CHAPTER 36

ON THE FITTING OF JONSWAP SPECTRA TO MEASURED SEA STATES

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ABSTRACT
A new technique has been described for the optimal fitting of JONSWAP model spectra to measured sea states. This technique, validated by numerical simulations, can provide enhanced estimations of the peak frequency, the peak enhancement factor and the significant wave height of natural seas.

1.0 INTRODUCTION

The variability of spectral estimates resulting from conventional variance spectral density analysis of relatively short wave records, is appreciable. It is, therefore, difficult to obtain from them a reliable appraisal of spectral parameters that reflect the properties of the sea state prevailing over a large area and over a long duration. Furthermore, wave data recorded in nature are frequently corrupted with background noise, such as transmission interference or data loss due to buoy submergence. As a result, the variance spectral densities of such wave data must usually be band-limited at arbitrarily chosen upper and lower cut-off frequencies. This truncation can have a significant effect on the values of estimated wave parameters, particularly those which depend on the calculation of spectral moments.

Various algorithms for the estimation of spectral wave parameters exist. Some of these are published in the IAHR/PIANC List of Sea State Parameters (1986). Other authors have also described methods for the determination of spectral sea state parameters (Houmb and Øyan, 1981). So far these techniques have been evaluated only by application to field recorded wave data, for which the expected values are not known accurately. It was therefore not possible to determine how well these methods succeeded in recovering the underlying process parameters from the analysis of relatively short samples of the sea state.

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Mansard and Funke (1988b) investigated a several of the known computational methods for spectral parameter estimation and evaluated their ability to recover the known process parameters by a numerical simulation of waves. This study used 200 numerically simulated wave records, each containing approximately 110 waves. These were derived from JONSWAP spectra with γ-values of 1 and 3.3, using the "random complex spectrum" method (Funke and Mansard, 1987). The JONSWAP parametric spectrum was chosen for this study because it is the most commonly used model, and it covers also the Pierson-Moskowitz as well as the Bretschneider (i.e. the ITTC or ISSC) spectra as special cases.

The "random complex spectrum" method of numerical wave synthesis is based on the Fourier transform, but is akin to a simulation of waves from long term "white noise" and is, therefore, a spectrally non-deterministic method. Miles and Funke (1988) demonstrated that the spectral domain characteristics of wave records produced by this method, mimic the natural variability of stationary, linear stochastic processes. This is to say, that individual realizations of synthesized waves exhibit the same variability of spectral estimates as would be encountered from filtered white noise. Whereas the properties of the underlying generating process are not known for natural sea states, they are known completely for numerically synthesized wave data. A numerical simulation of waves and subsequent spectrum analysis permits, therefore, the evaluation of conventional methods of parameter estimation by comparing the mean, the standard deviation, the maximum and the minimum values of the spectral parameters to the known simulation inputs.

Estimations of some spectral characteristics are linked to a basic assumption of the spectral shape characteristics. For deep water waves, one usually assumes a JONSWAP type of spectrum of which the Pierson-Moskowitz and the Bretschneider are special cases. For these parametric model spectra, one must know three fundamental parameters, i.e. the peak frequency, the significant wave height and a spectral width measure. (The peak width parameters σ_a and σ_b will be assumed in this study to have the same values as those suggested by JONSWAP, ie. 0.07 and 0.09 respectively).

Various spectral width measures have been proposed, such as the peakedness factor, Q_p, or the spectral width parameters ϵ_2 or ϵ_4 (IAHR/PIANC, 1986). Mansard and Funke (1988a) have shown that the peakedness factor is highly sensitive to the choice of cut-off frequencies. Because the spectral width parameters ϵ_n are derived from higher spectral moments, the same problem applies to these as well (see also Rye, 1977). In any case, for a JONSWAP spectrum, the most suitable spectral width measure would be the peak enhancement factor, γ. Knowledge of this parameter would permit matching of the parametric model spectrum to the measured sea state spectrum. LeBlond (1982) published a technique for this purpose, which will be described below.

The work presented both here and in Mansard and Funke (1988b), was undertaken to enhance computational methods for the discovery of the underlying spectral parameters of a sea state, and to provide the means of assigning confidence limits to such estimates. The authors consider it important that relatively short samples

of ocean waves, taken at a single point in an area of interest, should reveal the generally prevailing sea state over the storm area with as much confidence as possible. This would then allow the use of such wave data for the verification of hindcasting or forecasting techniques. It would also permit a more credible use of this data for design or simulation applications, where wave recording stations had been deployed at some distance from the site of interest.

A particular case in point is the situation which the authors encountered with the Ocean Ranger capsize investigations during 1982. At the time of the accident, only one wave recording buoy was operational and was located some 37 km from the accident site. However, prior to the accident, 3 buoys were operational, with the second 17 km from the Ocean Ranger and the third quite near the platform. Comparative analysis between the three buoys indicated considerable differences in peak frequency (Mogridge, 1985). A preliminary reanalysis of the data from two of these stations was carried out recently, using the enhanced method of analysis presented here. It was then found, that the differences were relatively small and that data from the two sites could be used interchangeably.

2.0 PRELIMINARY RESEARCH

By applying the technique of numerical wave simulation, Mansard and Funke (1988b) evaluated traditional algorithms for the estimation of three principal spectral parameters, i.e. the peak frequency, the peak enhancement factor and the significant wave height. The investigation was carried out for a range of JONSWAP γ-values varying from $1 \leq \gamma < 12$. From this it became apparent that the results of the analysis are:

- in most cases, highly dependent on the arbitrary higher and lower cut-off frequencies, which are used for the analysis;
- a function of the resolution of the spectral density analysis;
- a function of the γ-value of the JONSWAP spectrum;
- biased due to the skewed profile of wave spectra.

Because it was the ultimate objective of the study to devise a technique of optimally fitting a parametric model spectrum to the simulated sample spectra, and because it was known that any multi-parameter optimal fit relies heavily on the initial guesses of the parameters to be optimized, it was considered essential to improve on existing algorithms for the estimation of the three above mentioned spectral wave parameters before attempting the use of optimization procedures.

2.1 Estimation of the Peak Frequency

2.1.1 Initial guess of the peak frequency

Mansard and Funke (1988b) considered 5 algorithms for the estimation of the peak frequency. The simplest of these is the detection of that frequency that corresponds to the largest value of the sample spectrum, S_{max} (i.e. $f_p = f(S_{max})$). Because of the large variability of spectral estimators, this method also leads to a large variability

in the estimation of the peak frequency.

For this reason, some researchers prefer to use the "Delft" method (IAHR/PIANC List of Sea State Parameters, 1986). This is defined as the frequency computed as the centroid of the spectral band between the lower and the upper intercepts of the spectral density and the threshold which is 80% of $S_\eta(f_p)$, i.e.

$$f_{pD} = \int_{f_1}^{f_2} f \cdot S_\eta(f) \cdot df \Big/ \int_{f_1}^{f_2} S_\eta(f) \cdot df \tag{1}$$

where f_1 and f_2 are the upper and lower frequencies corresponding to the intercepts. This parameter is also referred to here as f_{pD8}.

A third option is similar to the second, except that the threshold is chosen to be 60% rather than 80% of the spectral peak value. It offers an advantage over the second method by including a larger portion of the spectrum, and thereby reducing variability. To distinguish it from the f_{pD8} parameter, it is referred to here as f_{pD6}.

The fourth and the fifth methods were used by Read (1986). The peak frequency estimator, referred as f_{p5} , is given by:

$$f_{p5} = \int_0^\infty f \cdot S^5(f) \cdot df \Big/ \int_0^\infty S^5(f) \cdot df \tag{2}$$

The fifth method yields f_{p8}, which is similar in definition to f_{p5}, except that the power coefficients are 8 instead of 5.

These five estimators of peak frequency (f_p , f_{pD8} , f_{pD6} , f_{p5} and f_{p8}) were evaluated by Mansard and Funke (1988b) through numerical simulation using a Pierson-Moskowitz model spectrum with a peak frequency of 0.55 Hz as a generating function. Using the random complex spectrum method, 200 wave trains, each 200 seconds long, were synthesized and then subjected to spectral analysis. The various estimates of peak frequency derived from these spectra were then subjected to statistical analysis. In order to evaluate the effect of spectral resolution on these statistics, three different resolutions, corresponding to 10, 20 and 30 degrees of freedom were also used. The main conclusions of this study are given below.

The traditional estimate of peak frequency (i.e. f_p) has by far the largest variability, which improves however, as the degrees of freedom of spectral analysis increase. The mean values of peak frequency from all five methods demonstrate a bias toward higher values. In terms of variability, the f_{p5} algorithm is superior to others and can be computed with relative simplicity, i.e. without bidirectional threshold detection, as required for f_{pDx} calculations. Because of this, f_{p5} was selected as the basis for the development of an enhanced wave parameter estimator, for which it will serve as the "initial guess".

2.1.2 The bias corrected estimate for the peak frequency

The JONSWAP parametric model spectrum is not symmetrical about its peak frequency f_0. The leading edge of the spectrum is steeper than its trailing edge. As

a result, any peak frequency definition based on centroid calculation is expected to provide a bias in the estimation of peak frequency. However, this bias does not constitute a serious concern since it can be corrected, and furthermore, it becomes insignificant as the peakedness of the spectrum increases.

The bias correction function for f_{p5} as a function of γ can be determined by applying the f_{p5}-calculation to smooth theoretical JONSWAP spectra, for which the peak frequency is predefined and therefore known. Such a calculation is free of variability. This was carried out and the result was subjected to a multivariate regression analysis, which in turn yielded to the following function:

$$C_f = 1.005 + 1 / [50.746 \cdot (\gamma - 0.2397)^2] \quad (3)$$

This function can only be evaluated once the γ-value is known. As this information is available only at a later stage of the analysis, C_f is initially estimated as $C_f \approx 1.02$ for the purpose of computing γ. This bias correction is applied as follows:

$$f'_{p5} = \frac{1}{C_f} \cdot \int_0^\infty f \cdot S^5(f) \cdot df \; / \; \int_0^\infty S^5(f) \cdot df \quad (4)$$

As will be described later, this bias corrected estimate will be used as the initial condition in an optimization procedure for the determination of an enhanced peak frequency estimator.

2.2 Estimation of the Peak Enhancement Factor.

2.2.1 The initial guess of the peak enhancement factor

It was shown in Mansard and Funke (1988b), that the peak enhancement factor γ may be estimated through the use of the parameter of a bivariate Rayleigh probability density, κ_f , given by Battjes and van Vledder, (1984). This is approximated by:

$$\kappa_f^2 \cdot \hat{m}_0^2 \approx [\int_{f_1}^{f_2} S(f) \cdot \cos(2\pi f\tau) \cdot df]^2 + [\int_{f_1}^{f_2} S(f) \cdot \sin(2\pi f\tau) \cdot df]^2 \quad (5)$$

where $S(f)$ is the spectral value, $\tau \approx \sqrt{\hat{m}_0/\hat{m}_2}$, and $\hat{m}_0$ and $\hat{m}_2$ are the estimators of the zeroth and the second spectral moment functions respectively. These were computed over the spectral range from $0.5f'_{p5}$ to $2.5f'_{p5}$. In this computation, f'_{p5} is evaluated using $C_f = 1.02$ in Equation (4).

By applying linear regression analysis to the κ_f-values, as derived from smooth JONSWAP parametric model spectra for various values of γ ranging from 1 to 12, a conversion formula was derived that established the relationship between κ_f and γ. This formula was given in Mansard and Funke (1988b) as:

$$\gamma_0 = 50.69 - 404.97\kappa_f + 1211.2\kappa_f^2 - 1599.6\kappa_f^3 + 817.26\kappa_f^4 \quad \text{for } \kappa_f \geq 0.4 \quad (6)$$
$$= 1 \quad \text{for } \kappa_f < 0.4$$

The results of applying this formula to simulated wave data were reported in Mansard and Funke (1988b).

2.2.2 Enhanced estimator of the peak enhancement factor

When applying Equation (6) to sample spectra derived from a 10-degrees of freedom spectral density analysis of 200 numerical simulation of wave trains, synthesized from JONSWAP class of spectra and using the "random complex spectrum" method of wave synthesis, it was found that an additional bias of the recovered γ-values for the range of $\gamma < 2.5$ occurred. Whereas the relation in Equation (6) was established from smoothed parametric model spectra, during the application of Equation (6) to numerically synthesized data, estimates of γ with values less than 1 were found to occur. By rounding these values up to 1, the mean value of the γ-estimator tends to be increased slightly. For this reason, an improvement to the relationship between κ_f and γ was derived here. This new peak enhancement factor is now given by:

$$\begin{aligned} \gamma' &= -0.835 + 1.797\gamma_0 - 0.2011\gamma_0^2 \quad \text{for } \gamma_0 < 2.5 \\ &= \gamma_0 - 0.10 \quad \text{for } \gamma_0 \geq 2.5 \end{aligned} \tag{7}$$

Applying this estimation formula to a numerical simulation leads to the statistics given in Table 1

REQUIRED	1.0	3.3	7.0
MEAN	1.13	3.33	7.09
STD. DEV.	0.26	1.06	2.14
MAX.	2.49	8.81	12.00
MIN.	1.00	1.00	1.56

TABLE 1. Variability of Enhanced γ-Values by Equation (7)

2.3 Estimation of the Significant Wave Height

2.3.1 Initial guess of the significant wave height

The significant wave height (H_{m0}) is computed from:

$$H_{m0} = 4\sqrt{m_0} \tag{8}$$

where m_0 is the zeroth moment, i.e. the area under the population variance spectral density function. Because of the presence of both high and low frequency noise, the zeroth moment is traditionally derived by spectral integration between some arbitrarily chosen upper and lower cut-off frequencies. As a result, the total variance is consistently underestimated by an amount corresponding to the spectral areas that have been eliminated by truncation. This energy loss can easily be

recovered as a cut-off frequency dependent bias correction. Because the authors have usually carried out wave data analysis for the band of $0.5f_p \leq f \leq 2.5f_p$, this was adopted for this investigation as well, using f'_{pS} for the peak frequency.

2.3.2 Enhanced estimate of the significant wave height

Mansard and Funke (1988b) determined the truncation induced bias by integrating smooth parametric JONSWAP class spectra over the range of $0.5f_p \leq f \leq 2.5f_p$. The resultant values of the zeroth moment were then subjected to regression analysis, which yielded a bias correction function. This was incorporated into an equation for the enhanced estimation of the significant wave height, namely:

$$m'_0 = C_m^2(\gamma') \cdot \int_{f_1}^{f_2} S_\eta(f) \cdot df \tag{9}$$

where:

$$C_m(\gamma') = 1.0015 + \frac{1}{[19.9178 \cdot (\gamma' + 2.6937)]} \tag{10}$$

The variability of the enhanced estimate of the significant wave height, H'_{m0}, obtained by Equations (8) and (9) is given in the following table for the numerical simulations described in Section 2.2.2.

	$\gamma = 1.0$	$\gamma = 3.3$	$\gamma = 7.0$
MEAN	.150	.150	.150
STD. DEV.	.007	.009	.011
MAX.	.167	.183	.193
MIN.	.134	.127	.122

TABLE 2. Variability of Enhanced Estimates of the Significant Wave Height, H'_{m0}

3.0 FITTING OF PARAMETRIC MODEL SPECTRA

The concept of optimally fitting a parametric model spectrum to spectral density estimates, as derived from the analysis of relatively short samples of a stochastic process, is founded on the hypothesis that knowledge of the spectral profile (i.e. a parametric model spectrum such as the JONSWAP class of spectrum), will improve the recovery of spectral parameters, because all spectral estimates are expected to contribute equally to the recovery process. Spectral fitting could be achieved as a multi-parameter or a single parameter optimization. In the former, several parameters are optimized concurrently, whereas only one is optimized at each pass in the latter case.

Optimization procedures must always start with some initial guess of the unknown parameters. The enhanced estimates of the spectral parameters described above,

such as the peak frequency f'_{p5}, the peak enhancement factor γ' and the spectrally derived significant wave height H'_{m0}, were considered to be the best possible initial guesses for this purpose. (Any spectral fit which uses just these initial guesses will be called as initial guess fit).

Optimization is achieved through the application of a criterion, which is generally computed from some error function between the measured spectral density and the theoretical spectral density evaluated with the optimally fitted parameters. The evaluation of the criterion at the end of the optimization process is also a measure of the goodness of fit obtained through the use of the particular criterion.

For this study, a 3-parameter optimization was attempted first and the fitting method used was the Simplex Algorithm (Caceci and Cacheris, 1984). This method has the advantage of permitting multiple parameter optimization with arbitrary optimization criteria. Four different criteria were investigated for this purpose.

After carrying out extensive experiments, it was found that in most cases, the optimal values achieved through a 3-parameter fit were different from those known to be correct. The only exception to this was the fitting of Pierson-Moskowitz spectra (i.e. for γ values of 1). At this stage of the development, it appears that multi-parameter optimization cannot be used. Since an extensive discussion of the optimizing criteria and the results of the multi-parameter fitting are beyond the scope of this paper, they will be reported separately in Mansard and Funke (1991).

As indicated earlier, optimization can also be employed by fixing one or two of the parameters, and then optimizing the remainder. This approach led to a very satisfactory result in the improvement of the estimation of the peak frequency.

The technique, which is now used, computes the intial guesses for all three spectral wave parameters, as described above. The best guess for H_{m0} and γ is then fixed, (i.e. H'_{m0} and γ') and optimal fitting is applied to find the best peak frequency f'_0. The optimizing criterion used for this purpose can be defined as follows:

$$\sum_{j=1}^{N} [\ (\sqrt{S_m(f_j)} - \sqrt{S_s(f_j)}\ ) \cdot \sqrt{W_j}\ ]^2 \ \textit{is to be a minimum} \tag{11}$$

where $S_m(f_j)$ is the fitted model spectrum, $S_s(f_j)$ is the sample spectrum and W_j is the weighting function. The weighting function can be defined as follows:

$$\begin{aligned} W_j &= \frac{S_m(f_0)}{S_m(f_j)} \quad \text{if } S_m(f_0) < 100 \cdot S_m(f_j) \\ &= 0 \qquad\qquad \text{otherwise} \end{aligned} \tag{12}$$

where f_0 is the peak frequency.

A brief discussion of this criterion can be found in Mansard and Funke (1988b).

The results of this one parameter fitting are shown in Figure 1 for γ-values from 1 to 7, together with the results obtained from traditional and bias-corrected estimations. From this it is evident, that the spectral fitting, in combination with a carefully selected criterion function, can significantly enhance the estimation of a process parameter.

Generally, the spectral resolutions used in this fitting procedure correspond to 10 degrees of freedom (DOF). By using a small resolution such as this, a large number of statistically independent spectral estimates are included in the optimization procedure.

Figure 1 shows that for broad spectra (i.e. $\gamma = 1$), the standard deviation of peak frequency obtained by the traditional estimate can be improved by a factor of about 7 through spectral fitting. Preliminary results indicate that, even if a coarser resolution such as DOF = 30 is used in the traditional estimate of f_p, the improvements that can be achieved by the spectral fitting would be in the order of 6. Effects of the resolution will be discussed separately in Mansard and Funke (1991).

Figure 2 illustrates an example of one sample spectrum, derived by synthesis of a random wave train, and subsequently submitted to spectral analysis with 10 degrees of freedom. The figure also shows the population spectral density (JONSWAP with $\gamma = 1$), the "initial guess" spectral fit, and the optimally fitted spectrum using criterion given in Equation (11). This example illustrates the improvements which may be achieved over the more traditional methods of analysis.

4.0 THE LEBLOND METHOD FOR THE ESTIMATION OF THE PEAK ENHANCEMENT FACTOR

LeBlond (1982) developed and used a technique for the estimation of the JONSWAP peak enhancement factor. This technique is based on an assumption that a certain relationship exists between the spectra's Phillip's constant α and the significant wave height H_{m0} (Mitsuyasu et al, 1980). The method is also limited to γ-values that are less than 4.

LeBlond analyzed his wave data with very low resolution, corresponding to 30 DOF, and determined the spectral peak frequency by the traditional method; that is, by taking the frequency of the largest spectral value. After this, LeBlond integrated the spectrum from 0.05 to 0.5 Hz (full scale units) to get the significant wave height, and used the formula:

$$\gamma = \left[\frac{S(f_p)}{\frac{5}{16 f_p} H_{m0}^2 e^{-5/4}} \right]^{3/2} \tag{13}$$

to compute the JONSWAP peak enhancement factor.

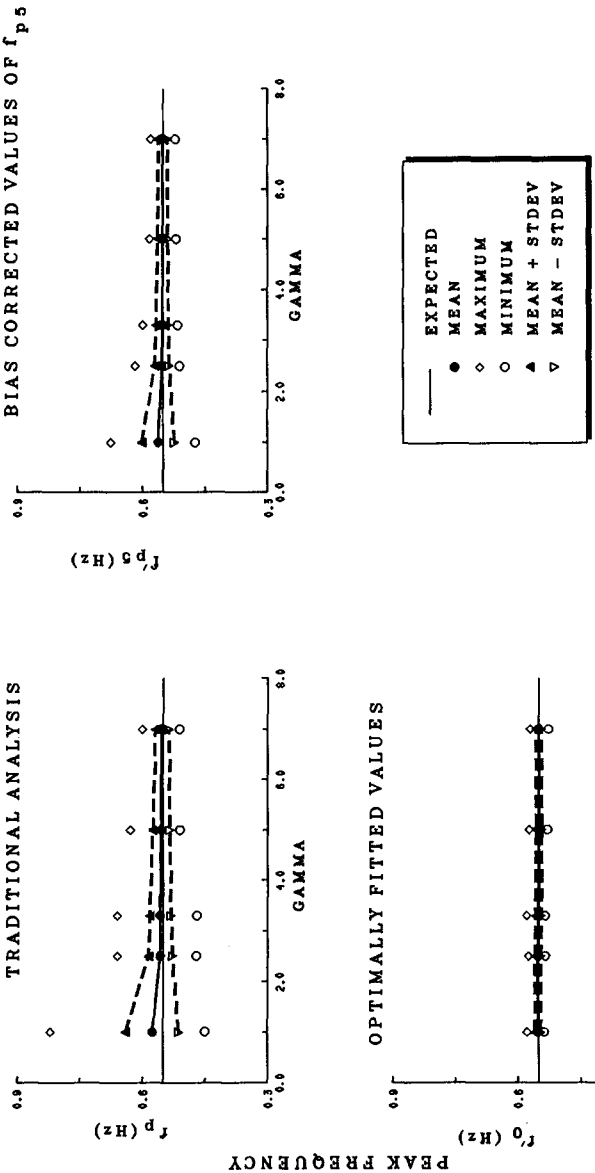

FIGURE 1

RANGE OF ESTIMATES FOR PEAK FREQUENCY UNDER DIFFERENT COMPUTATIONAL METHODS
(RANDOM COMPLEX SPECTRUM METHOD OF SYNTHESIS)
(T_R=200s f_0=0.55 Hz Hm_0=0.15m N_R=200 DOF =10)

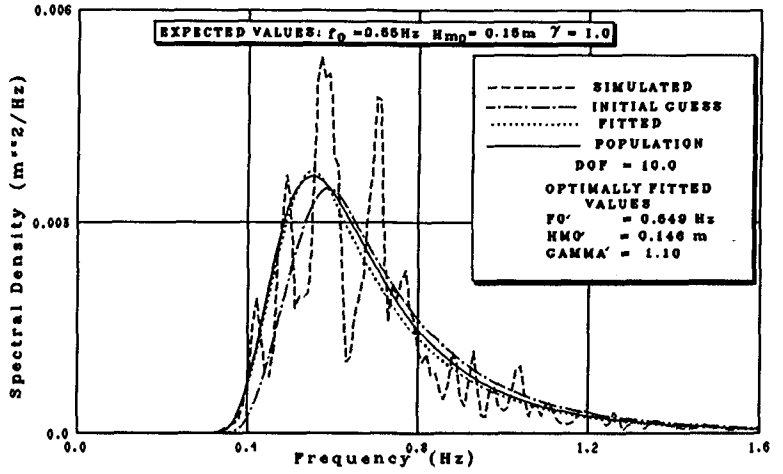

Fig.2 EXAMPLE OF AN OPTIMAL FIT

Table 3 summarizes the result of an analysis carried out by the LeBlond method, using 200 numerically simulated waves with approximately 110 waves derived from a JONSWAP spectrum with a peak enhancement factor γ=3.3 and a peak period of 0.55 Hz.

	PEAK FREQUENCY f_0' (HZ)	EST. SIG. WAVE HEIGHT H_{m0}' (m)	PEAK ENHANCEMENT FACTOR, γ'
MEAN	.557	.150	2.817
STD. DEV.	.018	.009	0.689
MAX.	.595	.183	4.849
MAX.	.490	.127	1.177

TABLE 3. Results of LeBlond Analysis

The important result from Table 3 is the fact that the standard deviation of the LeBlond estimator for γ-values is only 65% of that obtained with the authors' method. The mean value of γ and peak frequency values are, however, not as satisfactory, and the significant wave height estimators look good only because the numerical simulation used here was free of noise, and the integration limits could be extended from 0 to the Nyquist frequency.

However, the positive result from the LeBlond example suggests that further improvements in the estimation of the peak enhancement factor may be possible.

5.0 EXAMPLE ANALYSIS OF OCEAN WAVE DATA

Figure 3 compares the peak frequency, estimated by the spectral fitting, for two wave recording stations at the Ocean Ranger and the Sedco 706, nearly 17 km apart. Although there were 3 sites near the Ocean Ranger platform, only these two stations were chosen for this preliminary investigation since continuous wave records were available from them. The third station, Zapata Ugland, which was 30 km away from the Ocean Ranger, had records only at 3 hour intervals during that period.

For purpose of comparison, the traditional estimates of peak frequency and the bias-corrected values of f_{pS} are also shown in Figure 3. This figure clearly illustrates the improvements that can be achieved in the interpretation of prototype data through optimal fitting. It can be seen from this figure that, while the traditional and bias-corrected estimates show considerable differences in the peak frequencies at these stations, the optimally fitted values indicate the similarity of the two sites during the time of the storm. For these two stations, the variability of peak frequency between consecutive records of 20 minutes is reduced considerably when spectral fitting is applied.

6.0 CONCLUSIONS

An enhanced method for estimation of the spectral peak frequency for ocean wave data has been presented. The method was tested by numerical simulation of short wave records. For JONSWAP spectra with $\gamma = 1$, the variability of the peak frequency estimate can be improved by a factor of 7. Two methods for the estimation of JONSWAP peak enhancement factors are presented, but neither is considered satisfactory.

Further work will seek improvements in the estimation of the JONSWAP peak enhancement factor, establish suitable bias correction functions for the significant wave height as a function of spectral cut-off frequencies, and establish convenient confidence bands for the estimation of the three principal spectral wave parameters. Improvements in computational efficiency and the effects of spectral resolution will also be investigated.

The new method is believed to improve the validation of hindcasting and forecasting numerical models. At this time, the method applies only to the deep water situation, where spectral estimates of contributing frequencies are statistically independent. Future development may be able to address the intermediate and shallow water depth situation as well, where the spectral estimates are interdependent because of nonlinear interactions.

7.0 REFERENCES

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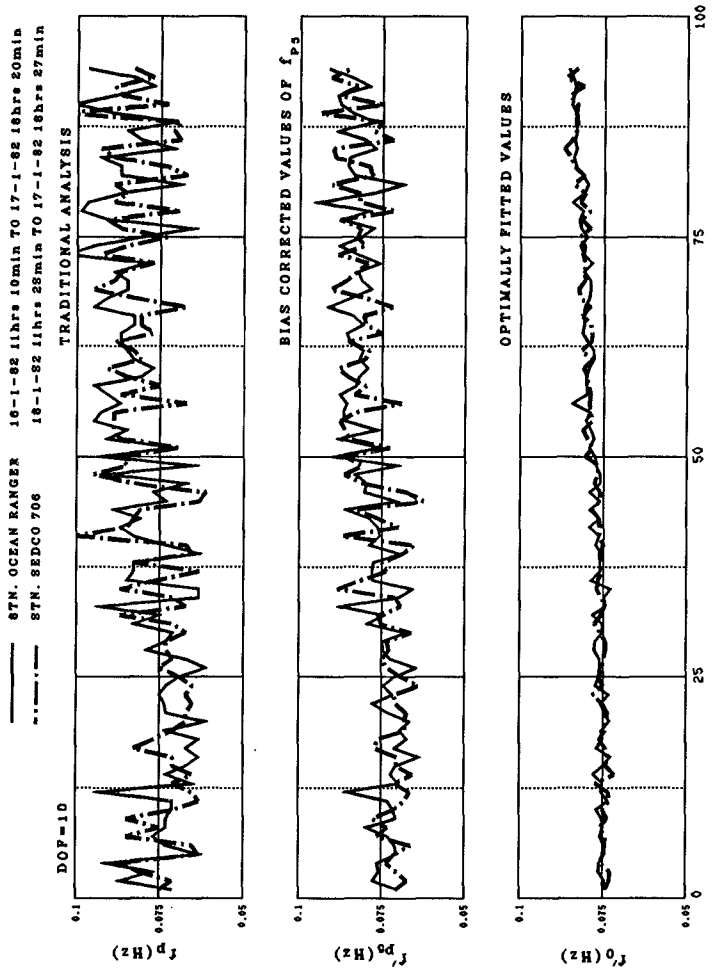

FIGURE 3
ESTIMATES OF PEAK FREQUENCY AT STATIONS OCEAN RANGER AND SEDCO

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