CHAPTER 30

IMPROVED CALCULATION OF THE SHOALING WAVE FIELD

Masataro Hattori[1] and Tetsuyuki Katsuragawa[2]

Abstract

Nonlinear wave theories was improved to describe the asymmetry in time histories of the wave elevation and associated particle velocities under relatively steep nonbreaking waves. The improvement was based on the idea that the asymmetry causes both the amplitude changes of the primary and harmonic Fourier components, and the relative phase shifts between the primary and harmonic components.

Relationships between the asymmetry and local wave parameters were prerequisite to the improved calculation , and were derived by using experimental data. The improved calculation proposed was made for a specified local wave condition, without aids of direct numerical simulation techniques. Good agreements of the improved calculation with measurements confirmed applicability of the calculation to the practical use.

1. INTRODUCTION

Coastal engineers have been fully aware of the fact that nonlinearity as well as irregularity in wave motions plays an essential part in various coastal processes, such as the cross-shore sediment transport and the wave-structure interaction. In examining such problems, especially under relatively steep waves, time-dependent characteristics of the kinematic field of wave motions at a certain location are usually predicted from flat bottom solutions of nonlinear theories of permanent waves.

When a surface wave advances over a gently rising slope, it is subjected various nonlinear effects due to convective inertia and bottom friction. The inertia effects bring about asymmetric wave forms both about the horizontal and vertical planes, as shown in Fig. 1. Nonlinear theories of constant wave forms describe reasonably well the vertical asymmetry in wave form about mean water level, but they do not allow the horizontal asymmetry about the wave crest, the front slopes of the waves become steeper than the back slopes. In this sense, no appropriate nonlinear wave theory exists for practical use.

The present paper discusses the asymmetry in shoaling wave profiles, with taking into account the bottom slope effect as well as the incident wave property. Based on wave elevation records , empirical

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1.Professor, Department of Civil Engineering, Chuo University, Kasuga 1-13-27, Bunkyo-ku, Tokyo, 112, Japan.
2. Research Engineer, Toyo Construction Co., Ltd., Kanda Nishikicho 3-7-1, Chiyoda-ku, Tokyo, 101, Japan.

equations for the asymmetry in the wave form are derived. Improvement of nonlinear wave theories is made by using the empirical equations. This study is concerned with modified versions of the Stokes 5th order wave theory, ST 5,(Isobe, Nishimura, and Horikawa, 1978; Fenton, 1985) and Dean's 10th order stream function representation of theoretical waves, SFMB 10 (1965), because they are compuationally simpler.

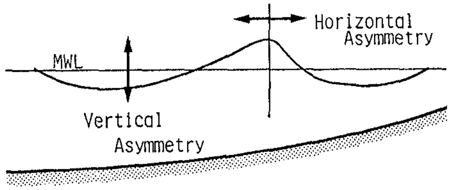


Fig. 1 Asymmetry in the wave profile.

2. OBJECTIVE

Objective of this study is to develop an improved calculation of the wave elevation and particle velocities, with respect to time, under relatively steep nonbreaking waves. From a practical viewpoint, the improved calculation is made for a specified local wave condition: wave height H , wave period T, water depth D, and bottom slope m. In so doing, four sub-objectives are defined:(1) To examine the transformation of shoaling waves by experiments,(2) to obtain empirical relationships of the wave profile asymmetry in terms of the local wave properties and the bottom slope, (3) to modify the theoretical solutions so as to describe the horizontal asymmetry ,and (4) to evaluate applicability of the improved calculation by comparisons with measurements.

3. EXPERIMENTAL SETUP AND MEASUREMENTS

Experiments were made in a glass-walled wave flume, 0.30m wide, 0.55m high, and 20m long. Regular waves were generated by a reflection-absorbing type wave maker. Model beaches with slopes of 1:10 and 1:20 were used for all the surface and velocity measurements. The water depth in the flat portion of the flume was between 0.33m and 0.43m.

Simultaneous measurements of the wave elevation and particle velocities were made at various locations over sloping bottom. Wave elevations were measured with a resistance-type wave gage having excellent linearity and high frequency response. Horizontal and vertical particle velocities at the wave measurement section were measured with a two-component LDA velocimeter. These outputs were recorded on a digital recorder with a sampling frequency of 200Hz for extensive processing by computer.

Individual waves in a time series of wave elevation at a certain location were defined by means of the zero-up-crossing method. Based on the individual wave data, the time series of wave elevation and particle velocities were averaged 50 waves with respect to the phase. The harmonic amplitudes were computed from the time series by fast Fourier transform (FFT).

4. EVALUATION OF THE IMPROVED CALCULATION

Applicability of the improved calculation was examined by the overall root-mean-square E of difference between the measurement and calculation of wave elevation and particle velocities for one wave cycle, given by Eq. (1).

$$E=\left[\sum_{i=1}^{N}(X_i-Y_i)^2/\sum_{i=1}^{N}X_i^2\right]^{1/2} \qquad (1)$$

, in which X_i and Y_i are the measured and calculated values at evenly spaced phases($\Delta\theta$=N/T) for one wave period of the time history, determined by ensemble averaging technique. Taking account of the experimental validity range of water wave theories (Hattori, 1986), the applicability limit of the improved calculation was set as $E < 1.0$.

5 EXPRESSION OF THE SKEWED WAVE FORM

5.1 Brief review of previous studies.

A number of experimental and theoretical studies have been made to examine physical processes in the skewed kinematic field under steep waves propagating on the sloping bottom. However, physical processes of the horizontal asymmetry, wave front faces are steeper than back faces, are still poorly understood.

As to the asymmetric characteristics, Adeyemo(1968 and 70) conducted extensive experiments focused on the asymmetry in wave forms and particle velocity changes with respect to time. He tried to parametrize the asymmetry with respect to various incident wave characteristics. However, he did not success to derive any synthetic relationship between the asymmetry and wave parameters.

Hedges and Kirkoz (1981), based on wave elevation data, proposed the transformation zone, which is the region between the point where waves start to loss horizontal symmetry and the breaking point. They found that the rate of horizontal asymmetry transformation increases as the bottom slopes get steeper (Galvin, 1967). In addition, the transformation zone tends to broaden with decreasing in the incident wave steepness.

Flick, Guza, and Inman(1981) presented a very interesting approach for examining the horizontal asymmetry in shoaling wave profiles. Fourier analysis of measured wave elevation records revealed the two following facts, on which the improvement is based:

(1) Harmonic amplitude data provide the principal control over the vertical asymmetry, and are predicted well by the nonlinear theories of permanent waves.

(2) The skewed wave profile causes relative phase shifts ζ_n between the primary and nth harmonic components, given by Eq. (2).

$$\zeta_n=\arctan(S_n/C_n)-n\cdot\arctan(S_1/C_1), \qquad (2)$$

where S_n and C_n are the sine and cosine Fourier coefficients. Thus, the measured wave elevation $\eta(t)$ as a function of time can be written as

$$\eta(t)=a_1\cos\omega t+\sum_{n=2}^{M}a_n\cos(n\omega t+\zeta_n), \qquad (3)$$

in which a_1 and a_n:the amplitudes at primary and nth harmonic frequencies, and ω:the frequency of the primary wave. According to Flick et al.(1981), the harmonic phase shifts ζ_n start to increase from zero as the waves advance over a rising slope, and increase toward the asymptotic values $\zeta_n=(\pi/2)(n-1)$.

5.2 Formulation of the harmonic phase shift ζ_n

The previous studies disclosed the horizontally asymmetric or skewed wave profile is closely related to the local wave parameters and the bottom slope. After careful examination of the experiments by the authors (m = 1/10 & 1/20) and Flick et al.(m = 1/25), we found that Goda's nonlinearity parameter Π(1983), given by Eq. (4), is of the most acceptable, for representing synthetically effects of the incident wave properties on the harmonic phase change inside the transformation zone (Hedges and Kirkoz, 1981).

$$\Pi = (H/L)\cot^3 kD, \tag{4}$$

where H is the wave height, L is the wavelength, computed from Airy wave theory, $k(=2\pi/L)$ is the wave number, and D is the water depth. As a consequence, the harmonic phase shifts ζ_n are given by Eq. (5), in terms of the Π parameter and the bottom slope m.

$$\left.\begin{array}{l} \text{For } \Pi < 10^{-(B_n/A_n)} \\ \quad \zeta_n/(n-1) = 0, \\ \text{and for } \Pi > 10^{-(B_n/A_n)}, \\ \quad \zeta_n/(n-1) = A_n \log \Pi + B_n, \end{array}\right\} \tag{5}$$

in which $A_n = (n-1)/30+0.60$ and $B_n = 1.33 \cdot m^{1/4}$.

Figures 2 and 3 are comparisons of Eq. (5) with the harmonic phases ζ_n computed from the wave elevation records for the bottom slopes of 1/10, and 1/25. Figure 3 is depicted by utilizing the experimental data of Flick et al.(1981; Fig. 5). The fairly good agreements persuade that the asymmetric wave profile is expressible by substituting Eq. (5) into the analytical solutions of nonlinear wave theories.

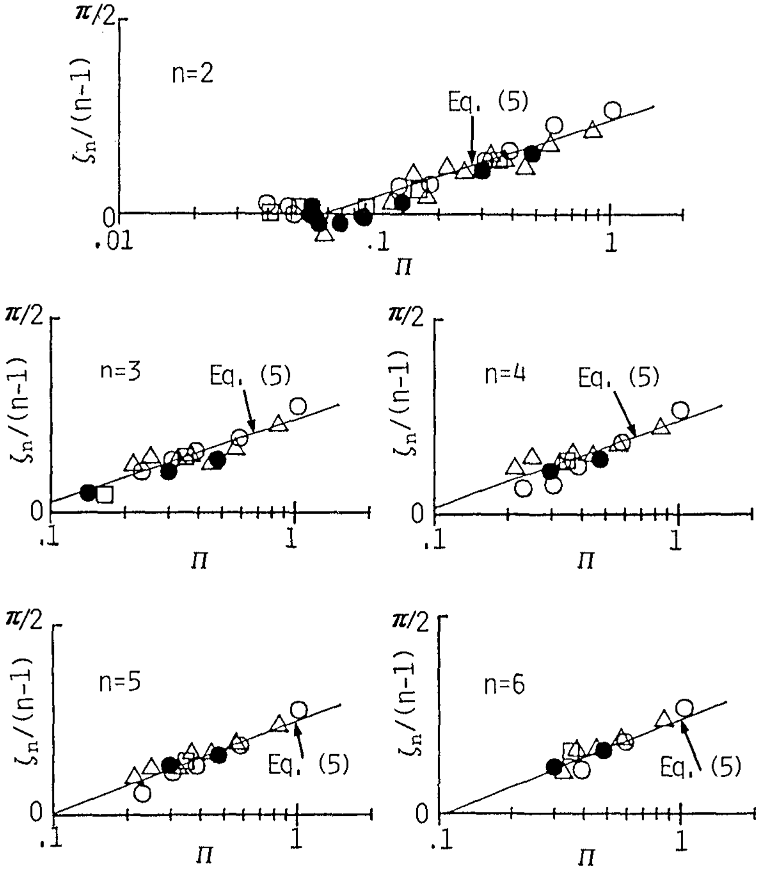


Fig. 2 Relative phase shifts $\zeta_n/(n-1)$ and Goda's Π.
[n=2-6, m=1/10]

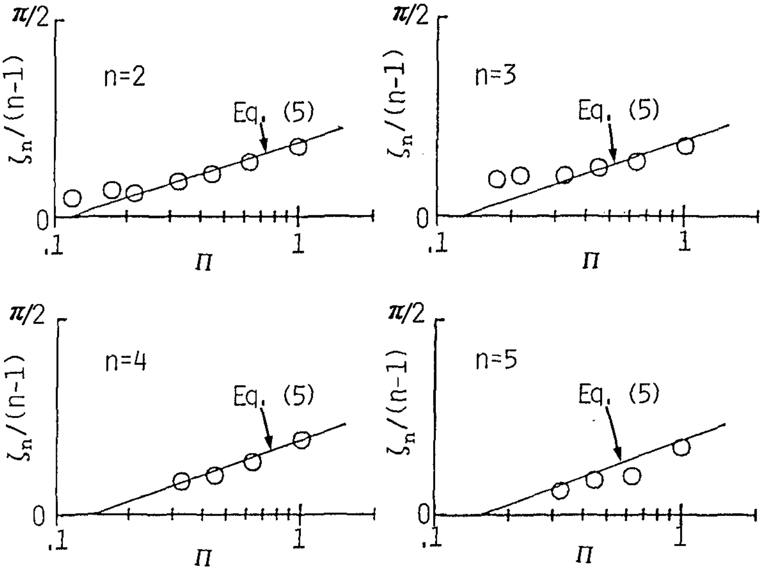


Fig. 3 Relative phase shifts $\zeta_n/(n-1)$ and Goda's Π.
[n=2-5, m = 1/25 ; Flicks et al., 1981]

6. IMPROVED CALCULATIONS

The improved calculation is based on the idea that the horizontal asymmetry in wave elevations and particle velocities, as a function of time, is expressible by the nonlinear wave theory, when the relative harmonic phase shifts are taken into account (Flick et al., 1981). Therefore, this study is concerned with the improvement in the Stokes 5th order wave theory, ST 5, and Dean's 10th order stream function representation of theoretical waves, SFMB 10 (1965). The cnoidal wave theory is excluded from the present study, because the cnoidal solutions are analyzed into the Fourier components in order to generate the horizontal asymmetry by means of the harmonic phase shift. This is very tremendous and barely suitable for practical use.

6.1 Improved Stokes 5th order theory, ST' 5

Figure 4 shows the details of improved calculation of the Stokes 5th order theory. After inputting a specified local wave condition and bottom slope, ST 5 starts to calculate the wave elevation and to determine the primary and harmonic amplitudes. In parallel with this step, the harmonic phase shifts ζ_n are computed from Eq. (5).

Substitution of ζ_n into the phase functions of ST 5 solutions, yields the first approximation of skewed free surface elevation. However, it usually brings about a slight deviation of wave height from the inputted one. Thus, the wave height is iteratively corrected by changing only the primary wave amplitude, because the harmonic amplitudes and their contribution to the wave height deviation are relatively smaller than these of the primary. After determining the skewed wave elevation, the horizontal and vertical velocities are calculated.

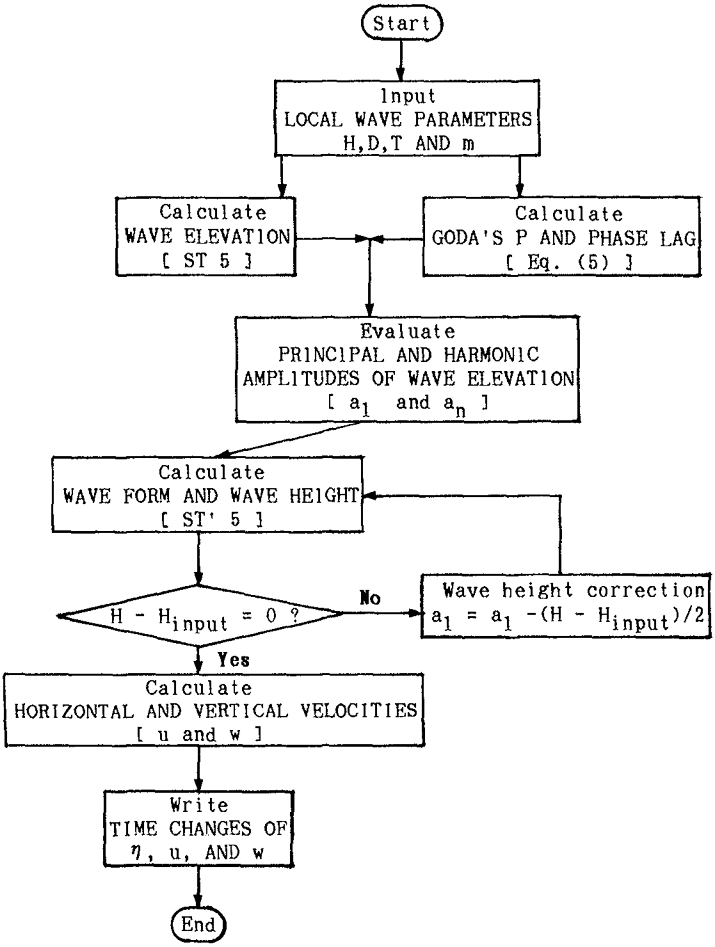
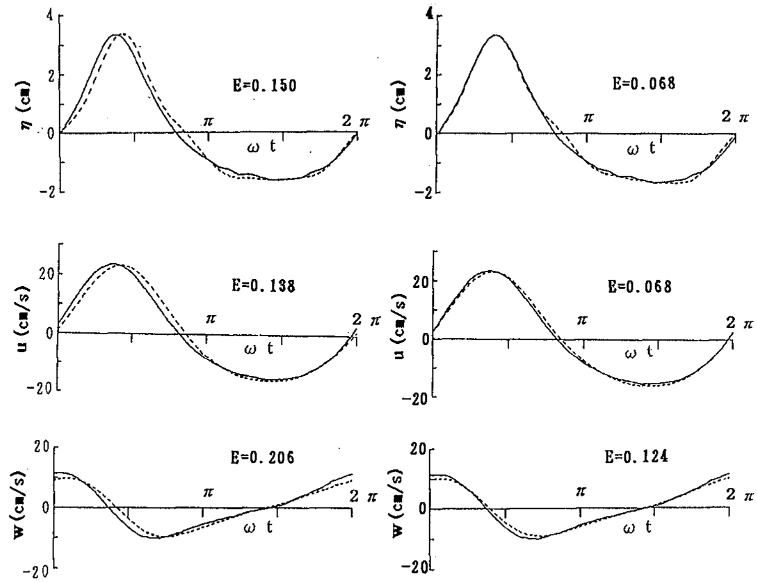


Fig. 4 Flow chart showing the details of ST' 5

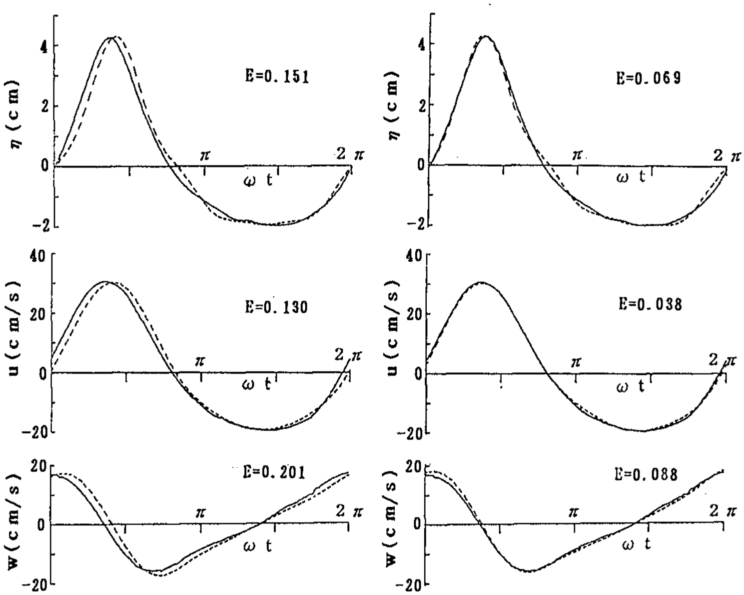
Figures5 and 6 are examples of the comparisons of ST 5 and ST' 5 with measurements of the wave elevation, η , and particle velocities, u and w. The solid and broken lines represent the measured and calculated. Since the local wave conditions are very close to the convergency limit of the Stokes wave theory (Dean and Darlimple, 1984), instability is clearly observed in the calculated wave elevation after the crest phase. Nevertheless, the ST' 5 yields better agreements with the measurements than the ST 5.

Equation (5) for the harmonic phase shift was derived only from the wave elevation data. It is, therefore, necessary to survey its applicability to the particle velocities. Figure 7 illustrates comparisons between the calculated and measured velocities at the middle and near bottom levels of the water column, for the same case shown in Fig. 6. As in Figs. 6 and 7, ST' 5 predicts fairly well the measured velocity changes in the wave crest, where their profiles is steeper and significantly asymmetric. The agreement, in visual, appears to be reasonably



ST 5 ST' 5

Fig. 5 Comparison of ST 5 and ST' 5 with the measurements.
(H=5.03 cm, T=0.84 s, D=10.0 cm, z/D=-0.40, Ur=28.4, m=1/20)
[—— : measured, ---- : calculated]



ST 5 ST' 5

Fig. 6 Comparison of ST 5 and ST' 5 with the measurements.
(H=6.15 cm, T=0.80 s, D=11.0 cm, z/D=-0.18, Ur=25.1, m=1/20)

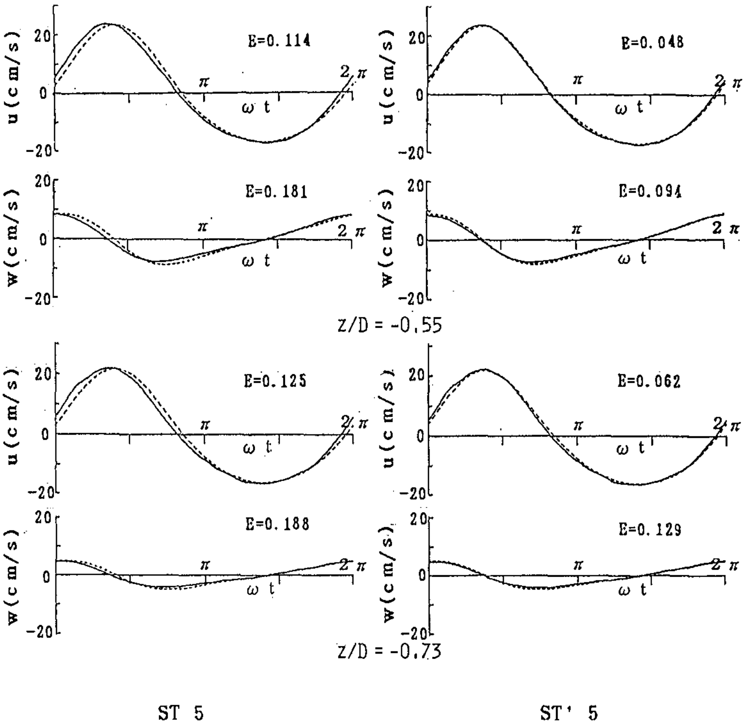


Fig. 7 Comparisons between the measured and calculated particle velocities by ST 5 and ST' 5. (the same case for Fig. 6)

good. However, the overall-rms value E increases gradually toward the bottom. This implies that (1) since the velocity profile get the symmetry with the depth, ζ_n for particle velocities changes at different rates , and (2) decreasing the vertical velocity magnitude toward the bottom gives rise to the increase in E value, even for very small difference between the measured and calculated velocities.

6.2 Improved Stream Function: SFMB' 10 and SFMC' 10

As the wave shoals and wave form becomes more asymmetric, the SFMB tends to overestimate the wave elevation and horizontal particle velocity around wave crest phase (Dean,1965). To reduce such stream function errors , Saito and Isobe (1987) proposed a modified version of SFMB , using the wave crest height η_c above the mean water level, instead of the wave height (SFMC). Better representation of the wave surface profile is prerequisite to the improvement of the SFMB. Thus, an improved version of SFMC is proposed as an alternative to the SFMB.

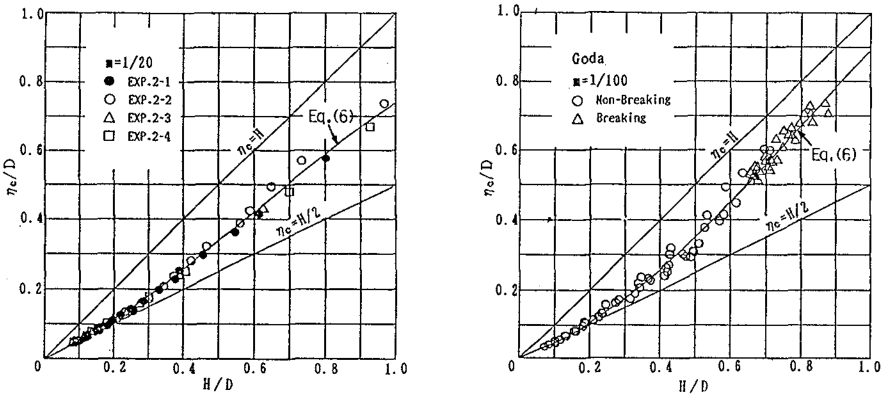
In calculations by the SFMC, the wave crest height needs to evaluate from the given wave condition. Steepening of the wave crest as well as skewing the wave form relates both to the wave property and the bottom slope. Empirical equation for the relative wave crest height, η_c/D, with respect to the relative wave height H/D and the bottom slope m, is determined by using the measuring data of the present authors(m=1/10 & 1/20), Eagleson(m=1/15, 1956), Hansen and Svendsen(m=1/34, 1979), and Goda(m=1/100, 1964).

$$\eta_c/D \fallingdotseq m^{f_1(H/D)} \cdot \mathrm{Exp}[f_2(H/D)], \tag{6}$$

in which

$$\left.\begin{aligned} &f_1(H/D)=(H/D)\{2.44(H/D)^2-9.24(H/D)+3.18\}\times10^{-2}, \\ \text{and}\quad &\\ &f_2(H/D)=(H/D)\{-1.93(H/D)^2+1.05(H/D)+5.58\}\times10^{-1}. \end{aligned}\right\} \tag{7}$$

Figure 8 is comparisons of Eq. (6) with the measurements for the bottom slopes of 1/20 and 1/100, and shows fairly good agreements. The solid lines labeled η_c=H and η_c=H/2 correspond to the solitary and Airy waves.



(m=1/20) (m=1/100)

Fig. 8 Relative crest height η_c/D and H/D.

The 10th order stream function method, SFMB 10, can be improved by the almost similar procedure adopted to the ST 5 theory . Figure 9 shows the details of the improved stream function methods, SFMB'10 and SFMC' 10. After determining the asymmetric wave profile with aids of the relative phase shifts ζ_n, the velocities associated are calculated with the aid of SFMA 10 for irregular waves (Dean, 1965).

Comparisons between the measured and calculated wave elevations near wave breaking, as shown in Figs. 10 and 11, indicate superiority of the SFMC' 10 over SFMB' 10. SFMC' 10, and represent reasonably well the asymmetry as well as the maximum elevation of the wave profile in wave crest phase.

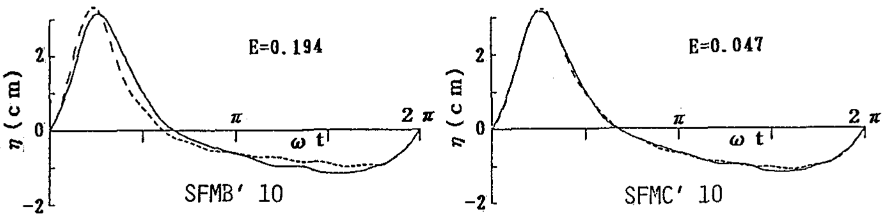


Fig. 10 Comparison of SFMB' 10 and SFMC' 10
with the measured wave elevation.
(H=4.3 cm, T=0.99 s, D=7.0 cm, Ur=84.3, m=1/20)

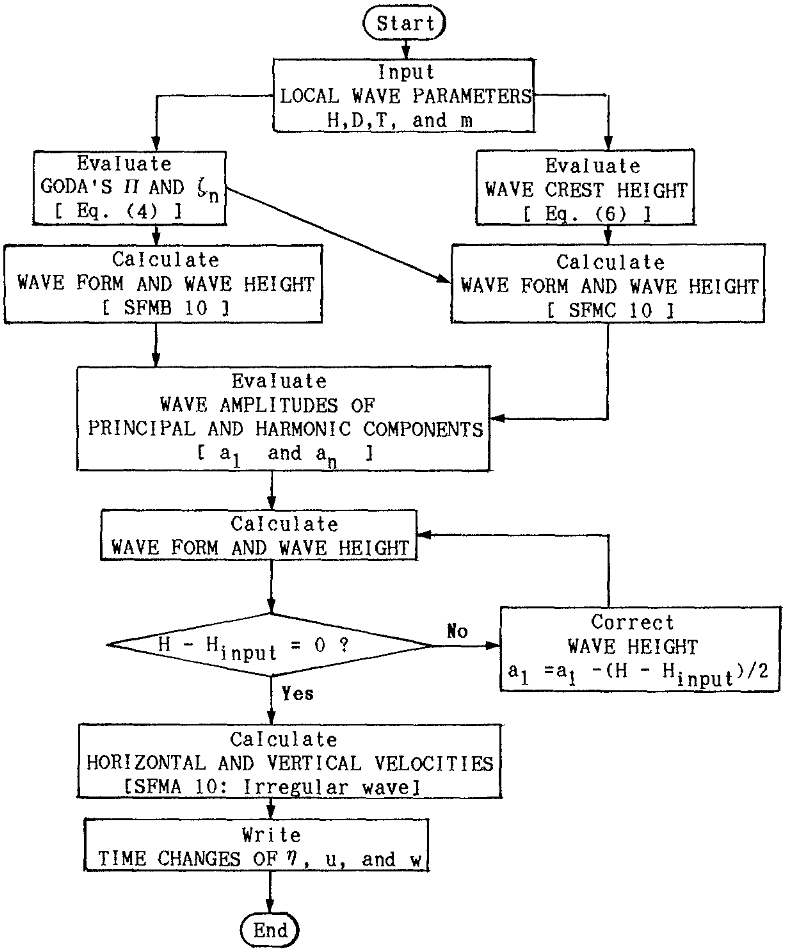


Fig. 9 Flow chart showing the details of SFMB' 10 and SFMC' 10.

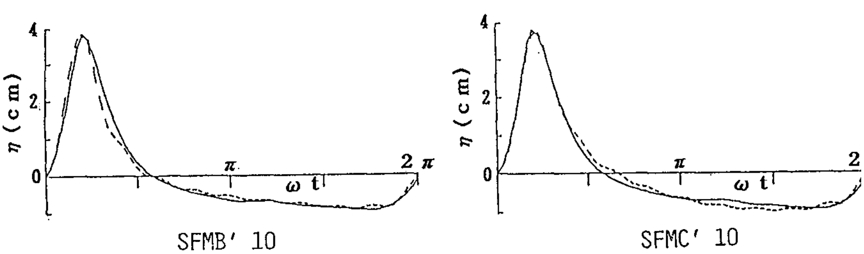


Fig. 11 Comparison of SFMB' 10 and SFMC' 10
with the measured wave form near wave breaking.
(Buhr Hansen & Svendsen,1979)
(H=4.7 cm, T=1.0 s, D=6.0 cm, Ur=127.9, m=1/34)
[---- : calculated, —— : measured]

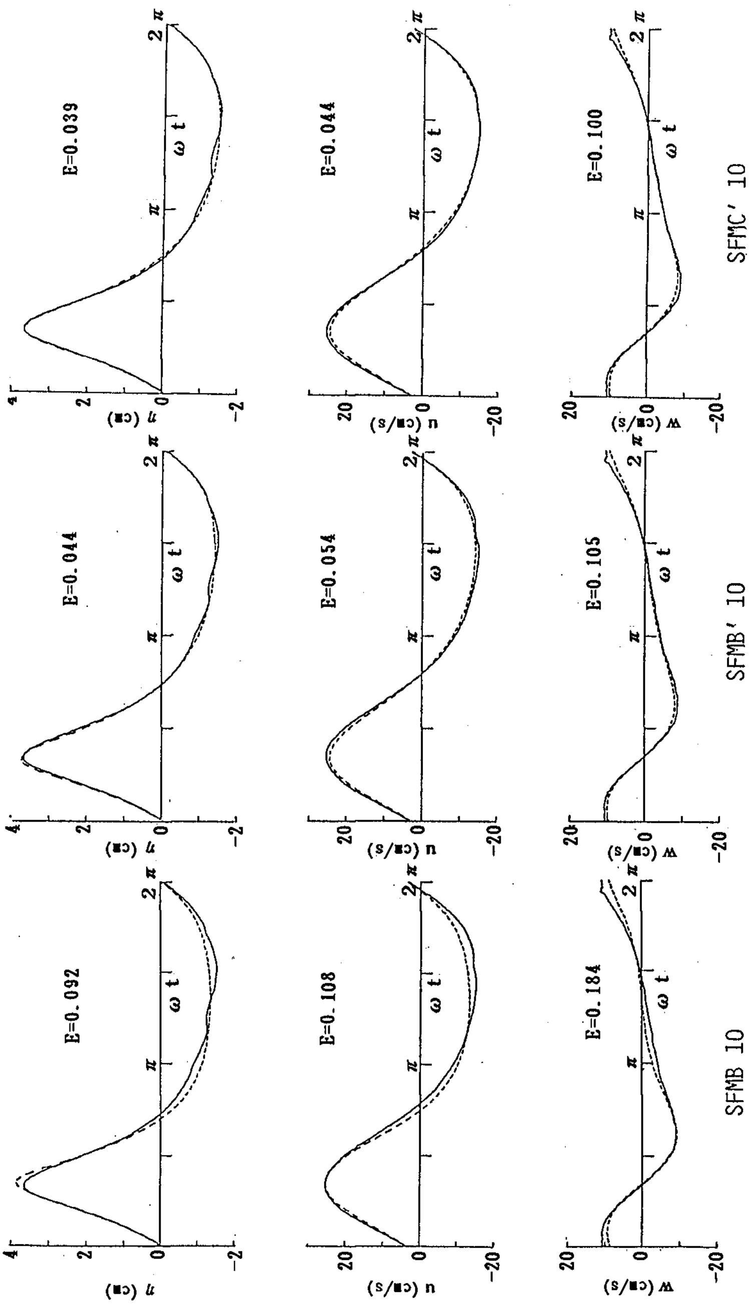


Fig. 12 Comparisons of SFMB 10, SFMB' 10, and SFMC' 10 with the measured wave profile and particle velocities. (H=5.2 cm, T=0.84 s, D=9.0 cm, z/D=-0.444, Ur=44.4) (——: measured, ----: calculated)

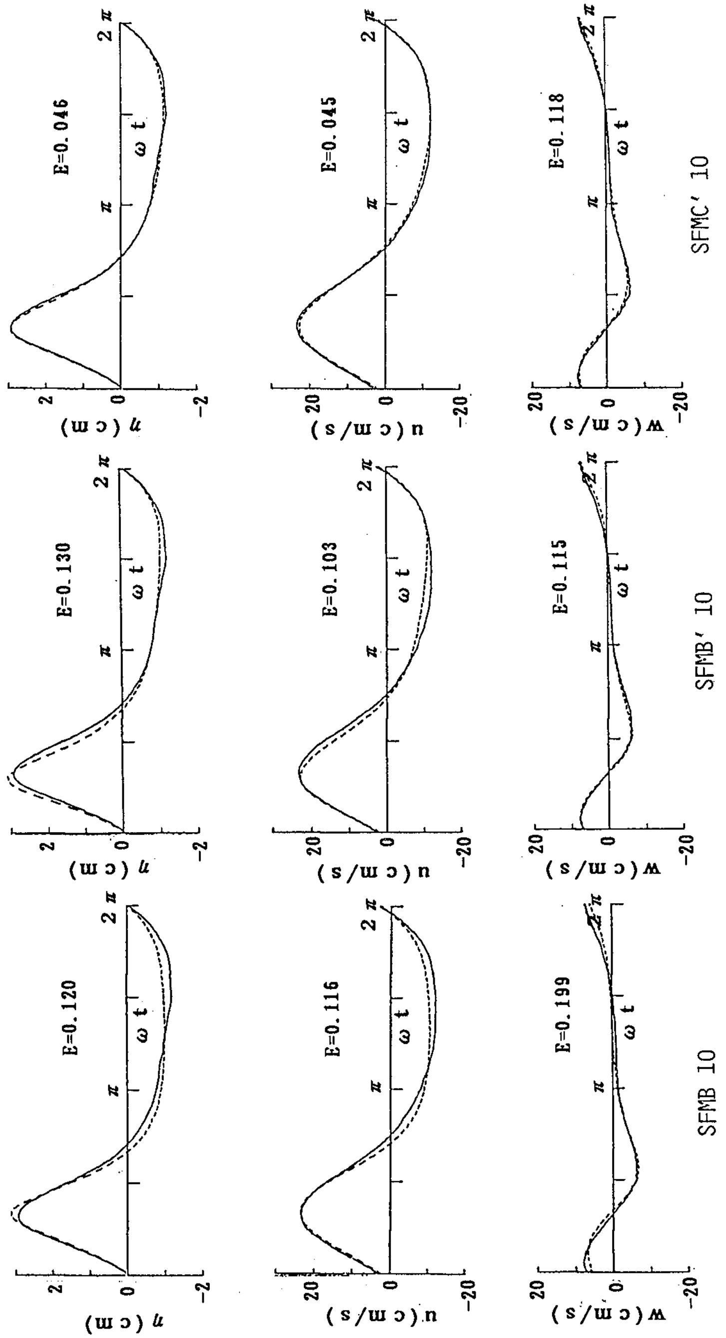


Fig. 13 Comparisons of SFMB 10, SFMB' 10, and SFMC' 10 with the measured wave profile and particle velocities.
(H=4.3 cm, T=0.99 s, D=7.0 cm, z/D=-0.786, Ur=84.3)
(——: measured, ----: calculated)

Figures 12 and 13 illustrate comparisons of the measured wave elevation and particle velocities with the computed ones by SFMB 10, SFMB' 10, and SFMC' 10. Measured velocities under breaking waves tend to lag the surface elevation(Adeyemo, 1972; Flick et al., 1981), while these of the nonbreaking wave occur in phase with the wave profile, as seen in Figs. 11 and 12. This supports that the particle velocities are predicted with aid of the SFMA 10, using the calculated wave elevation. The good agreement of SFMC' 10 with the measurements around the crest phase brings about the better agreement in the trough phase.

It is concluded that the improvement in SFMC' 10 successfully achieves demonstration of the highly asymmetric variation of the wave kinematics even near the wave breaking. Moreover, The improved calculation yields the reasonable prediction both of the magnitude and phase of the peak velocities, which are of fundamental importance to the cross-shore sediment transport.

7. CONCLUSIONS

(1) The improved calculation of the kinematic field under relatively steep and nonbreaking waves was developed to describe asymmetric characteristics of wave kinematics. The improvement is made for the Stokes 5th order theory and Dean's 10th order stream function theory.
(2) The improvement is based on the idea that the vertical asymmetry in the wave elevation can be described by the harmonic amplitude of the nonlinear solution, and the horizontal asymmetry is caused by the relative phase shifts between the primary and harmonic wave component.
(3) Based on the experiments, the harmonic phase shifts are determined as a function of Goda's nonlinearity parameter Π and the bottom slope.
(4) Agreements of the calculations with the measured wave elevation and particle velocities confirm that the improved calculations, ST' 5, SFMB' 10, and SFMC' 10, predict reasonably the temporal changes in one wave period.

In concluding, the improved calculation can predict the strongly nonlinear wave field for specified local wave conditions, without aids of direct numerical simulation techniques.

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