

CHAPTER 35

Velocities and Bed Friction in Combined Flows

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Abstract

Velocity measurements have been made with a laser doppler anemometer in a steady flow flume with an oscillating bed. The velocity profiles in these tests, for which the oscillation was at right angles to the steady current, were found to be qualitatively similar to those observed in previous measurements with currents collinear with the direction of oscillation. Measured time-mean velocity profiles show moderate agreement with the predictions of the models of Grant & Madsen (1979) and Christoffersen & Jonsson (1985) in the outer layer (the 'current boundary layer') but poor agreement close to the bed (the 'wave boundary layer').

1. Introduction

The problem of wave/current interactions is of importance in many areas of coastal engineering. The present paper is concerned with the velocity profiles, and hence the shear stresses, in the boundary layer near the bed. There have been a great many theoretical studies of this problem (see, for example, Sleath, 1990) and also several experimental studies for current collinear with the wave direction (Kemp & Simons, 1982, 1983, Van Doorn, 1981). But, as far as the writer is aware, there have been no systematic boundary layer measurements for waves at right angles to the current. The aim of the present study was to provide such measurements although, as will be seen from the next Section, the experimental situation investigated was not quite the same as that of waves at right angles to a current.

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2. Experimental equipment

One of the reasons for the absence of systematic laboratory measurements of boundary layer profiles for waves at right angles to a current is that a very large apparatus is required. It is difficult to obtain in a wave tank Reynolds numbers high enough to ensure fully-developed turbulence in the wave-induced boundary layers. In order to get round this problem it was decided to replace the wave motion by an oscillatory flow produced by oscillating the bed. It may be shown that, relative to axes fixed in the oscillating bed, the velocity profiles obtained in this way are identical with those which would be obtained in an oscillatory flow water tunnel with a steady current at right angles to the direction of oscillation. In other words, the oscillatory flow is that which would be produced in the vicinity of the bed by waves of infinite length. Second-order effects, such as the wave-induced mass transport current, are not reproduced.

Figure 1 shows a plan view of the experimental apparatus. It consists, essentially, of a steady flow re-circulating flume with a section of bed replaced by an oscillating plate. The direction of oscillation of the plate is at right angles to that of the steady current. Its length is 0.81 m in the direction of the steady flow and it extends across the full width of the flume, through slits in the walls, into outer chambers fitted with compensating cylinders. The oscillation of the plate is produced by a Scotch Yoke mechanism driven by a variable speed motor. The amplitude of oscillation can be varied from 0 to 0.19 m and the period from 0.5 to 6 sec. The steady flow flume is of length 5 m and width 1.2 m. Meshes are fitted at the upstream end to produce a uniform entry flow. In these tests the depth of water was usually held at about 0.26 m. Under these circumstances the pumps can produce mean velocities in the test section of up to 0.2 m/s.

Three different bed roughnesses were investigated: smooth stainless steel plate, sand of median diameter D equal to 1.64 mm, and gravel of median diameter 8.1 mm. The sand and gravel were glued to the oscillating section of bed. A thin layer of contact adhesive, diluted with petrol, was spread uniformly over the bed and then the sediment was sprinkled on top. The same roughness was spread for a distance of 1.7 m upstream of the oscillating section of bed and for a distance of 1.0 m downstream. The Nikuradse roughness length k_s was determined from the steady flow velocity profiles on the assumption that for rough beds the zero intercept of the

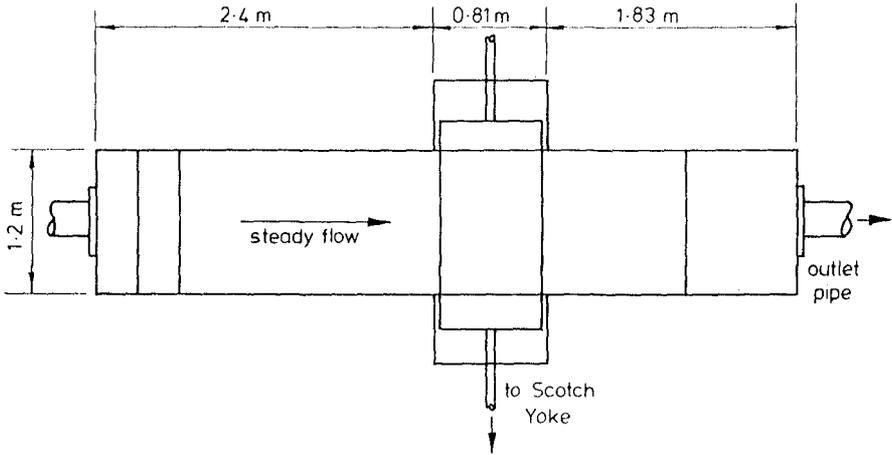


Figure 1. Plan view of the flume

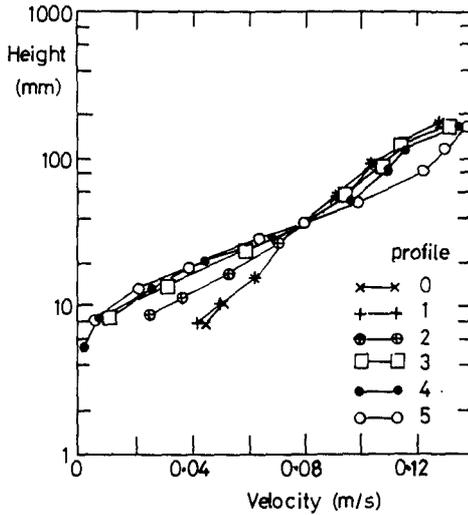


Figure 2. Steady current velocity profiles on the centre line of the flume. Distances downstream of the upstream edge of the oscillating bed are: (0) - 0.05 m, (1) - 0.05 m, (2) 0.09 m, (3) 0.22 m, (4) 0.47 m, (5) 0.72 m. For all profiles except (0) the cross-channel oscillation of the bed was of amplitude ± 0.14 m and period 2.9 sec.

logarithmic velocity profile is equal to $k_s/30.2$. The values of k_s obtained in this way were 20.3 mm for the 8.1 mm gravel and 1.49 mm for the 1.64 mm sand. The explanation for the surprisingly low values of k_s for the 1.64 mm sand is probably that the grains were very rounded and that the glue tended to fill the voids between grains.

The velocities were measured with a single component fibre optic laser doppler anemometer manufactured by DANTEC. The probe was operated in back-scatter mode with a 30 mW He-Ne laser. Further details of this instrument and of the flume are given by Lee Young (1989).

3. Boundary layer growth with downstream distance

Most of the velocity profiles in the main test series were measured, on the centre line of the flume, at a distance of 0.65 m from the upstream edge of the oscillating plate. Since a new boundary layer will grow from the edge of the oscillating plate it is reasonable to ask whether this distance is large enough for the velocity profile to have reached an equilibrium value. Figure 2 shows examples of the steady current velocity profiles at various distances from the upstream edge of the oscillating plate. Profile (0) was measured without oscillation of the plate whereas all of the other profiles were measured with a plate oscillation of amplitude 0.14 m and period 2.9 sec. The mean flow rate was the same for all profiles. Profiles (0) and (1) were each measured at a distance of 0.05 m upstream of the oscillating plate. The fact that these two profiles are almost identical suggests that the upstream influence of the plate oscillation is very slight. On the other hand, the effect of plate oscillation on the steady current profiles downstream of the upstream edge is significant. As might be expected, the change in velocity profile with downstream distance is most rapid just downstream of the leading edge of the oscillating plate. At 0.47 m from the upstream edge the velocity profile appears to have reached an equilibrium up to a height of about 40 mm from the bed.

4. Time-mean currents and shear stresses

The time-mean velocities and shear stresses discussed in this Section are in the direction of the mean current. With the present experimental apparatus time-mean quantities perpendicular to this direction are zero.

4.1 Smooth beds

The velocity profiles in Figure 2 were measured with the 8.1 mm gravel. Very similar results are obtained with the 1.64 mm sand but the smooth bed profiles are

different. Figure 3 shows two smooth bed time-mean velocity profiles measured on the centre line of the flume 0.65 m from the leading edge of the oscillating plate. The pump setting was the same for both profiles but in one case there was a plate oscillation of amplitude 0.125 m and period 2.41 sec and in the other there was no oscillation. It is clear that oscillation of the smooth plate has a negligible effect on the time-mean velocity profile. The reason is probably that the oscillatory flow is not at a sufficiently high Reynolds number to generate additional turbulence with this smooth plate. On the other hand, oscillation of the rough beds generates significant additional turbulence and it is this which changes the time-mean velocity profile, as shown in Figure 2.

4.2 Rough beds

One of the aims of the present study was to provide new data with which to test the predictions of theoretical models. Figure 4 shows a typical comparison of two measured time-mean velocity profiles with the curve obtained from the model of Grant & Madsen (1979). We see that there is quite good agreement between the measurements and the predictions of the model in the outer layer (the 'current boundary layer') but that the agreement is much less good close to the bed (the 'wave boundary layer'). Similar discrepancies are found with other models. This suggests that the assumptions of existing models for the wave boundary layer are not adequate.

Figure 5 shows how the measured values of the apparent roughness k_a compare with the values predicted by the models of Grant & Madsen (1979) and Christoffersen & Jonsson (1985). The predicted value has been obtained by assuming that the measured and predicted velocities are the same at a height of 25.4 mm above the bed. This allows k_a and the time-mean shear velocity to be calculated independently.

We see that the theoretical models predict the value of k_a/k_s quite well at low values of this parameter but that the agreement is less good at high values. It should be emphasized, however, that there is considerable uncertainty in the estimation of k_a even when, as in the present case, a least-squares technique is adopted. This is because the velocity profile is only logarithmic over a restricted range of heights so that experimental error in a velocity measurement can cause an appreciable change in the apparent value of k_a .

On the whole, Christoffersen & Jonsson's model appears to give closer agreement with experiment when $n = 2.0$ and $r = 0.925$ than when $n = 0.367$ and $r = 0.450$.

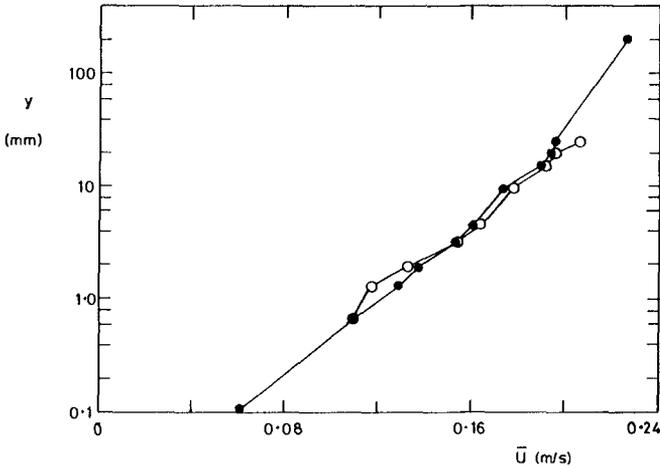


Figure 3. Time-mean velocity profiles over a smooth bed.
 O—O:— oscillation amplitude = + 0.125 m, period = 2.41 sec. ●—●:— no oscillation

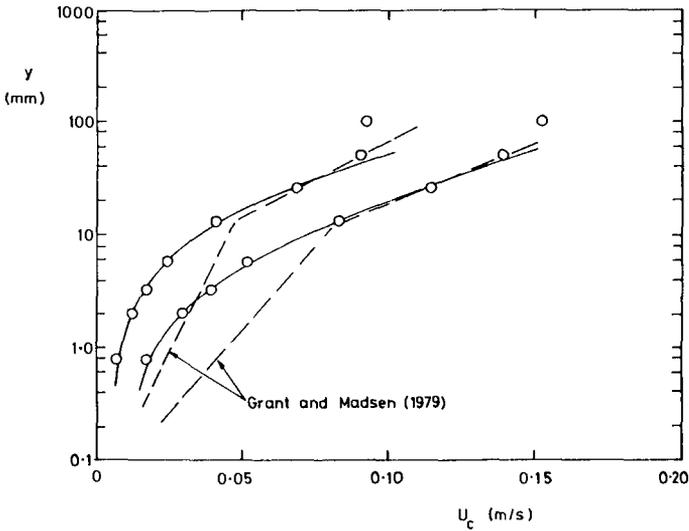


Figure 4. Comparison of two measured time-mean velocity profiles with predictions of the model of Grant & Madsen (1979). Oscillation amplitude = + 0.14 m, period = 1.76 sec. 1.64 mm sand

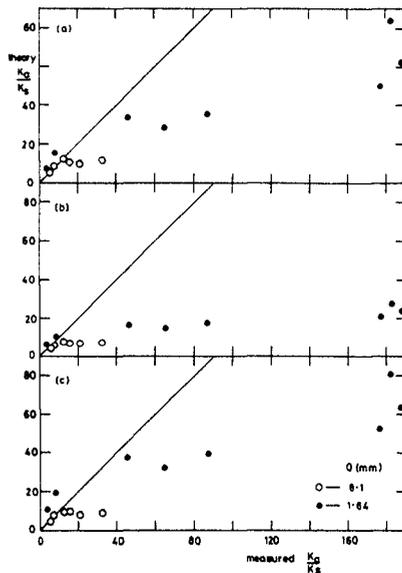


Figure 5. Comparison of predicted and measured values of combined flow roughness k_a divided by steady flow roughness k_s , (a) Grant & Madsen (1979) (b) Christoffersen & Jonsson (1985) with $n = 0.367$, $r = 0.450$ (c) Christoffersen & Jonsson (1985) with $n = 2.0$, $r = 0.925$

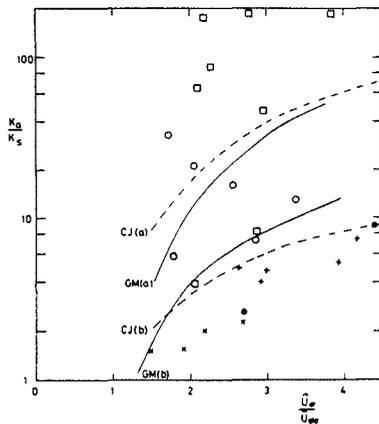


Figure 6. k_a/k_s versus \hat{u}_* / \bar{u}_{*c} . GM and CJ represent the predicted curves of Grant & Madsen (1979) and Christoffersen & Jonsson (1985) for (a) $a/k = 94.8$ and (b) $a/k = 6.9$. Symbols are: \square - present tests $D = 1.64$ mm, O - present tests $D = 8.1$ mm, \bullet - Van Doorn (1981), x - Kemp & Simons (1982), $+$ - Kemp & Simons (1983)

A different way of predicting the apparent roughness k_a was suggested by Coffey & Nielsen (1986). They observed that existing laboratory measurements could be represented by a single curve if k_a/k_s was plotted against \hat{u}_*/\bar{u}_{*c} , where \hat{u}_* is the amplitude of the fluctuation in shear velocity at the bed and \bar{u}_{*c} is the time-mean value. Figure 6 suggests that the same curve would not be valid for the present measurements. This may be because the present measurements are for currents at right angles to the oscillation whereas the previous measurements were for currents collinear with the oscillation or it may be that another parameter is important. The models of Grant & Madsen and Christoffersen & Jonsson both suggest that the ratio of orbital amplitude a to Nikuradse roughness k_s is important. Figure 6 shows the curves predicted by these models for the two bed roughnesses. In the present tests $a/k_s = 6.9$ for the 8.1 mm gravel and 94.8 for the 1.64 mm sand. We see that the predicted change in k_a/k_s with a/k_s is quite large.

Another quantity of interest which may be obtained from the slope of the time-mean velocity profiles in the outer region is the time-mean shear velocity \bar{u}_{*c} . This may be used to calculate the time-mean friction coefficient:

$$\bar{f}_c = \frac{2 \bar{u}_{*c}^2}{U_o^2}, \quad (1)$$

where U_o is the amplitude of the velocity of the oscillating plate. Figure 7 shows how the measured values of \bar{f}_c for the present tests compare with the values predicted by the models of Grant & Madsen (1979) and Christoffersen & Jonsson (1985). It is difficult to draw definite conclusions because the experimental scatter is considerable. On the whole, the agreement between theory and experiment is closer than for the apparent roughness k_a . However, there does seem to be a consistent tendency for the theory to underestimate the friction factor at large values of this parameter. In this case the choice of the values of n and r in Christoffersen & Jonsson's model appears to make little difference to the overall comparison.

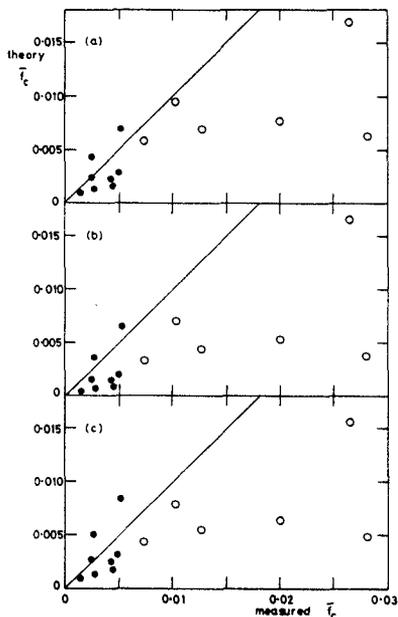


Figure 7. Comparison of predicted and measured values of time-mean friction factor \bar{f}_c . (a) Grant & Madsen (1979) (b) Christoffersen & Jonsson (1985) with $n = 0.367$, $r = 0.450$ (c) Christoffersen & Jonsson (1985) with $n = 2.0$, $r = 0.925$. Symbols as for Figure 5.

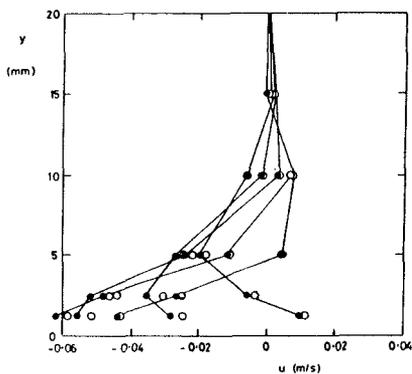


Figure 8. Oscillatory velocity profiles at phase intervals of 45° . Oscillation amplitude = $+ 0.14$ m, period = 2.75 sec in each case. $D = 1.64$ mm. \bullet - no steady current, \circ - steady current with $\bar{u}_{*c} = 0.0094$ m/s.

5. Oscillatory velocities and shear stresses

5.1 Velocities and shear stresses parallel to the direction of oscillation

In the present tests the influence of the steady current on the oscillatory velocity profile was much less marked than the influence of the oscillation on the steady current. This is illustrated in Figure 8 which shows typical velocity profiles at various phases of oscillation with, and without, a steady current.

The same situation is shown in Figure 9 which represents the ratio of the oscillatory friction factors with and without a steady current. In this figure

$$f_w = \frac{\hat{\tau}_{\omega t}}{\frac{1}{2}\rho U_o^2} \quad (2)$$

where $\hat{\tau}_{\omega t}$ is the amplitude of the shear stress at the bed in the absence of a steady current. The definition of f_{wC} is the same except that the shear stress amplitude is that in the presence of a steady current. The values of f_w and f_{wC} used in this Figure are those determined from the experimental measurements, with the aid of the momentum integral. Figure 9 shows that for this range of \bar{u}_{*c}/U_o the experimental results are in good agreement with the predictions of Grant & Madsen and Christoffersen & Jonsson.

5.2 Oscillatory velocities and shear stresses parallel to the mean current

The oscillation of the rough plates also generates an oscillatory velocity, and hence a shear stress, in the direction of the mean current. This is because the additional turbulence generated by the oscillatory flow fluctuates in intensity during the course of the cycle (Sleath, 1987). If the angular frequency of the oscillation is ω the frequency of the fluctuation in turbulence intensity is 2ω . Consequently the fundamental frequency of the oscillatory velocity and shear stress in the direction of the mean current is also 2ω . Interaction between the various oscillatory components also produces higher-order harmonics both parallel and perpendicular to the mean current direction but the magnitude of these harmonics is much less than that of the fundamental components.

The fluctuating component of velocity in the mean current direction is relatively weak but the shear stress it generates at the bed is not negligible, as indicated in Figure 10. We see that the amplitude $\hat{\tau}_{2\omega t}$ of the

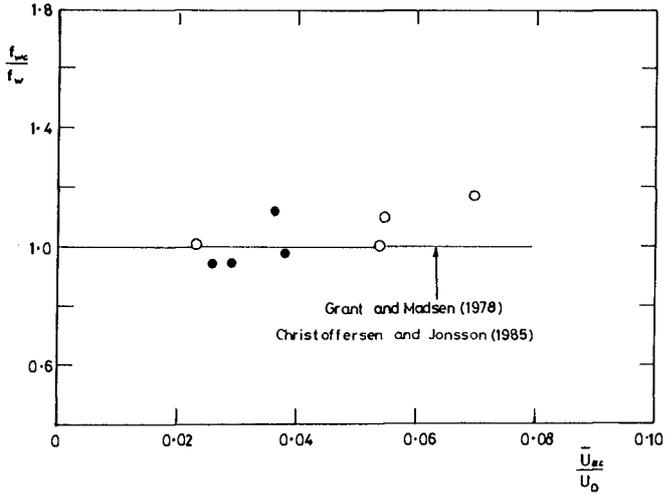


Figure 9. Ratio of oscillatory friction factors measured with and without a superimposed steady current.
 O - 8.1 mm gravel, ● - 1.64 mm sand

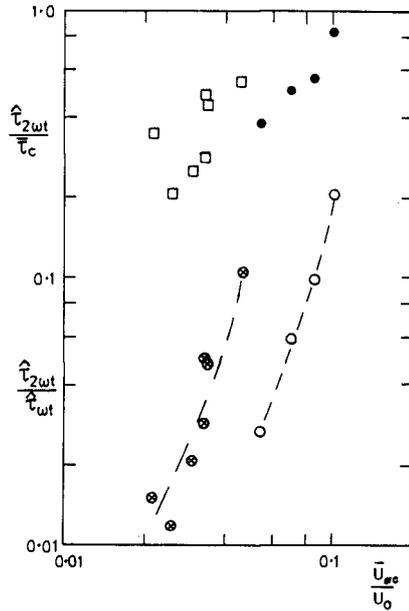


Figure 10. Amplitude of oscillatory shear stress $\hat{\tau}_{2wt}$ in the mean current direction.
 O, ● - 8.1 mm gravel; ⊗, □ - 1.64 mm sand

fluctuating shear stress ranges from about 20 % to 80 % of the time-mean shear stress $\bar{\tau}_c$. However, this fluctuating stress in the mean current direction is still small compared with the amplitude $\hat{\tau}_{\omega t}$ of the fluctuating shear stress in the direction of oscillation. As shown in Figure 10, the ratio of these two shear stresses was usually significantly less than 10 % in these tests.

In Figure 10, the shear stress $\hat{\tau}_{\omega t}$ in the direction of oscillation was calculated from Jonsson's (1963) expression for friction factor whereas the component $\hat{\tau}_{2\omega t}$ in the mean current direction was determined from the momentum integral

$$\tau = \int_0^{\infty} \frac{\partial}{\partial t} (u_{\infty} - u) dy$$

where u is the fluid velocity at height y and u_{∞} is the fluctuating velocity outside the wave boundary layer. In these tests u_{∞} was either zero or very nearly so. It is well known that calculations of shear stress based on the momentum integral tend to be inaccurate and this is particularly true in the present case because the boundary layer for the 2ω component of velocity is very thin. The values of $\hat{\tau}_{2\omega t}$ in Figure 10 should consequently be treated with caution.

6. Conclusions

(1) Velocity measurements show that equilibrium profiles are established relatively quickly above the oscillating section of bed. For example, 0.47 m downstream of the leading edge of the oscillating bed the velocity profile was in equilibrium to a height of about 0.04 m above the bed.

(2) The measurements with smooth beds showed no significant effect of the oscillation on the velocity profile of the steady current. This lack of interaction is probably because the oscillatory Reynolds numbers were not high enough for additional turbulence generation with smooth beds.

(3) In contrast, the measurements with the rough beds showed significant modification of the time-mean velocity profile when an oscillation was superimposed on the flow. The modification was qualitatively similar to that observed, by previous investigators, for steady currents parallel to the direction of oscillation.

(4) The effect of the steady current on the oscillatory flow was small, even for the rough beds, for the present test conditions.

(5) Measured time-mean velocity profiles are in quite

good agreement with the profile predicted by Grant & Madsen's (1979) model in the outer layer (the 'current boundary layer') but in poor agreement close to the bed (the 'wave boundary layer'). Measured outer layer quantities such as apparent roughness and mean shear velocity also show moderate agreement with Grant & Madsen's model and with that of Christoffersen & Jonsson (1985) with $n = 2.0$ and $r = 0.925$.

(6) With the rough beds it was possible to observe a fluctuating shear stress at right angles to the direction of oscillation. The amplitude of this shear stress was small compared with the shear stress in the direction of oscillation but quite significant compared with the time-mean shear stress.

7. Acknowledgements

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amplitude (m)	period (s)	temperature (°C)	\bar{u}_{*c} (m/s)	k_a/k_s	$\frac{\hat{\tau}_{2\omega t}}{\bar{\tau}_c}$	$\frac{U_o}{(\omega v)^{1/2}}$
$D = 1.64 \text{ mm}, k_s = 1.49 \text{ mm}$						
0.141	2.22	18.3	0.019	177	0.22	231
0.141	2.19	21.2	0.020	65	0.45	242
0.141	2.18	21.6	0.011	183	0.35	244
0.141	4.58	19.0	0.007	8.1	-	162
0.141	4.49	21.1	0.010	3.9	0.54	169
0.141	4.44	21.8	0.007	46	0.25	171
0.141	1.76	19.3	0.023	87	0.49	263
0.141	1.75	20.0	0.019	188	0.28	266
$D = 8.1 \text{ mm}, k_s = 20.25 \text{ mm}$						
0.140	2.43	14.2	0.026	7.5	0.50	208
0.140	2.43	14.7	0.022	13	0.38	210
0.140	4.39	17.4	0.023	5.8	0.83	161
0.140	4.39	17.9	0.020	21	0.55	162
0.140	2.92	18.9	0.024	16	-	202
0.140	2.82	19.8	0.037	33	-	208

Table I. Test conditions for the measurements in line with the mean current direction