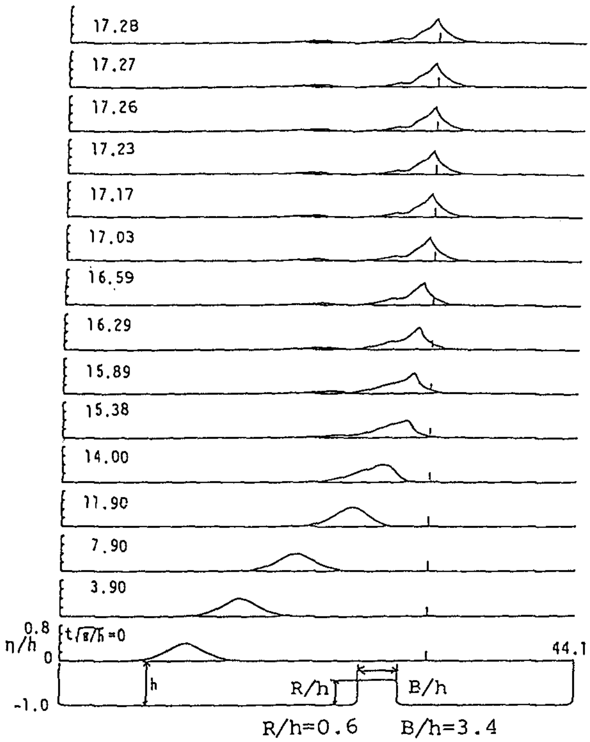


Fig.1 Transformation of a solitary wave passing over a submerged break-water

solitary wave makes its front face overturn at the location of P.3, the reflected wave propagates backward as a small but noticeable solitary wave.

Figure 2 describes the temporal changes of the crest-height η_c/h and the horizontal water particle velocity $U_c/\sqrt{gh}$ and the gradient angle of the front face θ at the top of the solitary wave shown in Fig.1. It is found that the gradient angle θ reaches -90 degrees and the front face becomes vertical at the onset when the top of the wave passes through the location of P.3 where the formation of a jet was observed in the experiment. Hence, the onset of overturning can be defined as the breaking point in the computational model.

Figure 3 shows the comparisons of the temporal surface elevation of the solitary wave shown in Fig.1 between the results computed by the model and the experimental results measured with the wave gauges installed at P.1, P.2 and P.3. The computed and measured profiles can not be quite distinguished up to the overturning location(P.3),nevertheless the formation of vortex was observed behind the end of submerged breakwater. This remarks that the numerical

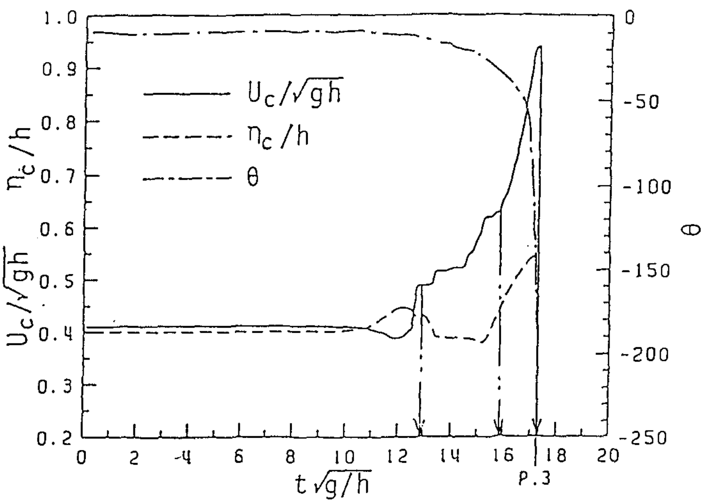


Fig.2 Temporal changes of the quantities at the top of the wave, η_c/h, $U_c/\sqrt{gh}$ and θ.

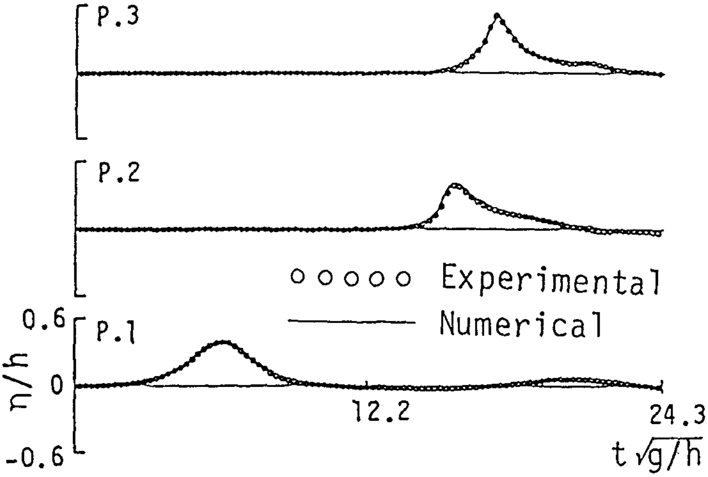


Fig.3 Comparisons with temporal water surface elevation between numerical and experimental results

results for the transformation of steep solitary waves over submerged obstacles are in excellent agreement with tank measurements up to overturning and the present model makes it possible not only to describe the transformation including overturning but also to predict the breaking point.

Further, in order to demonstrate the accuracy of the numerical solution obtained by using the present scheme, we carried out some numerical simulations of solitary waves propagating over a gently sloping bottom under the same initial wave-height H/h and bottom slope tan θ with Papanicolaou and Richlen's experiment(1988). The breaker wave-height H_b/h was compared between the numerical results, where the breaker point is defined as the onset of overturning and experimental results by Papanicolaou and Richlen (for brevity P-R). Table 1 shows the results of the comparisons. It can be easily recognized from this table that the breaker wave-heights computed with the present model agree with those obtained by P-R within the error degree of 1%. This remarks that the breaking criteria could be established for the solitary waves without carring out experimental works in a wave tank.

Table 1 Comparison with the breaker wave-height between the present numerical results and experimental ones by P-R

H /h	$\tan\theta$	H_b/h	
		Experi. (P-R)	Numeri.
0.2	0.0141	1.222	1.218
0.3	0.0126	1.086	1.077
0.4	0.0126	1.071	1.080

4. DEFORMATION UP TO BREAKING AND REFLECTION

Figure 4 shows. the propagation processes of solitary waves up to breaking points. While the wave profile at the breaker point shown in Fig.4(a) seems to be a spilling breaker, the wave profile shown in Fig.4(b) could be regarded clearly as a plunging breaker. From these results, we can be convinced that both breaker types of spilling and plunging occur even in the case except for sloping bottoms and depend on a parameter $\xi_s{}^{*}$ defined by authors (Yasuda,Hara & Sakakibara 1990) as

$$\xi_s^* = (R/h)/(H/h)^{2/5} \ , \tag{2}$$

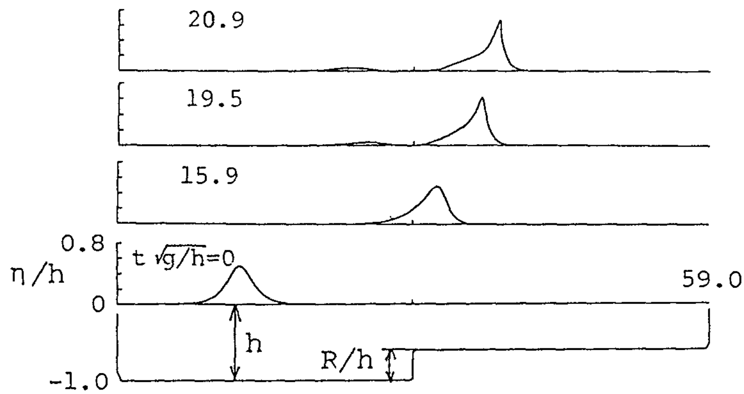
where H denotes the initial wave-height of an incident solitary wave, h the still water depth in front of the rectangular step and R its height.

Further, we define a horizontally asymmetric parameter β_4

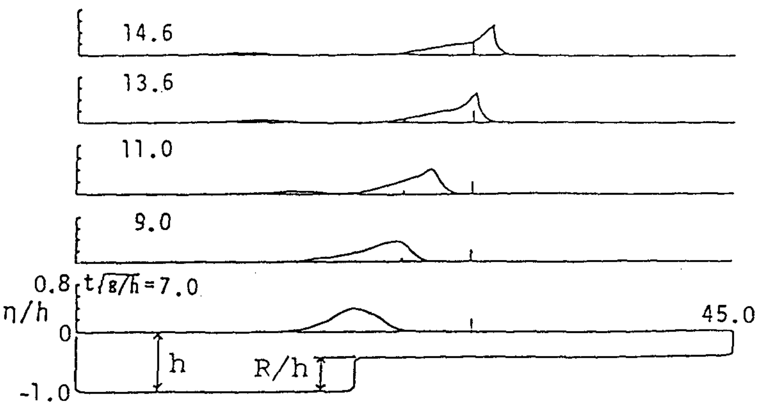
$$\beta_4 = \frac{-1}{2B}\int_{-B}^{B} \eta_x dx \ , \tag{3}$$

as the breaker type index, since the breaker type could be supposed to correspond directly to horizontal asymmetry. Here, the integral region B in the equation is indicated in Fig.5 Figure 6 shows the relation between the values of the parameter $\xi_s{}^{*}$ of incident solitary waves and the parameter β_4 calculated from their wave profiles at breaker points.

Figure 7 indicates the backward propagation process of a reflected wave from a rectangular step . The reflected wave also seems to propagate as a steady solitary wave. Hence, we can easily define the reflection coefficient Kr of a solitary wave as a ratio



(a) Spilling breaker
(H/h=0.50, R/h=0.4, ξ_s' =0.032)



(b) Plunging breaker
(H/h=0.36, R/h=0.6, ξ_s' =0.100)

Fig.4 Temporal changes of wave profile of a solitary wave passing over a step

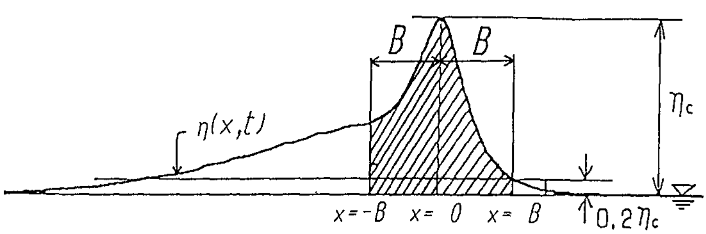


Fig.5 Region of integration for a horizontally asymmetric parameter β_4

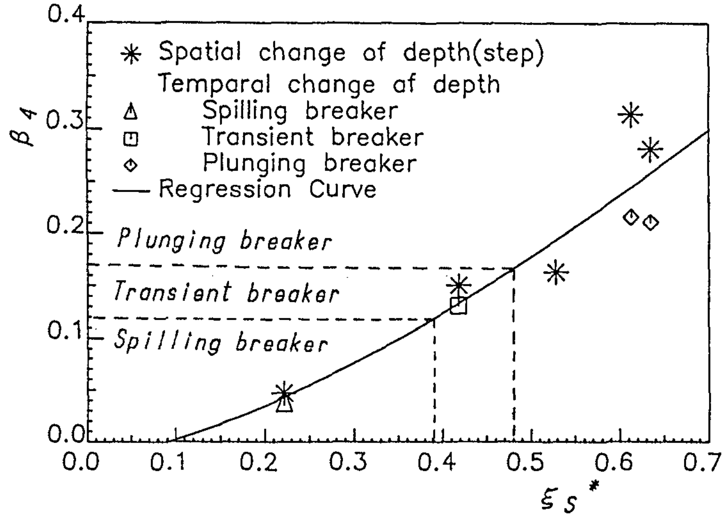


Fig.6 Relation with the breaking of a solitary wave caused by a step between $\xi_s{}^*$ and β_4

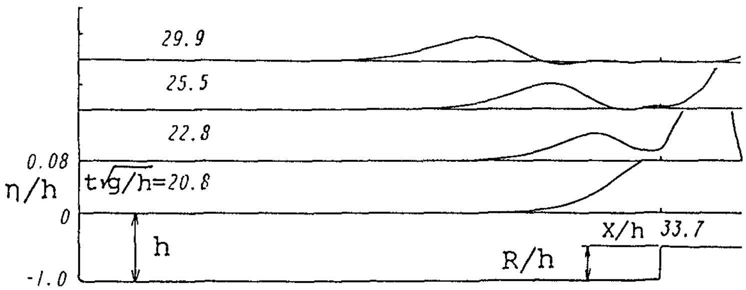


Fig.7 Backward propagation of a reflected wave from a step

of an incident wave-height H to a wave-height of the reflected solitary wave. Figure 8 shows the comparison of the relation of the reflection coefficient Kr to the relative step-height R/h between the computed results by the present model and the experimental ones by Seabra-Santos et al(1987). In the figure, a solid line indicates the result calculated by Lamb's formula

$$K_r = \frac{1-\sqrt{1/(1-R/h)}}{1+\sqrt{1/(1-R/h)}} \quad , \tag{4}$$

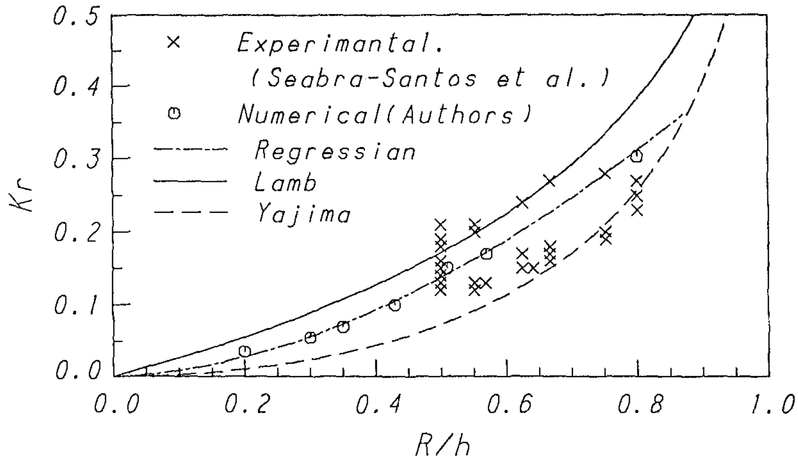


Fig.8 Relation of the reflection coefficient Kr from a step to its relative step height R/h

and a broken line denotes the theoretical result suggested by Yajima(1984) who solved the reflection problem of the KdV soliton using the inverse scattering method.

$$K_r = \frac{1}{4}\left\{\sqrt{1+8\frac{\sqrt{1/(1-R/h)}-1}{\sqrt{1/(1-R/h)}+1}}-1\right\}^2 , \tag{5}$$

The regression curve is drawn with the following equation

$$K_r = 0.460(R/h)^{1.745} , \tag{6}$$

It is made clear that the reflection coefficients obtained by the present model are independent of the amplitudes of incident waves as well as both the results of Lamb and Yajima, although they reveal the intermediate characteristics between eq.(4) and eq.(5). This remarks that the reflection coefficient of steady waves from a step is almost independent of the order of their nonlinearity but mainly depends on the relative step-height R/h alone.

5. BREAKING CRITERIA

Figure 9 shows the relation between the wave-height H/h of incident solitary wave of which front face

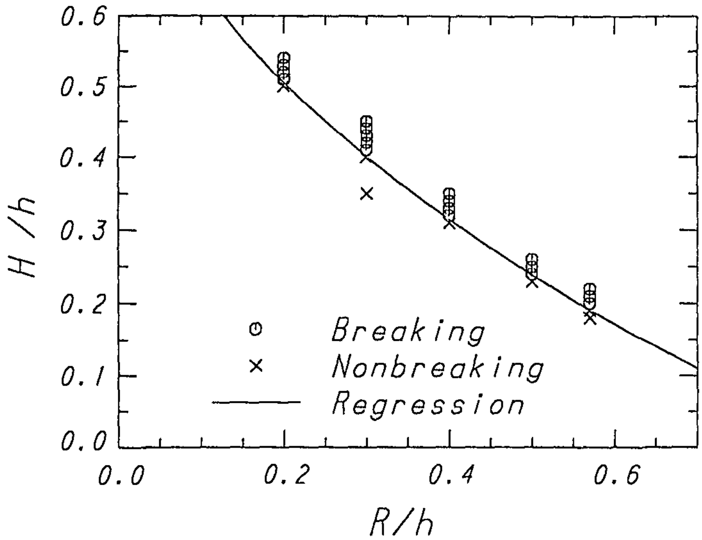


Fig.9 Relation of the presence of breaking caused by a step to the initial wave-height of incident solitatry wave H/h and relative step-height R/h

overturns or not on a step and the relative step-height R/h. The solid line indicates the limiting wave-height H_c/h of the solitary wave which can be transmitted without breaking. The limiting wave-height is almost inversely proportional to the step-height. From the relation between H/h and R/h, the value of the critical step-height comes to light against each incident solitary wave which propagates over the step without breaking.

Figure 10 shows the breaking criteria of a solitary wave over a step, that is, the relation of the crest-height at the onset of breaking η_b/d and the limiting wave-height H_c/d to the relative step-height R/h. Here, d is the water depth on the step and is equal to h-R. It should be noted that the value of η_b/d is almost constant and is nearly 0.9, independently of the step-height. This remarks that the solitary wave having the crest-height under about 0.9 does not break on the step, even if the crest-height exceeds 0.78 which is the maximum crest-height of the steady solitary wave on a flat bottom. The solitary wave having the crest-height between 0.78 and 0.9 suffers fission on the

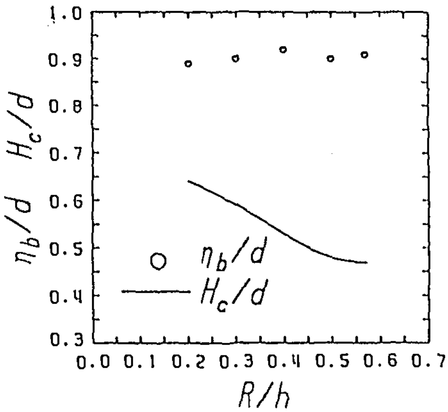


Fig.10 Breaking criterion of a solitary wave over a step

step, instead of breaking.

6. CONCLUDING REMARKS

It is verified through carefully controlled experiments that the present model enables us to compute very accurately the transformation of a steep solitary wave leading to breaking. Many numerical simulations using the model yield the breaker criterion and reflection coefficients of steep solitary waves over a step and further make clear the characteristics of reflected waves consisting of the positive solitary wave from the submerged breakwater.

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