CHAPTER 49

Bottom Shear Stress and Friction Factor Due to the Asymmetric Wave Action

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Abstract

A micro shear gauge was used to measured the bottom shear stress under the shallow water wave on the smooth bed, the data show that bottom shear stress have a asymmertic form similar to the surface water elevation due to the non-linearity of wave. The bottom friction factor under wave creat and wave trough were very different with that defined by Jonsson's and the results were contrary when the bottom particle velocity were defined by linear and non-linear wave theory respectively. Empirical formula were obtained for practical use.

Introduction

Because of the development of ecnomic and increase of population, in recent years, utilization of coastal areas has steadily been increasing for human activities such as transportation and industry. All of the development were taken in the nearshore area and the wave phenomena occured in this area almost influced by the bottom shear stress, directly or indirectly.

Therefore wave induced bottom shear stress and friction has an impacted relativity with energy decay、sediment transport rate and wave induced current, etc. About two decade, there were many researchers paid their attention to the bottom boundary layer problems, such as Riedel(1972), Jonsson(1976), Kamphius(1976)

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, Iwagaki (1974), John (1982), Kuo & Chuan (1981) etc. Though they have obtained some basic results, those researchs haven't disscussed the variation of bottom shear stress and friction affected with the asymmetry of wave profile in shollow water; therefore, the phenomena described by the above may have an important parameter to determine the sediment moving direction and the on-offshore sediment transport rate.

Sawaragi(1974), Tanaka(1987), Iwagaki(1987),etc. have presented that the non-linear effect of shollow water wave could be made a obviously difference on bottom shear stress and friction factor under the wave crest and wave trough passed respectively. Tanaka (1987) used the stream function theory to calculate the shear stress under non-linear wave and took their results to derive the on-offshore sediment transport rate. In spite of there have a few paper menationed about the bottom shear stress influenceed by the non-linear wave, those research haven't analyzed the shear stress under crest and trough detailly, also they haven't measured the bottom shear stress to verify their results.

In this paper, we use a micro shear gauge to measure the bottom shear stress under shollow water wave acting, the data collected from this experiment were taken to analyze the stress under wave crest and trough respectively, the results were compared with the stress that calculate from linear wave theory. Also we use the non-linear wave theory to calculate the bottom friction factor, and the analyzed results show a visible difference which compared to the results that obtained from linear wave theory by Jonsson etc.

Experimental Facilities and Conditions

The experiment of this paper is taken in a 40 m long, 1.0 m wide, and 1.0 m deep wave flume. The bottom shear stress is measured by a micro shear gauge which was set up in a steely box and installed into the bottom of flume. The sketch of the experimental equipments were show in Fig. 1. The maximum capaicity of this shear gauge is 0.2 g and the diameter of the sensible plate is only 1 cm, therefore the sensitive area is too small when compared to the shear plate , so the measured error which occurred at the shear plate; such as sensitive accuracy of the gauge and the

pressure gradient difference at the each side of plate , etc. could be reduce to minimum; therefore the data measured by this micro shear gauge may be more reasonable.

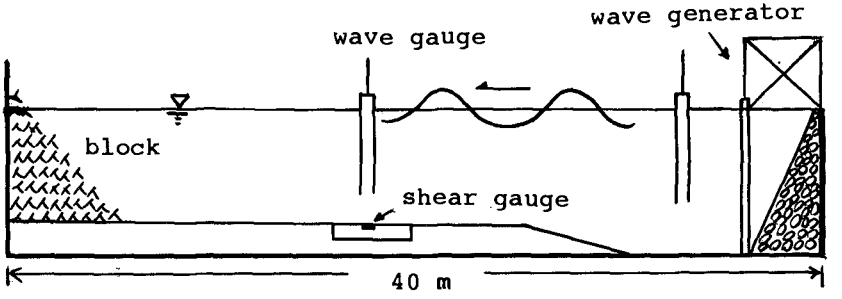

Fig. 1 Sketch of experimental wave flume.

Fig. 2 shows the sketch of the micro shear gauge used by this paper.

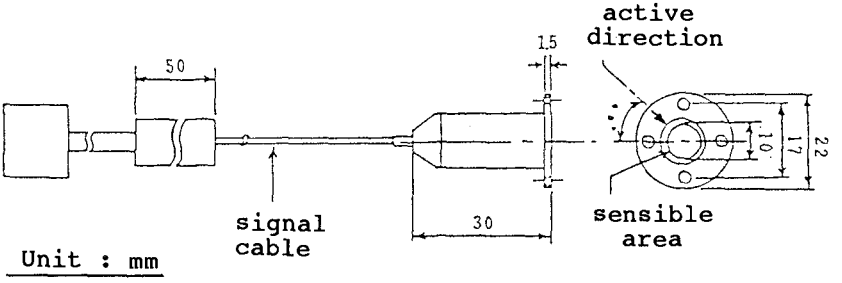

Fig. 2 Sketch of shear gauge.

The test conditions of this experiment are :

(1) bottom type : smooth bed.
(2) water depth : 20 cm, 25 cm, 30 cm, 35 cm
(3) wave period : 1.0, 1.2, 1.4, 1.6, 1.8, 2.0 sec
(4) wave height : enlarged gradually from 2 cm to before breaking

Fig. 3 shows a example of time series of surface wave height and bottom shear force that just measuring from experiment. In this figure, we could find the asymmetry of bottom shear stress were related to the surface wave profile.

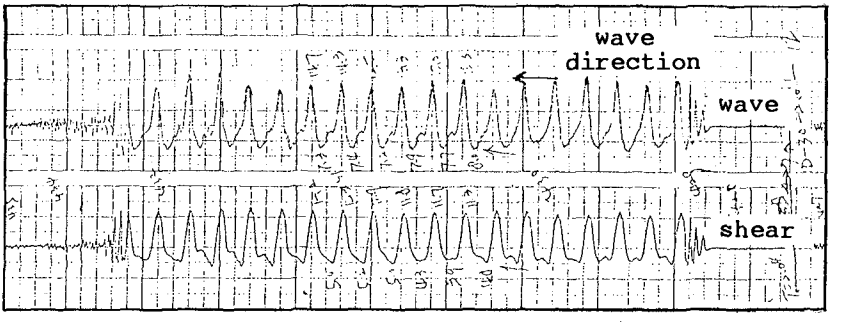

Fig. 3 Time history of measured wave and shear.

Experimental Results and Discussion

(1) Shear stress

In general case, the maximum bottom shear stress τ_{omax} is defined as

$$\tau_{omax}=\mu(\partial u/\partial z)_{z=-h} \qquad \cdots\cdots\cdots\cdots\cdots (1)$$

If the horizontal particle velocity is calculated by linear wave theory and we transformed Eq. 1 to a non-dimensional form, then a dimensionless equation can be represented as Eq. 2

$$\tau_{omax}/\rho gH=\sqrt{2}/[g\sinh(kh)](\pi/T)^{3/2} \qquad \cdots\cdots\cdots(2)$$

where h is water depth, H wave height, k wave number, T wave period, ρ water density, ν is kinematic viscosity and τ_{omax} is the maximum bottom shear stress calculated from Eq. 2.

Fig. 4 represents the non-dimensional measured maximum shear stress under the wave crest (postive direction) and wave trough (negative direction) passed related to the parameter h/L_o , and the curves on the diagram represent the value that obtained from Eq. 2 which was derived by linear wave theory. From this diagram, we could find that the postive maxmium shear stress are large than the negative maximum shear stress obviously, and the measured positive value are large than the value were got from linear wave theory and the measured negative value are small than τ_{omax} , the reason is that in shollow water, wave profile

are no longer be maintained a sine form, it just has a sharp crest and a flat trough, this non-linear effect influence on bottom directly, so the shear stress on bottom also has a profile similar to the surface water elevation. The fluctuation of the measured data were due to the constant value of ν in Eq. 2 and the different bottom friction in each test condition.

The symbol of τ_c , τ_t in the figure represents the measured bottom shear stress under wave crest and and wave trough respectively.

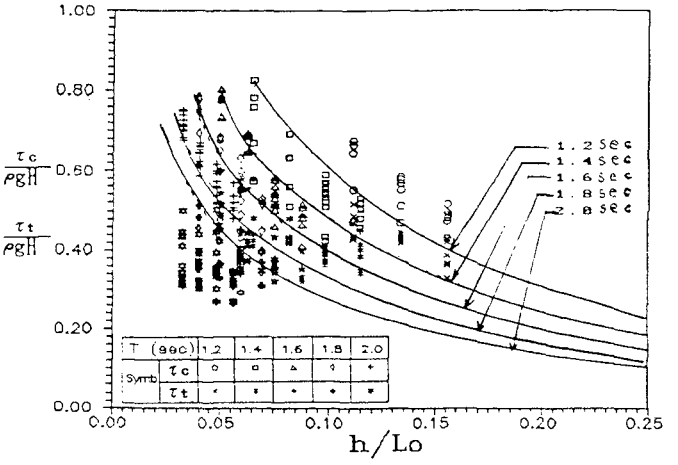

Fig. 4 Relation between $\tau_c/\rho gH$, $\tau_t/\rho gH$ and h/L_o.

Fig. 5 shows the relation between τ_c/τ_a , τ_t/τ_a and Ursell number Ur, the data in the figure indicate that the ratio of τ_c/τ_a are great than 1 , τ_t/τ_a are less 1 , the reason were due to the asymmetric wave action, that just described above. There also exist a obvious tendency of that τ_c/τ_a , τ_t/τ_a were enlarged and reduced gradually related to the increase of Ur respectively, this phenomenon could emphansized that when wave non-linearity increased, the postive shear stress also increased, but the negative shear stress were decreased. The increment of postive direction are more obvious than negative direction. This may has an important effect on the initial motion of sand and the on-offshore sediment transport rate, so this topic will be studied more detail. The regressive equation from data is expressed as Eq. 3 for practical use.

$$\tau_c/\tau_a = 0.903 Ur^{0.091} \quad , \quad \tau_t/\tau_a = 1.211 Ur^{-0.149} \quad ....(3)$$

Here $\tau_a = f_\omega u_a^2/2$ and f_ω is the friction factor, u_a the maximum bottomhorizontal particle velocity by linear theory. Ursell number Ur was defined as

$$Ur = HL^2/h^3 \quad .....(4)$$

where L is the wave length at water depth h.

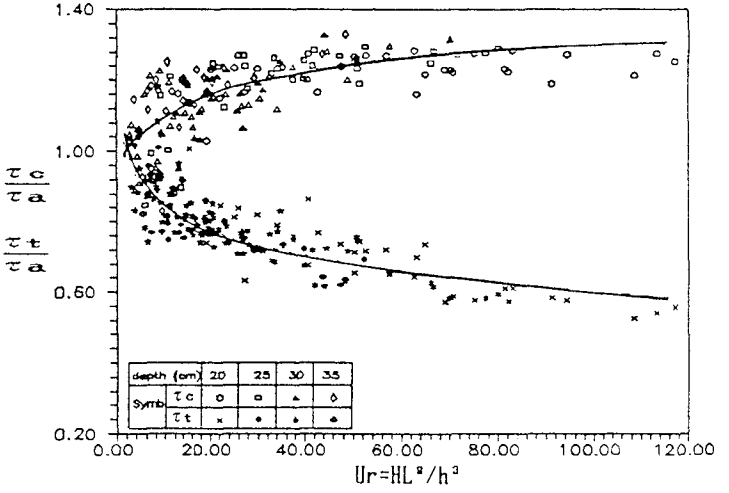

Fig. 5 Relation between τ_c/τ_a , τ_t/τ_a and Ur.

(2) Bottom Friction Factor

Fig. 6 shows the relation between Reynold number (RE) and bottom friction factor f_ω, f_{ac}, f_{at} Where f_ω is defined by Jonsson etc., f_{ac} 、f_{at} respresent the bottom friction factor under wave crest and wave trough action. Where

$$f_\omega = 2./\sqrt{RE} \quad , \quad f_{ac} = 2.\tau_c/\rho u a^2 \quad , \quad f_{at} = 2.\tau_t/\rho u a^2 \quad ....(5)$$

From this diagram, we can find that when linear wave theory was defined to compute the horizontal bottom velocity, the friction f_{ac} is larger than f_ω, and f_{at} is smaller than f_ω, furthermore, this tendency is more visible when RE increased. The experimental data can be regressed by two empirical formula like Jonsson's ,

$$f_{ac}=1.386RE^{-0.448} \quad , \quad f_{at}=4.093RE^{-0.605} \qquad ....(6)$$

Also, we could express f_{ac}, f_{at} in another form, just like Eq. 7

$$f_{ac}=0.724f_{\omega}^{0.875} \quad , \quad f_{at}=1.611f_{\omega}^{1.183} \qquad ....(7)$$

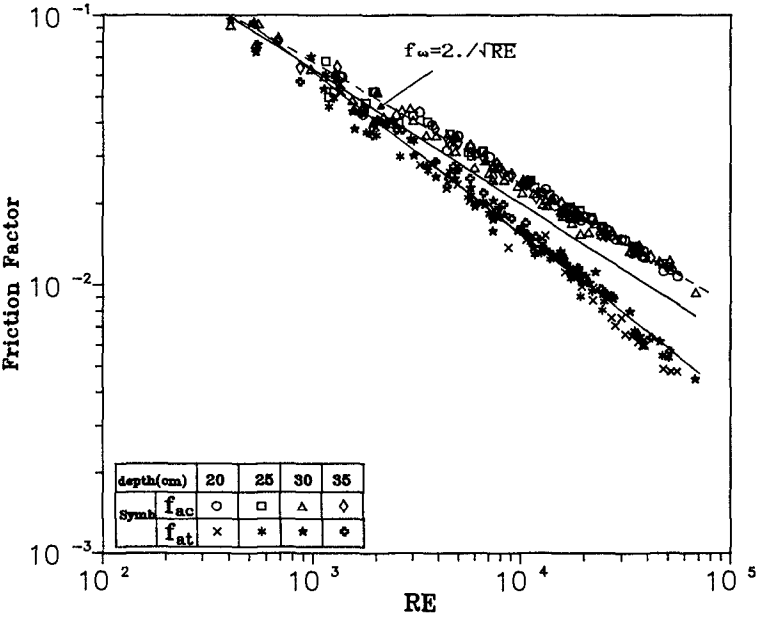

Fig. 6 Relation between friction factor and RE

In nearshore area, linear wave theory is unsuitable to describe the wave phenomena. For more realistic conditions, we use the non-linear wave method to define the bottom particle velocity, and use those to calculate the bottom friction factor under wave crest and trough, which were defined as Eq. 8

$$f_{sc}=2\tau_c/\rho u_{sc}^2 \quad , \quad f_{st}=2\tau_t/\rho u_{st}^2 \qquad .....(8)$$

where

$$u_s=c\{F_1\cosh[k(h+z)\cos(kx-\sigma t)+F_2\cosh[2k(h+z)\cos2(kx-\sigma t)+F_3\cosh[3k(h+z)]\cos3(kx-\sigma t)]\,|_{z=-h}$$

$$F_1=ak/[\sinh(kh)]$$

$$F_2=3a^2k^2/[4\sinh^4(kh)]$$

$$F_3=3a^3k^3[13-4\cosh^2(kh)]/[64\sinh^7(kh)]$$

When the friction factor were defined by Eq. 8, the relationship between f_{sc}, f_{st} and RE were show in Fig. 7. It shows that f_{sc} is smaller than f_{st} and f_{ω}, but f_{st} is larger than f_{sc} and f_{ω}. These results are just contrary to those obtained from linear wave theory which were defined in Eq. 5, the reason is that when velocity is calculated by finite amplitude wave theory ,there were a sharply velocity under wave creast and a flatly velocity under wave trough due to the wave non-linearity in shollow water. So how are the real bottom friction factor change with wave condition must be studied detailly. For avoid the complex calculation , the data shows in Fig. 7 can be expressed by a empirical formula as Eq. 9 or Eq. 10

$$f_{sc}=2.596RE^{-0.540} \quad , \quad f_{st}=0.613RE^{-0.341} \qquad ...(9)$$

or

$$f_{sc}=0.724f_{\omega}^{0.875} \quad , \quad f_{st}=1.611f_{\omega}^{1.183} \qquad ...(10)$$

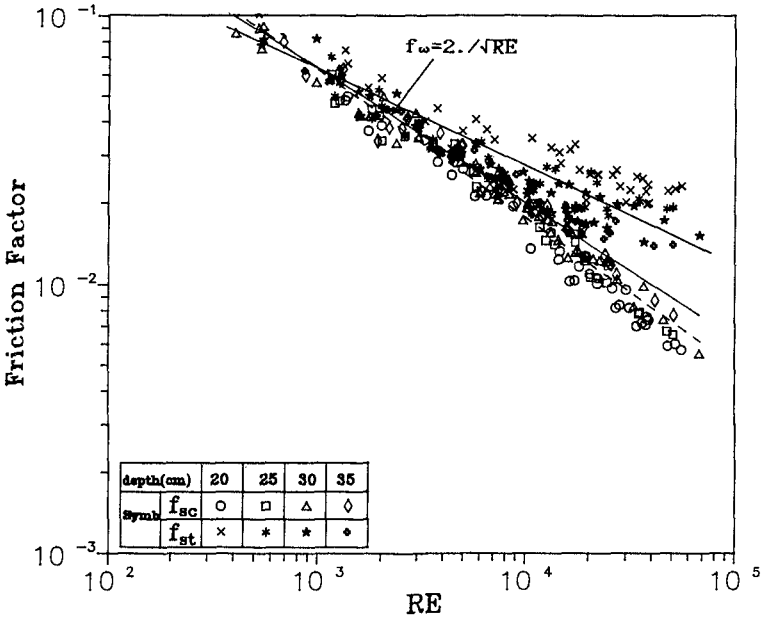

Fig. 7 Relation between friction factor and RE.

Fig. 8 shows the comparisions of f_{ω}, f_{ac}, f_{at}, f_{sc}, f_{st} from this diagram the differences between these factors can be viewed more clearly. When RE

were near transitional flow condition , the data begin scattered away from f_w and f_{st} scattered more obviously, the reason were due to the slowly increased of bottom velocity under wave trough action. There exist a 10% $\sim$ 40% difference between f_{sc} , f_{st} and f_w, the variation were depended on the Reynolds number. If we have measured the velocity near bottom simultaneously , then how the friction factor varied may be verified more accurately.

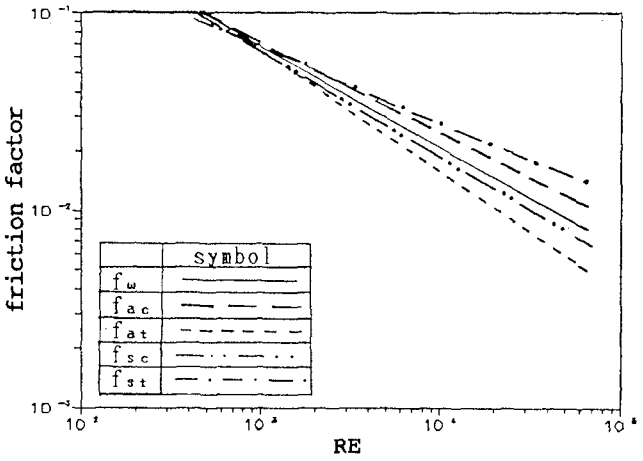

Fig. 8 Comprisions of friction factor and RE.

Conclusion and Suggestion

It shows that the bottom shear stress has a asymmetric form similar to the surface wave profile in shallow water. The postive maximum shear stress (under wave crest) are much larger than the negative maximum shear stress (under wave trough) and those computed from linear wave theory. In addition, the friction factor were varied with what the wave theory was adopted, thers show a constray results when the bottom velocity were defined by the linear and non-linear wave respectively. Phenomena which induced by wave just mentioned above are an important parameter on wave energy decay, nearshore current and may have a determination on the on-offshore sediment transport rate and the particle moving mechanics etc., so how they are changed by the asymmetric wave profile must be study more detailly. If the wave profile, bottom

velocity and shear stress are mearsured simultaneously , the advanced discussion may get a more reasonable explanation. The next work for us are to improve the experiment and extend to the rough bed condition.

Moreover, we attempt to research a function to express the shear stress and friction factor under a asymmetric wave action and to correlate the results with the on-offshore sediment transport rate.

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