CHAPTER 24

ANOTHER QUASI-3D MODEL FOR SURF-ZONE FLOWS

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Abstract

In this report, a quasi-3D model for nearshore circulation is presented. The aim of the model is an economic simulation of surf- zone flow features.

This model relies on the efficient integration of three main modules: i) Wave propagation, ii) Depth-uniform currents, and iii) 1DV model for current-profiles calculation.

The basic equations are vertically integrated up to the trough level (z_{tr}), assuming at this level the existence of a rigid-lid, which permits to replace the free surface elevation by an equivalent pressure.

This device allows a continuous description of the flux below z_{tr}, making possible the obtention of the flux components in the same vertical solution domain.

The model is still under development. The results obtained, although insufficient to validate the code, serve to explore its capabilities.

1.-Introduction

A new model for surf-zone flow analysis has been developed at the University of Catalonia U.P.C.. It's a versatile tool, valid for a wide range of coastal engineering problems, that maintains cost/accuracy in the range considered nowadays reasonable (for desk-top computers). This is mainly due to the use of a Quasi-3D scheme (3D codes are still too expensive). In the development of the code, it has been prefered to keep some degree of vertical resolution to make possible, for instance, sediment transport computations.

The code (ALF) is composed of:

- Wave propagation module.
- Rigid-lid 2DH module.
- 1DV module for the vertical profiles.

The last two modules will allow a reasonably accurate 3D resolution, with nearly the cost of a 2DH model.

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In spite of the general formulation, herein we'll focus on the nearshore circulation, with emphasis on the internal consistency of the equations and associated simplifications.

The solution domain (vertical and horizontal) of the uniform ($\vec{u}$) and the depth-varying ($\tilde{\vec{u}}$) current components is the same. The water column is divided into three layers, coupled by mass/momentum transfers. The surface layer (above wave trough level) is not solved, and it's effects are considered through boundary conditions identical for $\vec{u}$ and $\tilde{\vec{u}}$. The middle layer goes from the top of the bottom boundary layer to z_{tr} for both variables. The third layer corresponds to the bottom boundary layer. To begin with and for simplicity, this layer will not be solved explicitly, its effects considered through continuity conditions for $\tilde{\vec{u}}$ and $\frac{\partial \tilde{\vec{u}}}{\partial z}$ at the interphase. Alternatively, and depending on the fit obtained in the selected test cases, the bottom boundary layer could be included in the solution domain. In this case, the boundary conditions become even simpler (for instance, no-slip at the bottom), although some additional complications appear due to the need to make "closure" assumptions for waves and turbulence near the bottom.

The driving terms here considered are basically the incident wind waves, described in terms of frequency (f) and angle of incidence (θ). Although it has been shown that the inclusion of randomness is essential for a rigorous treatment of wind-wave phenomena, for now it has been prefered to run a model lumped in f and θ, assuming that the wave climate is well defined by these two parameters. An additional reason is that most available expressions for D (the rate of wave energy dissipation per unit area, closely related to surf-zone circulation features) have been derived for waves described in this lumped manner. The wave velocity field, $\vec{u}$, is obtained from linear theory.

With respect to the numerical discretization, suffice it to say that in this initial stage, the simplest possible technique has been selected to concentrate on the physics of the problem.

In what follows, the set of equations and simplifications involved in the model will be exposed, together with some preliminary results.

2.- Current Modelling

2.1- Assumptions

- Incompressible - newtonian fluid.
- z_{tr} , z_b and $< \eta >$ are time-independent, (<> is the time-averaging operator at the wave-scale), see fig. 1.

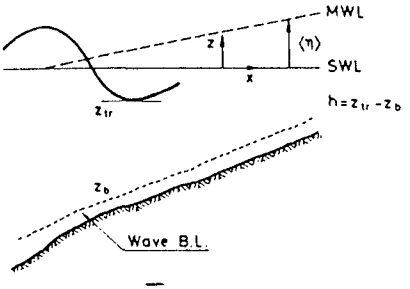

Figure 1.

- Horizontal gradients of z_{tr} and z_b are much smaller than the corresponding Reynolds stresses.
- Horizontal gradients of vertical velocities are much smaller than vertical gradients

of horizontal velocities.

- The current flow is quasi-horizontal (i.e. vertical accelerations are negligible. Thus, the pressure is given by:

$$p = p_{hydrostatic} - \rho < \tilde{w}^2 > \tag{1}$$

- There is no mass flow through the z_b and η boundaries, but there certainly is through the z_{tr} level.

Once all these assumptions have been made, the operators $\int_{z_b}^{z_{tr}}() \, dz$ and $<>$ will be applied to the mass and momentum equations. In order to avoid discontinuity problems of the variables describing the surface layer, the following decomposition will be used:

$$\vec{u} = \bar{\vec{u}} + \tilde{\vec{u}} \tag{2}$$

where:

$\vec{u}$: current velocity.

$\bar{\vec{u}}$: depth-invariant component.

$\tilde{\vec{u}}$: z-dependent component.

The $\tilde{\vec{u}}$ component satisfies:

$$\int_{z_b}^{z_{tr}} \tilde{\vec{u}} \, dz = 0 \tag{3}$$

From now on, the overbar symbol on a variable will be used to denote its mean value in the middle layer. The symbol " $\tilde{}$ " on a variable has the effect of centering the variable around its mean. So,it will always be:

$$\int_{z_b}^{z_{tr}} \tilde{\vec{f}} \, dz = \underline{0}, \; for \; every \; variable \; \vec{f} \tag{4}$$

2.2- Mass conservation Equation

The continuity equation, obtained after applying the depth-integration and time-averaging operators, is:

$$\frac{\partial(h\bar{u})}{\partial x} + \frac{\partial(h\bar{v})}{\partial y} = -G \tag{5}$$

where G is the net volume flux over z_{tr}, which reads:

$$\begin{aligned} G &= \frac{\partial}{\partial x} < \int_{z_{tr}}^{\eta} (u + \tilde{u}) \, dz > + \frac{\partial}{\partial y} < \int_{z_{tr}}^{\eta} (v + \tilde{v}) \, dz > \\ &= \nabla_h \cdot < \vec{Q}_s > \end{aligned} \tag{6}$$

($h = z_{tr} - z_b$ is the thickness of the middle layer)

Invoking the starting assumptions, G is also seen to satisfy:

$$G = [w - u\frac{\partial z_{tr}}{\partial x} - v\frac{\partial z_{tr}}{\partial y}](z_{tr}) \tag{7}$$

The wave contribution to the volume flux above z_{tr} is calculated with an expression deduced by Svendsen (1984b), which has been later used by other authors (e.g. Stive and De Vriend, 1987):

$$< \vec{Q}_s >_w = (1 + \frac{7\,d}{L})\frac{E}{\rho c} \tag{8}$$

d : Water depth.

L : Wavelength.

c : Wave celerity.

E : Wave energy density.

(The second term of the parenthesis is only considered inside the surf zone)

The current contribution to z_{tr} is more difficult to evaluate, due to the loss of physical meaning of this concept in a wave averaged time scale, because in this layer there are then "dry" and "wet" instants. Nevertheless, in the currents time-scale it looks reasonable to expect values of this contribution to be proportional to the values of the current in the middle layer (at $z = z_{tr}$) and the mean width of the surface layer. This term will be thus modelled by:

$$< \vec{Q}_s >_c = \alpha(< \eta > - z_{tr})\vec{u} \tag{9}$$

Where α is a parameter to be evaluated.

2.3- Momentum Conservation equation for $\vec{u}$

The momentum equation obtained integrating between z_b and z_{tr} and applying the wave time-averaging operator is for the x-component (analogous for y):

$$\frac{\partial(h\bar{u})}{\partial t} + \frac{\partial(h\bar{u}^2)}{\partial x} + \frac{\partial(h\bar{u}\bar{v})}{\partial y} - f(h\bar{v}) + (\bar{u} + \hat{u}(z_{tr}))G$$

$$= -\frac{h}{\rho}\frac{\partial p_t}{\partial x} + \frac{\partial}{\partial x}(\breve{R}_{xx} + \hat{R}_{xx}) + \frac{\partial}{\partial y}(\breve{R}_{xy} + \hat{R}_{xy}) + \frac{< \hat{\tau}_{tr} - \hat{\tau}_b >_x}{\rho} + \tilde{W}_x \tag{10}$$

Where:

$p_t = \rho\, g\, < \eta >$,which shows the formal equivalence between pressure and set-up/down.

$\breve{R}_{ij} = h\,\bar{\nu}_t\,(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i})$, stresses which take account of the horizontal momentum transfer due to $\bar{\nu}_t$.

$\hat{R}_{ij} = \int_{z_b}^{z_{tr}} [\hat{\nu}_t\,(\frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i}) - \hat{u}_i\hat{u}_j]\; dz$,term which takes account of the horizontal momentum transfer due $\hat{\nu}_t$ and the interaction effects of the z-dependent flow in the uniform flow.

$\tilde{W}_x = -\frac{\partial}{\partial x}\int_{z_b}^{z_{tr}} < \tilde{u}^2 - \tilde{w}^2 > dz - \frac{\partial}{\partial y}\int_{z_b}^{z_{tr}} < \tilde{u}\,\tilde{v} > dz$, which represents the horizontal gradient of driving terms due to the correlations of the wave-component velocities. This term doesn't exactly coincide with the usual definition of the radiation stress tensor S_{ij} (in our formulation vertical integration has been made up to z_{tr}).

Comparing (10) with the equation obtained integrating up to the free surface and disregarding time and space derivatives of the current over z_{tr} (and other terms that, according to the assumptions, are negligible), the following expression is obtained:

$$\tilde{W}_i + < \tau_{tr} >_i = -\frac{\partial \breve{S}_{ij}}{\partial x_j} + (\bar{u}_i + \hat{u}_i(z_{tr}))G \tag{11}$$

Replacing this expression in the momentum equations for $\bar{u}$ and $\bar{v}$, the usual horizontal gradients of the radiation stress tensor are recovered. This approach requires however variables evaluated at the trough level which is, in the surf-zone, particularly difficult. An interesting alternative, at present under analysis, is the direct calculation of $\vec{W}$ and G, with a suitable wave theory together with a semi-empirical model for $< \tau_{tr} >$ as a function of D similar to the formulation used by De Vriend and Stive (1987). This D formulation has been already used for the description of the z-dependent flow.

With all this, the momentum equation can be written (similarly for the y-component):

$$\frac{\partial(h\bar{u})}{\partial t} + \frac{\partial(h\bar{u}^2)}{\partial x} + \frac{\partial(h\bar{u}\bar{v})}{\partial y} - f(h\bar{v})$$

$$= -\frac{h}{\rho}\frac{\partial p_t}{\partial x} + \frac{\partial}{\partial x}(S_{xx} + \bar{R}_{xx} + \hat{R}_{xx}) + \frac{\partial}{\partial y}(S_{xy} + \bar{R}_{xy} + \hat{R}_{xy}) - \frac{< \hat{\tau}_b >_x}{\rho} \qquad (12)$$

These equations, together with the continuity equation (5) , become the usual ones for the calculation of free surfaces flows upon neglection of the z-dependent component of the current and horizontal gradients of the volume flux over z_{tr} and identifying h as the total depth.

2.4- Momentum conservation equation for $\vec{\hat{u}}$

Considering that:

- Wave stresses are aproximately uniform in the vertical (Stive and Wind, 1986)
- Horizontal gradients of turbulent stresses as well as interaction terms can be neglected (as suggested from an order- of-magnitude analysis)
- $\vec{\bar{u}}/\vec{\hat{u}}$ interaction terms associated to convective accelerations do not appear to play an essential role. Thus it has been prefered to retain only the terms which make possible an efficient 1DV model. These terms which preserve the order of magnitude of convective terms, are for the x-component (similarly for y):

$$\hat{u}\frac{\partial \bar{u}}{\partial x} + \hat{v}\frac{\partial \bar{u}}{\partial y} \qquad (13)$$

- The coriolis term is also neglected, since its effects (e.g. Ekman's layer) are usually masked by surf-zone turbulence.

Taking all this into account, the resulting equation for $\hat{u}$ is:

$$\frac{\partial \hat{u}}{\partial t} + \hat{u}\frac{\partial \bar{u}}{\partial x} + \hat{v}\frac{\partial \bar{u}}{\partial y} - \hat{u}(z_{tr})G - \frac{\partial}{\partial z}(\nu_t\frac{\partial \hat{u}}{\partial z}) + \frac{< \hat{\tau}_{tr} - \hat{\tau}_b >_x}{\rho h} = Res_x \qquad (14)$$

The term $\frac{\partial}{\partial t}$ is retained to allow transient computations (in the future) and in order to use t as a marching variable for stationary cases (at present).

The $\vec{\hat{u}}$ equation, though parabollic in t-z, is of elliptic-type in z. Once it has been discretized in time, the resulting equation is of second-order in z. Thus, only 2 independent boundary conditions are needed to have a "well-posed" problem. Nevertheless, at least formally, there is a larger number of available boundary conditions, as pointed out by Battjes et al (1988). The choice is not obvious, and this remains a still open problem.

The term $\hat{u}(z_{tr})G$ is evaluated lagged backwards one time-step (error $O(\Delta t)$).

The term $Res_x(x, y, z)$ is a residual, which includes:

- All neglected terms (related or not to $\hat{u}/\hat{v}$).

- Errors (incompatibilities) due to closure submodels.
- Errors due to the discretization.

As for now, to simplify the computations, the term associated to G, will be included inside Res_x. With all this, the $\hat{u}$ equations can be written in a compact manner as:

$$\frac{\partial \vec{\hat{u}}}{\partial t} + A\vec{\hat{u}} - \frac{\partial}{\partial z}(\nu_t \frac{\partial \vec{\hat{u}}}{\partial z}) + \vec{T} = \vec{Res} \qquad z_b \leq z \leq z_{tr} \tag{15}$$

Where:

$$A = \begin{pmatrix} \frac{\partial \bar{u}}{\partial x} & \frac{\partial \bar{u}}{\partial y} \\ \frac{\partial \bar{v}}{\partial x} & \frac{\partial \bar{v}}{\partial y} \end{pmatrix} \quad ; \quad T = \frac{1}{\rho h} \begin{pmatrix} < \hat{\tau}_{tr} - \hat{\tau}_b >_x \\ < \hat{\tau}_{tr} - \hat{\tau}_b >_y \end{pmatrix}$$

These equations (after simplifying some terms) turn out to be similar to the usual ones. For instance, for waves normally incident to the coast, the expression obtained adding the $\bar{u}$ and $\hat{u}$ components is analogous to the one used by Svendsen (1987) in the calculation of undertow.

2.5- Boundary/Initial Conditions

i) **Equations for $\bar{u}/\bar{v}$**

- Any initial condition compatible with the continuity equation, as shown in figure 2.

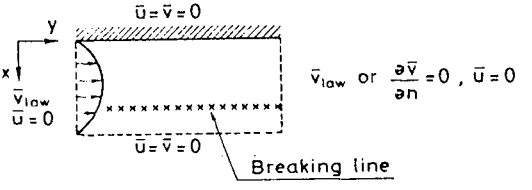

Figure 2.

- Boundary conditions are the same as those required by the 2D Navier-Stokes equations, i.e., Dirichlet or Neumann for $\bar{u}$ and $\bar{v}$ at each boundary point (compatible with the continuity equation).

ü) **Equations for $\hat{u}/\hat{v}$**

- Initial conditions: zero velocities.

$$\vec{\hat{u}} = 0, \ \forall z \in [z_b, z_{tr}].$$

- As for boundary conditions, a summary of possible choices (some of them suggested in publications by authors such as Svendsen, De Vriend and Stive) is:

At $z = z_{tr}$:

$\nu_t \frac{\partial \hat{u}}{\partial z} = \frac{<\hat{\tau}>}{\rho}$; $< \hat{\tau} >$, being related to the wave-height decay over z_{tr}, is also strongly related to the shape of $\vec{\hat{u}}(z)$. Furthermore, there is experimental evidence suggesting a non-negligible value of $\frac{\partial \vec{\hat{u}}}{\partial z}$ at z_{tr} (Hansen et al,1984; Nadaoka,1986).

$\vec{\hat{u}}(z_{tr})$; is left free, since it doesn't exist any reliable information on this parameter in the state-of-art.

At $z = z_b$:

If z_b coincides with the upper limit of the bottom boundary layer, then $\vec{\tilde{u}}(z_b)$ and $\frac{\partial \vec{\tilde{u}}}{\partial z}(z_b)$ are obtained from a closure submodel for the bottom boundary layer imposing continuity at z_b.

If z_b coincides with the bottom (zero-intercept level), the no-slip condition $\vec{\tilde{u}}(z_b) = 0$ appears naturally, although its use requires an explicit specification, inside the boundary layer, of poorly-known variables such as the eddy viscosity profile and the correlation (product) of wave velocities.

For now, in order to simplify the numerical development attention will be focussed on the middle layer. The available boundary conditions are (according to what has just been presented) $\vec{\tilde{u}}(z_b)$, $\frac{\partial \vec{\tilde{u}}}{\partial z}(z_b)$ and $\frac{\partial \vec{\tilde{u}}}{\partial z}(z_{tr})$, all of them obtained with closure models. These condition plus an externally obtained $\vec{Res}$, give an over-determined problem. Assuming known values for $\bar{u}/\bar{v}$ and ν_t there are 3 possible options to solve this problem:

a) Choosing 2 boundary conditions and assuming $\vec{Res} = \vec{0}$

b) Parameterizing $\vec{\tilde{u}}(z_b)$ and $\frac{\partial \vec{\tilde{u}}}{\partial z}(z_b)$ as a function of $\vec{u}_*$ (shear velocity) and using this free parameter to satisfy the third boundary condition (see Stive and De Vriend, 1987).

c) Inserting in the definition of $\vec{Res}$ an extra degree of freedom in order to accomodate the third boundary condition.

Option c) has been initially selected, since it seems the cheapest way to apply the three boundary conditions. This choice requires $\vec{Res}$ to be kept in the permissible range, which in practical terms, means relatively small values that must verify at the same time the compatibility condition $\int_{z_b}^{z_{tr}} \vec{Res}\, dz = 0$, which appears integrating vertically the momentum equation for $\vec{\tilde{u}}$. In fact, defining $\vec{I} = \int_{z_b}^{z_{tr}} \vec{\tilde{u}}\, dz$, the depth-integrated equation for $\vec{\tilde{u}}$ gives:

$$\frac{\partial \vec{I}}{\partial t} + A\,\vec{I} - \left(\nu_t \frac{\partial \vec{\tilde{u}}}{\partial z}\right)\Big|_{z_b}^{z_{tr}} + \vec{T}\,h = \int_{z_b}^{z_{tr}} \vec{Res}\, dz \qquad (16)$$

Assuming that $\vec{\tilde{u}}(t=0) = 0$ and considering $\nu_t \frac{\partial \vec{\tilde{u}}}{\partial z} = \frac{\vec{\tau}}{\rho}$ in z_b and z_{tr}, $\forall t$ (2 Neumann-type boundary conditions), it can be seen that a necessary and sufficient condition for $\vec{I} = \vec{0}$ (the integral condition that must satisfy $\vec{\tilde{u}}$ by definition) is $\int_{z_b}^{z_{tr}} \vec{Res}\, dz = 0$.

3.-Numerical Solution

3.1.- Equations for $\vec{\tilde{u}}$

The unknowns are $\bar{u}, \bar{v}$ and p_t (equivalently $< \eta >$). The available equations are continuity and the x and y momentum equations (vertically integrated and time-averaged). The equations are solved with an explicit finite-differences method based on a MAC-type grid (see figure 3). The solution algorithm uses an upwind and Euler-type discretization (SOLA-type) allowing for variable h and ν_t .

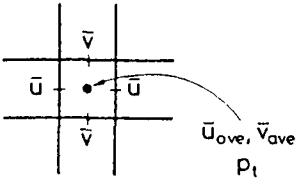

Figure 3.

The $\tilde{\hat{u}}/\bar{\hat{u}}$ interaction effects are considered via $\hat{R}_{ij}$ and $< \hat{\tau} >$. Initially, all terms related to $\hat{u}/\hat{v}$ are assumed zero, when solving the $\bar{\hat{u}}$ equations. The possible a posteriori correction effects have not been evaluated yet, leaving this as a still open point.

3.2- Equations for $\bar{\hat{u}}$

Using a forward Euler-type discretization for the time-derivative (error $O(\Delta t)$):

$$\frac{\partial u}{\partial t} = \frac{\bar{\hat{u}}^{n+1} - \bar{\hat{u}}^{n}}{\Delta t} \tag{17}$$

We obtain an equation for $\bar{\hat{u}}^{n+1}=\bar{\hat{u}}((n+1)\Delta t)$ which turns out to be a set of linear coupled second order ordinary differential equations for $\hat{u}/\hat{v}$.

$$A_t\ \bar{\hat{u}}^{n+1} - \frac{\partial}{\partial z}(\Delta t\ \nu_t \frac{\partial \bar{\hat{u}}^{n+1}}{\partial z}) = \vec{R}^n(z) \qquad z_b \leq z \leq z_{tr} \tag{18}$$

Where:

$$\begin{aligned} A_t &= I + \Delta t\ A \\ \vec{R}^n &= \bar{\hat{u}}^n + \Delta t\ (-\vec{T}^n + \vec{Res}^n) \end{aligned} \tag{19}$$

The solution technique for this set of equations relies on essentially, three ideas:

i) Uncoupling the $\hat{u}/\hat{v}$, equations with an adequate change of variable $w = Vu$, so that:

$$A_t = V\ \Lambda\ V^{-1} \qquad with \qquad \Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix};\ \lambda_1, \lambda_2 \epsilon R. \tag{20}$$

This change of variables may not always be possible, with cases in which the A_t eigenvalues can be complex or where it cannot be diagonal (in which case it should be replaced by the corresponding Jordan matrix). However, the numerical treatment is essentially identical in all cases, so that without a significant loss of generality, it will be suposed that this kind of situation will never arise in practice. On the other hand if $\frac{\partial \hat{u}}{\partial y} = 0$ or $\frac{\partial \hat{v}}{\partial x} = 0$, the equations become automatically uncoupled. The resulting equations can thus be written as:

$$a\ w_i + \frac{\partial}{\partial z}(K_t(z)\frac{\partial w_i}{\partial z}) = r_i(z) \qquad i = 1, 2 \tag{21}$$

ii) The use of a power series descomposition to reproduce the vertical variation of variables such as:

$$u_i(z) = \sum_{j=0}^{n} a_j\ z^j \tag{22}$$

$$\nu_t(z) = \sum_{j=0}^{p} b_j\ z^j \tag{23}$$

$$\vec{Res}(x, y, z) = \vec{Res}(x, y)\ (\frac{z_b + z_{tr}}{2} - z) \tag{24}$$

$\vec{Res}(x,y)$ represents the extra degree of freedom inserted to adjust the redundant boundary condition since it is mainly associated to neglected terms. The variation of $\vec{Res}(x,y,z)$ with z has been assumed linear (simplest possible solution satisfying the integral condition).

The power series technique is, among other possible choices, one of the simplest and cheapest, presenting advantages over a finite-differences scheme in z. The reason is that with a power series approach an analytical solution is obtained, whose precission and cost are controlled by the number of terms considered allowing an increase of the vertical resolution where desired in line with the quasi-3D philosophy. An additional advantage is that this approach allows recoverying expressions comparable to the polynomial solutions $\hat{u}/\hat{v}$ proposed elsewhere in the literature (see De Vriend and Stive, 1987; Svendsen, '87 '88 '89)

iii) Aplication of the power series approach to the uncoupled ordinary differential equations associated to the new variables w_i. With this, a recurrent relationship is obtained for the w coefficients:

$$a_{i+2} = f(a_{i+1}, a_i) \qquad i = 0, 1, ..., n-2 \tag{25}$$

These are n-1 equations with n+1 unknowns. Additionally there are three extra equations arising from the boundary conditions for $< \hat{\tau}_{tr} >_i$, w_i and $< \hat{\tau}_b >_i$. The missing unknown, needed to balance the number of equations and unknowns is provided by the degree of freedom associated to $Res_i(x,y)$.

This system of equations is only slightly more expensive than existing 1DV models, since if the equations are adequately ordered, the matrix associated to a_i's unknowns is aproximately lower-triangular.

4.-Closure Submodels

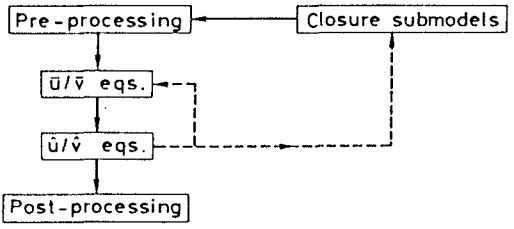

Figure 4.

In this section, some closure submodels will be briefly described, since they are similar to other existing state-of-art models. These submodels, although related, will be separately presented (see figure 4). All closure submodels not mentioned below can be considered identical to those used in De Vriend and Stive (1987).

4.1- Wave Propagation

Wave propagation properties are computed with a pair of equations for the wave number vector $\vec{K}$, obtained from the Kinematic Conservation Principle, together with a third coupled equation for the amplitude "a", obtained from the energy balance equation (see Yoo, 1986; Yoo and O'Connor, '86a,b). The numerical solution is based on a classical finite differences scheme, with a mesh as schematized in figure 5. This closure submodel reproduces adequately and at reasonable cost wave/current interactions, refraction/diffraction phenomena, and energy dissipation. The current driving terms are considered to be proportional to D (Dingemans et al,1987), which in turn is computed by means of the formula proposed by Battjes and Janssen (1978). The trough level, z_{tr}, is obtained using cnoidal theory.

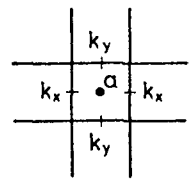

Figure 5.

4.2- Eddy Viscosity

To model $\nu_t(z)$ the simplest possible approach (based on an algebraic model) has been once again selected. This algebraic model (see for a review De Vriend and Kitou, 1990), includes the contributions of breaking waves and currents (influenced by the presence of the waves). The ν_t value satisfies the relation; ${\nu_t}^2 = {\nu_c}^2 + {\nu_{br}}^2$, equivalent to adding the corresponding values of K (turbulent kinetic energy) for waves and currents (Deigaard et al,1986). Following De Vriend and Stive (1987), the vertical profile of $\nu_c(z)$ is assumed constant in the upper half of the middle layer and quadratic in the lower half, although in both cases the profile is modelled as a function of the shear velocity enhanced by wave-effects. The profile of $\nu_{br}(z)$ is assumed constant in the whole middle layer, as a function of $(\frac{D}{\rho})^{\frac{1}{3}}$ (Svendsen,1987).

The evaluation of turbulent stresses remains nevertheless, an open problem that must be solved in a near future, in view of their strong influence on ν_t. The crucial effect of ν_t on the vertical profile of $\vec{u}$ can be easily illustrated by solving the homogeneus equation for w_i, which looks as:

$$\lambda\, w + \frac{\partial}{\partial z}(K_t(z)\frac{\partial w}{\partial z}) = 0 \tag{26}$$

Table 1 shows a summary of the wide range of possible expressions for $w(z)$.

Tabla 1: Illustration via of homegeneous eq. for w

$K_t(z)$	$\longrightarrow\lambda$	w(z)
ctnt > 0	> 0	Exponential
ctnt > 0	< 0	Sinusoidad
Linear	=0	logarithmic
cuadratic	Varying	Varying (dep. on BC and λ)

It is important to remark that although the order-of-magnitude analysis carried out in

previous sections assumed equal values of the eddy viscosity for the horizontal and vertical directions (see Svendsen, 1988), several authors have proposed different values. As a matter of fact, some references suggest the horizontal ν_{tH} two orders-of-magnitude larger than the vertical ν_{tV} (see De Vriend and Kitou, 1990).

4.3- Boundary conditions

The expression considered for the shear stress $< \hat{\tau}_{tr} >$ is very similar to the formulation proposed by De Vriend and Stive (1987).

The shear stress $< \hat{\tau}_b >$ is modelled using the formulation of Nishimura (1983), because of its ability to reproduce adequately the directional features of this stress (Yamaguchi, 1988).

To evaluate $\vec{\tilde{u}}(z_b)$ there are several available options, though none of them appears, at this stage, to be very convincing. Conceptually, it seems that the clearest solution would be to solve the middle and bottom boundary layers in a coupled way, imposing a non-slip condition at the real bottom. This approach, however, as previously indicated presents some additional difficulties. As a starting point, $\vec{\tilde{u}}(z_b)$ will be derived from the identity $\vec{u}(z_b) = \vec{\bar{u}} + \vec{\tilde{u}}(z_b)$, with $\vec{\bar{u}}$ obtained from the depth-averaged equations, and $\vec{u}(z_b)$ given by a wall-law profile:

$$u_i(z) = \frac{u_{*i}}{\kappa} log(\frac{u_{*i} E}{\nu_{mol}} z) \tag{27}$$

where:

$$\begin{aligned} u_{*i} &= (\frac{\tau_{bi}}{\rho})^{\frac{1}{2}} \\ E &= 9.0 \\ \kappa &= 0.41 \ (Von\ Karman's\ constant) \end{aligned} \tag{28}$$

5.-Validation

To begin with, it must be stressed that the calibration/validation processes is far from finished. The present status can be summarized as follows:

- The code for the 1DV model is already developed, but has not been validated yet.
- The two modules (2DH + 1DV) have not been run yet in a coupled way.
- For the time being, results can only be interpreted in a qualitative manner.

In any case, the list of test cases as follows:

2DH CODE ($\vec{\bar{u}}$):

- Normal and oblique incidence on a plane beach (setup,v_l) (Stive and Wind, 1982; De Vriend and Stive, 1987).
- Circulation behind a detached breakwater (Nishimura et al, 1985; Horikawa, 1987).
- Circulation on a variable bottom topography (Noda, 1974; Yamaguchi, 1986).

1DV CODE ($\vec{\tilde{u}}$):

- Undertow (Hansen and Svendsen, 1984; Stive and Wind, 1986; Nadaoka, 1986).
- Long-shore Current (Visser, 1984).

The only case here presented is the 2DH nearshore circulation over a symetric concave topography with normal wave incidence (Noda, 1974; Yamaguchy, 1986). It has been selected because it is a particularly interesting 2DH problem whose bathymetry can be numerically generated. The obtained results (see figure 6 and 7), are qualitatively similar to those reported

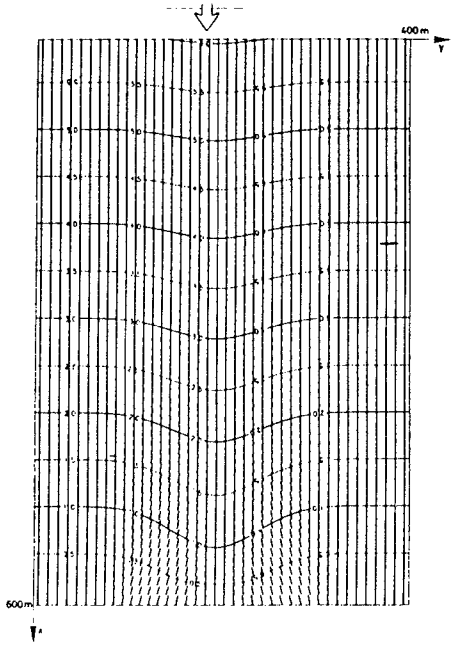

Figure 6

- Bottom topography and associated wave number vector field for the Noda, 1974 test case.

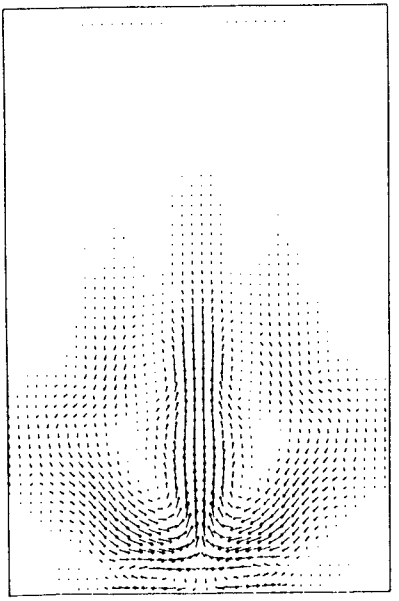

Figure 7.

- Current velocities field for the same case of figure 6.

by other authors, showing the presence of two circulation cells near the breaker line, together with another pair of flatter cells near the shoreline.

6.-Conclusions

A newly developed quasi-3D model for nearshore circulation has been presented. The most outstanding features are:

- Same solution domain and boundary conditions for both current-flow components, $\vec{\bar{u}}$ and $\vec{\tilde{u}}$ (so as to give a consistent physical and mathematical meaning to the algebraic sum of both components).

- Some degree of interaction between $\vec{\bar{u}}$ and $\vec{\tilde{u}}$ is retained.

- A new treatment for solving the depth-varying component is proposed (allowing local increases in resolution where desired).

- The same grid is used for waves and currents, avoiding thus, spurious effects due to interpolations.

Finally, it's important to point out that a certain number of points remain still open. Among them, the following are worthwhile mentioning:

- **Specific of the model:**

  a) Effects of $\vec{\bar{u}}$ / $\vec{\tilde{u}}$ interaction.

  b) Effects of G term (Volume flux over z_{tr}).

  c) Effects of introducing the residual $\vec{Res}$ (to accommodate the third boundary condition and other uncertainties).

  d) Effects of $\vec{\tilde{W}}$ evaluated up to z_{tr}

- **General for nearshore circulation code:**

  a) Empirical modelling of wave characteristics inside the surf-zone.

  b) Use and validation of an eddy viscosity model inside the surf-zone.

  c) Effects of solving in a coupled way the middle and bottom boundary layer.

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