PART I

Characteristics of Coastal Waves and Currents

CHAPTER 1

Distribution Function Fitting for Storm Wave Data

Yoshimi Goda[1] and Koji Kobune[2]

Abstract

New objective criteria are established for rejecting unfitting distribution functions to a sample of extreme wave data. Another criterion is also introduced to select the best fitting function among the eligible ones. Application of these criteria to storm wave data around Japan indicates a possibility of identifying the parent distributions for the regional population of storm wave data.

Introduction

Selection of design wave height is generally made on he basis of storm wave data fitting to some model distribution function. Methodology of distribution fitting has been discussed by many people including Goda [1988], and various techniques are used in design processes. However, selection of the distribution function for a given set of wave data among several candidates is still left to subjective judgment of a wave analyst or design engineer. Although the correlation coefficient between the extremal wave height and its reduced variate provides a measure of the degree of fitting, there is no guarantee that the best fitting distribution represents the true distribution of storm waves. The bootstrap technique as applied for storm wave data by Rossouw [1988] and Andrew and Hemsley [1990] provides an alternative means for distribution function selection, but it is not effective enough for distinguishing the true distribution from other candidate functions.

The first step to find the true distribution is to establish sound criteria by which inappropriate candidate functions can be rejected from distribution fitting to a given set of wave data. Application of such rejection criteria to a number

---

[1] Prof., Yokohama National Univ., Dept.Civil Engg., Hodogaya-Ku, Yokohama 240, Japan, M. ASCE.
[2] Dr. Eng., Chief, Marine Obs. Lab., Marine Hydrodyn.Div.,Port and Harbour Res.Inst., Nagase, Yokosuka 239, Japan.

of storm wave data sets within a particular region would yield an indication of the parent distribution pertinent to that region. The present paper describes the derivation of such rejection criteria based on the Monte Carlo simulation as well as a best-fitting criterion for the case of the least square method application. Examination of storm wave data around Japan with the newly derived rejection criteria indicates existence of regional parent distributions as discussed in the last part of this paper.

Candidate Distribution Functions and Plotting Position Formulas

The distribution functions examined herein are the Fisher-Tippett type I and II (hereafter denoted as FT-I and FT-II) and the Weibull distribution of the following form:

$$\text{FT-I}: \quad F(x) = \exp\{-\exp[-(x-B)/A]\} \tag{1}$$

$$\text{FT-II}: \quad F(x) = \exp\{-[1+(x-B)/kA]^{-k}\}$$
$$: k = 2.5,\ 3.33,\ 5.0,\ \text{and } 10.0 \tag{2}$$

$$\text{Weibull}: \quad F(x) = 1 - \exp\{-[(x-B)/A]^{k}\}$$
$$: k = 0.75,\ 1.0,\ 1.4,\ \text{and } 2.0 \tag{3}$$

where $F(x)$ denotes the distribution function of extreme data x, and A, B, and k are the scale, location, and shape parameters, respectively. The fixing of the shape parameter k of the Weibull distribution at the four values above is due to Goda [1988]. The functional form of the FT-II distribution is so selected to make it asymptotically approach to FT-I as $k \to \infty$. It is essentially same as the generalized extremal distribution by Jenkinson [1955]. The shape parameter k for FT-II is so set that $1/k$ would linearly increase from 0.1 to 0.4. In addition to the above distributions, the two-parameter log-normal distribution is also examined.

The present paper employs the least square method for parameter estimation for a sample of storm wave data, because of its simple algorithms, adaptability to censored data, and well-established information on confidence intervals of return wave heights based on a Monte Carlo simulation study (Goda 1988). The choice of the plotting position then becomes crucial to yield unbiased estimates of return wave heights. The unbiased plotting position formulas for the FT-I, Weibull, and log-normal distributions have been recommended by Goda [1988]. Another Monte Carlo simulation was carried out by Goda and Onozawa [1990] for the FT-II distribution with the sample size ranging from 10 to 200. For each condition, 10,000 samples were simulated and analyzed. Based on this simulation, the following plotting position formula has been derived:

$$\hat{F}_m = 1 - \frac{m - 0.11 - 0.52/k}{N_T + 0.12 - 0.11/k} \qquad (4)$$

where $\hat{F}_m$ is an estimate of the non-exceedance probability for the *m-th* largest variate, and N_T denotes the total number of storm wave events occurring during the period of data analysis.

The extremal analysis of storm wave data is usually made for the peak values of individual storms which exceed a certain threshold value. The number of storm wave data thus collected, N, is less than N_T. The ratio of N to N_T is called the censoring parameter (Goda 1988) and denoted by ν.

Equation (4) becomes same as the Gringorten formula for the FT-I distribution as $k \to \infty$. According to the simulation study with the above plotting position formula, the mean value of return wave height (averaged over 10,000 samples) estimated at the return period 10 times the sample duration showed a difference of -2.7% ~ +0.3% from the true value, depending on the sample size and shape parameter. For the shape parameter k = 5 and 10, the difference was -0.6%~+0.3%.

Confidence Interval of FT-II Distribution

Another simulation study was carried out to examine the sample variation of the return value of the FT-II distribution. Two sets of 10,000 sample runs were analyzed and their results were averaged. An empirical formula for the standard deviation of the estimated return value $\hat{x}_R$ at the return period R has been given as (Goda and Onozawa 1990)

$$\sigma(\hat{x}_R) = [1 + a(y_R - c + \alpha \ln \nu)^2]^{1/2} \sigma_x / N^{1/2} \qquad (5)$$

where ν is the censoring parameter defined as N/N_T, σ_x is the unbiased standard deviation of sample data, a, c, and α are empirical coefficients given by Eq.(6) and Table 1, and y_R is the reduced variate for the return period R expressed by Eq.(7) below:

$$a = a_1 \exp\{a_2[\ln(N \nu^{0.5}/N_0)]^2 - \kappa [\ln(\nu/\nu_0)]^2 \qquad (6)$$

$$y_R = k \{-\ln[1 - 1/(\lambda R)]^{-1/k} - 1\} \qquad (7)$$

in which λ denotes the mean rate, or the average number of storm events per year.

The senior author (Goda 1988) proposed empirical formulas for the correction of possible bias in the estimate of return wave height when the true distribution is unknown. He also presented other formulas to esimate standard errors under the

Table 1 Empirical Coefficients for The Standard Deviation of The FT-II Return Value

Shape Par.	a_1	a_2	N_0	κ	ν_0	c	α
k = 2.5	1.27	0.12	23	0.24	1.34	0.3	2.3
k = 3.33	1.23	0.09	25	0.36	0.66	0.2	1.9
k = 5.0	1.34	0.07	35	0.41	0.45	0.1	1.6
k =10.0	1.48	0.06	60	0.47	0.34	0	1.4

same situation. In the present paper no proposal is made however, because it is hoped that the use of new rejection criteria to be discussed below will diminish the possibility of choosing a distribution other than the true one.

Rejection Criterion Based on Outlier (DOL *Criterion*)

Another series of Monte Carlo simulations were carried out to explore the possibilities of establishing new criteria for distribution fitting. The sample size was from 10 to 400, the censoring parameter was set at 0.25, 0.50, and 1.0, and 10,000 sets of simulated data were produeced and analyzed for each combination of the distribution function, sample size, and censoring parameter.

In the extremal analysis of wave data, presence of an outlier or an abnormally large data often causes a trouble for analysts in data interpretation and distribution fitting. However, the outlier can provide a good measure of sample deviation from its population. Let x_1 be the value of largest data among a sample. Then its magnitude is measured with the following dimensionless deviation ξ :

$$\xi = (x_1 - \bar{x})/s \tag{8}$$

where $\bar{x}$ and s are the mean and the standard deviation of sample data.

In the statistical test of the normality of a sample, Thompson's test is sometimes used against its mean value in comparison with the overall means of many samples. By dividing the data set into a sample composed of the largest data only and that of remaining data, Thompson's test can be applied to the dimensionless deviation ξ. Then, the ξ value having the non-exceedance probability P is approximately given by

$$\xi_P = \left[\frac{(N-1)F(1, N-2:\alpha)}{N-2+F(1, N-2:\alpha)} \right]^{1/2} \tag{9}$$

where $F(1, N-2:\alpha)$ denotes the F distribution with the $(1, N-2)$ degrees of freedom at the exceedance probability α. For the largest data x_1 in a sample with the size N, α is given as $2(1-P^{1/N})$.

Analysis of simulated samples from the Normal distribution has yielded the cumulative distribution of ξ as shown in Fig. 1. Simulated data are in agreement with the theory by Eq.(9) except for the range of low ξ value. Difference is

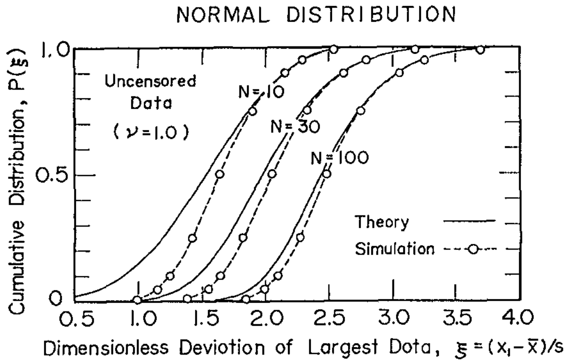

Fig.1 Cumulative distribution of ξ value.

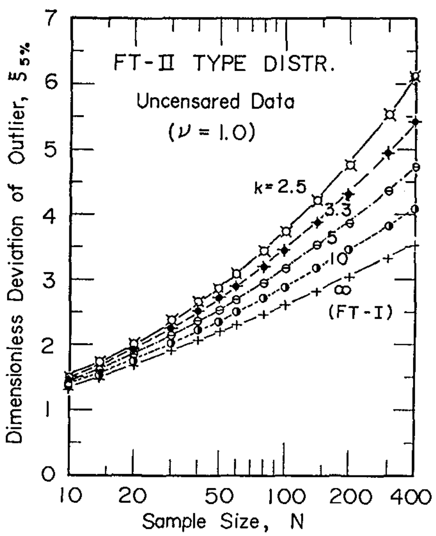

Fig.2 Examples of $\xi_{5\%}$ value.

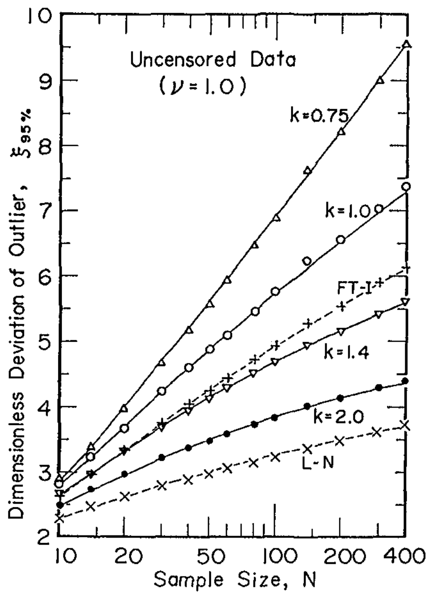

Fig.3 Examples of $\xi_{95\%}$ value.

attributed to the fact that use of Thompson's test for a single sample with the constraint of the largest data among the whole sample is beyond its range of applicability. Thus, the simulation data rather than theory were used in the following analysis of cumulative distribution of ξ value.

If the ξ value of a sample occupies a location at either the upper or lower tail of the cumulative ξ - curve of the distribution function being tested for fitting, the chance that the sample belongs to that population is slim and the fitting of that distribution could be rejected. The 5% and 95% cumulative ξ values were tentatively chosen as the threshold values for rejection. These threshold values were obtained from the simulation data for various sample sizes. Figures 2 and 3 shows examples of such ξ values for the case of uncensored samples. These results were formulated into the following expression:

$$\xi_{5\%},\ \xi_{95\%} = a + b\ \ln N + c\ (\ln N)^2 \qquad (10)$$

The coefficients a, b, and c have been expressed as the functions of the censoring parameter ν for each distribution function as listed in Tables 2 and 3. The difference between the estimates of $\xi_{5\%}$ and $\xi_{95\%}$ by Eq.(10) and the simulation data was less than 2%.

Fitting of a distribution function to a given sample could be rejected, if the ξ value of the sample is either greater than the $\xi_{95\%}$ value or less than the $\xi_{5\%}$ value of that distribution. This rejection criterion is hereby called the DOL (Deviation of OutLier) criterion. Note that the DOL criterion is applicable to any sample regardless of data fitting methods.

Table 2 Empirical Coefficients for The Lower DOL Criterion $\xi_{5\%}$

Distribution	Coefficient a	Coefficient b	Coef. c
FT-II (k = 2.5)	$1.481-0.126\nu^{1/4}$	$-0.331-0.031\nu^2$	0.192
FT-II (k = 3.3)	1.025	$-0.077-0.050\nu^2$	0.143
FT-II (k = 5.0)	$0.700+0.060\nu^2$	$0.139-0.076\nu^2$	0.100
FT-II (k =10.0)	$0.424+0.088\nu^2$	$0.329-0.094\nu^2$	0.061
FT-I	$0.257+0.133\nu^2$	$0.452-0.118\nu^2$	0.032
Weibull (k=0.75)	$0.534-0.162\nu$	$0.277+0.095\nu$	0.065
Weibull (k =1.0)	0.308	0.423	0.037
Weibull (k =1.4)	$0.192+0.126\nu^{3/2}$	$0.501-0.081\nu^{3/2}$	0.018
Weibull (k =2.0)	$0.050+0.182\nu^{3/2}$	$0.592-0.139\nu^{3/2}$	0
Log-normal	$0.042+0.270\nu$	$0.581-0.217\nu^{3/2}$	0

Table 3 Empirical Coefficients for The Upper DOL Criterion $\xi_{95\%}$

Distribution	Coefficient a	Coefficient b	Coef. c
FT-II (k = 2.5)	$4.653-1.076\nu^{1/2}$	$-2.047+0.307\nu^{1/2}$	0.635
FT-II (k = 3.3)	$3.217-1.216\nu^{1/4}$	$-0.903+0.294\nu^{1/4}$	0.427
FT-II (k = 5.0)	$0.599-0.038\nu^{2}$	$0.518-0.045\nu^{2}$	0.210
FT-II (k =10.0)	$-0.371+0.171\nu^{2}$	$1.283-0.133\nu^{2}$	0.045
FT-I	$-0.579+0.468\nu$	$1.496-0.227\nu^{2}$	-0.038
Weibull (k=0.75)	$-0.256-0.632\nu^{2}$	$1.269+0.254\nu^{2}$	0.037
Weibull (k =1.0)	-0.682	1.600	-0.045
Weibull (k =1.4)	$-0.548+0.452\nu^{1/2}$	$1.521-0.184\nu$	-0.065
Weibull (k =2.0)	$-0.322+0.641\nu^{1/2}$	$1.414-0.326\nu$	-0.069
Log-normal	$0.178+0.740\nu$	$1.148-0.480\nu^{3/2}$	-0.035

Rejection Criterion Based on Correlation Coefficient (REC *Criterion)*

Rejection of distribution fitting can also be made by using the absolute value of the correlation coefficient r between the extremal wave height x_m and its reduced variate y_m. Figure 4 shows examples of the cumulative distribution of the residue of r from 1, *i.e.*, $\Delta r = 1 - r$ for the case of the uncensored Weibull distribution with $k = 1.0$. If a sample is fitted to this distribution and the residue of correlation coefficient shows a value located at the upper tail of cumulative curve corresponding to the size of the sample, then the fitting of this distribution to that sample should be rejected. For quantitative analysis, the 95% exceedance value was taken as the threshold value and analyzed from the simulation data. Figure 5 shows examples of the variation of $\Delta r_{95\%}$ with respect to the sample size N. An empirical formulation has been made for $\Delta r_{95\%}$ as

$$\Delta r_{95\%} = \exp[a + b \ln N + c (\ln N)^2] \qquad (11)$$

The coefficients a, b, and c are expressed as the functions of censoring parameter for each distribution as listed in Table 4. The difference between the estimates of $\Delta r_{95\%}$ by Eq.(11) and the simulation data was mostly within ±3%.

The 95% exceedance value of the residue of correlation coefficient can be utilized as a reference for the rejection of distribution fitting. This is hereby called the REC (REsidue of Correlation coefficient) criterion. This rejection criterion is introduced primarily for the case of parameter esti-

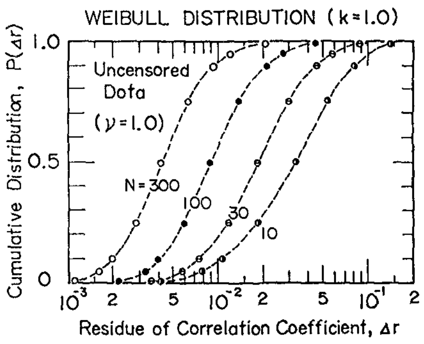

Fig. 4 Cumulative distribution of Δr.

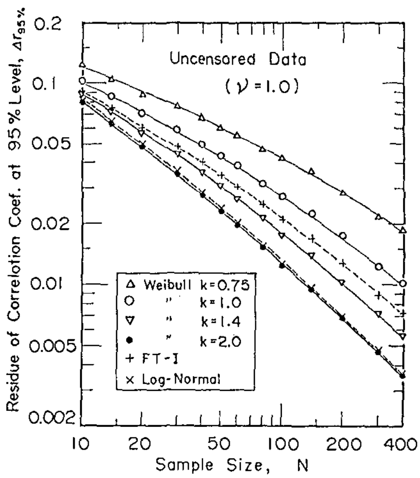

Fig. 5 Examples of $\Delta r_{95\%}$ versus N.

Table 4 Empirical Coefficients for $\Delta r_{95\%}$ in The REC Criterion

Distribution	Coefficient a	Coefficient b	Coef. c
FT-II (k = 2.5)	$-1.122-0.037\nu$	$-0.3298+0.0105\nu^{1/4}$	0.016
FT-II (k = 3.3)	$-1.306-0.105\nu^{3/2}$	$-0.3001+0.0404\nu^{1/2}$	0
FT-II (k = 5.0)	$-1.463-0.107\nu^{3/2}$	$-0.2716+0.0517\nu^{1/4}$	-0.018
FT-II (k =10.0)	$-1.490-0.073\nu$	$-0.2299-0.0099\nu^{5/2}$	-0.034
FT-I	-1.444	$-0.2733-0.0414\nu^{5/2}$	-0.045
Weibull (k=0.75)	$-1.473-0.049\nu^{2}$	$-0.2181+0.0505\nu^{2}$	-0.041
Weibull (k =1.0)	-1.433	-0.2679	-0.044
Weibull (k =1.4)	-1.312	$-0.3356-0.0449\nu$	-0.045
Weibull (k =2.0)	$-1.188+0.073\nu^{1/2}$	$-0.4401-0.0846\nu^{3/2}$	-0.039
Log-normal	$-1.362+0.360\nu^{1/2}$	$-0.3439-0.2185\nu^{1/2}$	-0.035

mation by the least square method, but it can also be applied to the case of other distribution fitting method after the parameters have been estimated.

Best Fitting Criterion Based on Correlation Coefficient (MIR *Criterion*)

After rejection of unfitting distribution functions, there arises the question of choosing the distribution closest to the true one. However, the parent distribution of the population of storm wave heights is unknown at present. When one set of extremal wave data at a particular location is analyzed for design purposes, a distribution function which seems best fitting to the sample is chosen as representative of the unknown true distribution. The senior author (Goda 1988) proposed to use the absolute value of the correlation coefficient between the extremal height x_m and its reduced variate y_m as the measure of best fitting: *i.e.*, to choose the distribution exhibiting the largest value of correlation coefficient.

Examination of the nature of the correlation coefficient, however, has indicated the fact that its mean value varies from one distribution to another. Figure 6 shows examples of the mean residual values of correlation coefficient Δr_{mean} of various distribution functions against the sample size N. The distribution with a long tail such as the Weibull with k = 0.75 yields a relatively low value or a large residual value of the correlation coefficient. It is therefore not fair to compare the degree of fitting of various distributions to a sample by means of the absolute value of correlation coefficient alone.

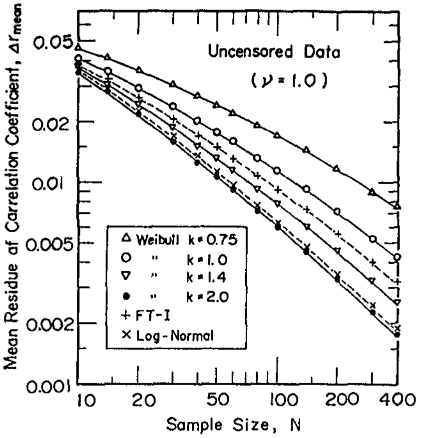

Fig. 6 Examples of Δr_{mean} versus N.

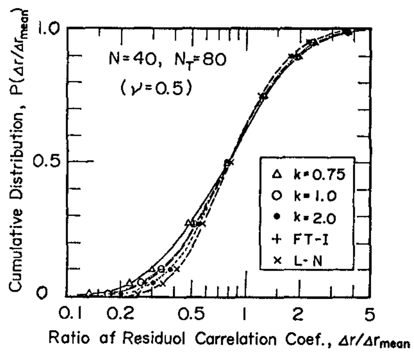

Fig. 7 Cumulative distribution of $\Delta r/\Delta r_{mean}$.

Differences in the absolute magnitudes of Δr among various distributions become insignificant, however, when their ratio to the respective mean values are compared. Figure 7 gives such an example, showing the cumulative distribution of $\Delta r/\Delta r_{mean}$ for the case of censored data with the sample size N = 40. Thus it is hereby proposed to employ the ratio $\Delta r/\Delta r_{mean}$ as the measure of best fitting and adopt the distribution which shows the minimum value of this ratio. This is called the MIR (MInimum Ratio of residual correlation coefficient) criterion for best fitting. It will be found in actual data analysis however that the MIR criterion is satisfied by most of distributions which show the largest correlation coefficient among various candidate functions.

The MIR criterion requires the formulation of Δr_{mean} for various distribution functions, sample sizes and the value of censoring parameter. An empirical expression same as Eq.(11) has been applied to the simulation data, *i.e.*,

$$\Delta r_{mean} = \exp[a + b \ln N + c (\ln N)^2] \tag{12}$$

and the coefficients *a*, *b*, and *c* have been formulated as listed in Table 5. The difference between the estimate of Δr_{mean} by Eq.(12) and the simulation data was mostly within ±3%.

Table 5 Empirical Coefficients for Δr_{mean} in The MIR Criterion

Distribution	Coefficient a	Coefficient b	Coef. c
FT-II (k = 2.5)	$-2.470+0.015\nu^{3/2}$	$-0.1530-0.0052\nu^{5/2}$	0
FT-II (k = 3.3)	$-2.462-0.009\nu^{2}$	$-0.1933-0.0037\nu^{5/2}$	-0.007
FT-II (k = 5.0)	-2.463	$-0.2110-0.0131\nu^{5/2}$	-0.019
FT-II (k =10.0)	$-2.437+0.028\nu^{5/2}$	$-0.2280-0.0300\nu^{5/2}$	-0.033
FT-I	$-2.364+0.054\nu^{5/2}$	$-0.2665-0.0457\nu^{5/2}$	-0.044
Weibull (k =0.75)	$-2.435-0.168\nu^{1/2}$	$-0.2083+0.1074\nu^{1/2}$	-0.047
Weibull (k =1.0)	-2.355	-0.2612	-0.043
Weibull (k =1.4)	$-2.277+0.056\nu^{1/2}$	$-0.3169-0.0499\nu$	-0.044
Weibull (k =2.0)	$-2.160+0.113\nu$	$-0.3788-0.0979\nu$	-0.041
Log-normal	$-2.153+0.059\nu^{2}$	$-0.2627-0.1716\nu^{1/4}$	-0.045

Application of New Criteria to Wave Data around Japan

Japanese islands have various wave climates along their coasts owing to their topography. As seen in Fig. 8, one side of the long stretched islands faces the Sea of Japan, the other side faces the Pacific, and the southwestern side is in the East China Sea. Waves in the Sea of Japan are fetch-limited wind waves, while waves from the Pacific are typhoon-generated seas and swell. Several dozens of nearshore wave stations at the depth 20 to 50 m have been operating with cable-connected acoustic wave recorders. Every 2 hours these wave sensors register surface wave records for 20 minutes. Most of the wave data which are under control of the Ministry of Transport are collected at the Port and Harbour Research Institute, analyzed and filed in a database. From this archives of wave records, extremal data of storm waves up to 1984 were selected and analyzed. (Further analysis with addition of recent data will be made in near future.)

Table 6 Distribution Fitting to Storm Wave Data around Japan

No.	Location	(1) K	(2) N	(3) ν	(4) ξ	Rejection Test of Dist. I II III IV	V	VI VII VIII IX
A	Monbetsu	7.4	75	0.42	3.62	×○○○	○	○○○○
B	Rumoi	13.1	284	0.44	3.70	▼▼▼▼	▽	▼▽○○
C	Setana	4.3	153	0.73	3.07	▼▼▼▼	▽	▼▼○○
D	Fukaura	4.2	156	0.78	3.58	▼▼▼○	○	▼○○○
E	Sakata	10.1	361	0.72	3.19	▼▼▼▼	▼	▼▼▼○
F	Hajiki-zaki	5.7	173	0.62	2.69	▼▼▼▼	▼	▼▼▼○
G	Wajima	5.9	196	0.68	3.13	▼▼▼▼	▽	▼▼▽○
H	Kanazawa	12.0	263	0.44	3.02	▼▼▼▼	▼	▼▼▼▽
I	Fukui	3.6	94	0.52	3.18	▼▼▽○	○	▽○○○
J	Tottori	4.9	143	0.60	4.75	×○○○	○	○○○△
K	Hamada	8.0	194	0.49	4.22	▼▼▼○	○	×○○○
L	Genkai-nada	3.0	78	0.52	3.00	▼▼▽○	○	▽○○○
M	Naze	7.6	166	0.44	3.16	▼▼▼▼	▼	▼▼○○
N	Naha	9.9	283	0.59	5.11	▼××○	○	×○○△
a	Tomakomai	13.9	213	0.47	4.78	▼○○○	○	○○○×
b	Mutsu-Ogawara	10.4	250	0.50	3.07	▼▼▼▼	▼	▼▼▽▽
c	Hachinohe	10.6	368	0.61	4.14	▼▼▼▽	○	▼○○×
d	Miyako	3.0	138	0.77	4.05	▼○○○	×	○○××
e	Kamaishi	3.3	81	0.43	2.91	▼▼▽▽	○	▽○○○
f	Sendai	5.3	176	0.57	3.94	▼▽▽○	○	○○○×
g	Onahama	3.7	102	0.49	3.48	▼▼○○	○	○○○×
h	Hitachi-Naka	4.8	121	0.43	3.22	▼▼▽▽	○	▽○○×
i	Kashima	9.4	407	0.73	4.71	▼▼▽○	○	▼○○×
j	Habu (Ohshima)	9.9	401	0.60	5.37	▼▼○○	○	○○×▲
k	Shiono-misaki	11.8	365	0.56	4.94	▼▼○○	○	○○○×
l	Kochi-offshore	3.6	149	0.75	5.08	○○○○	×	○××▲
m	Aburatsu	7.5	71	0.18	4.94	○○○○	○	○×△▲
n	Shibushi	4.5	91	0.41	5.22	○○○○	×	○×▲▲
p	Nakagusuku	8.9	270	0.68	4.74	▼▽○○	×	○○×▲

REMARKS:

Column (1) : Effective duration of observation in years.
(2) : Number of storm wave events above a threshold value.
(3) : Censoring parameter ($=N/N_T$).
(4) : ξ value of storm wave data.

Distribution Functions :

I = FT-II (k = 2.5), VI = Weibull (k = 0.75),
II = FT-II (k = 3.3), VII = Weibull (k = 1.0),
III = FT-II (k = 5.0), VIII = Weibull (k = 1.4),
IV = FT-II (k =10.0), IX = Weibull (k = 2.0).
V = FT-I ,

Distribution Fitting :

○ = Fitting acceptable,
△ = Rejected by the upper DOL but not by the REC criteria,
▲ = Rejected by both the upper DOL and the REC criteria,
▽ = Rejected by the lower DOL but not by the REC criteria,
▼ = Rejected by both the lower DOL and the REC criteria,
× = Rejected by the REC but not by DOL criteria.

Table 6 lists the location names of wave stations, the effective durations of wave observations in years excluding the periods of downtime, the number of storm wave data analyzed, and so on. The locations of these stations are shown in Fig. 8 with the alphabet. The average number of storm wave events at these stations was about 50 to 60 per year, and the threshold wave height for censoring varied from station to station; thus the censoring parameter ranged from 0.18 to 0.78. In total, wave data at 29 stations were analyzed, and the duration of wave observation ranged from 3.0 to 13.9 years.

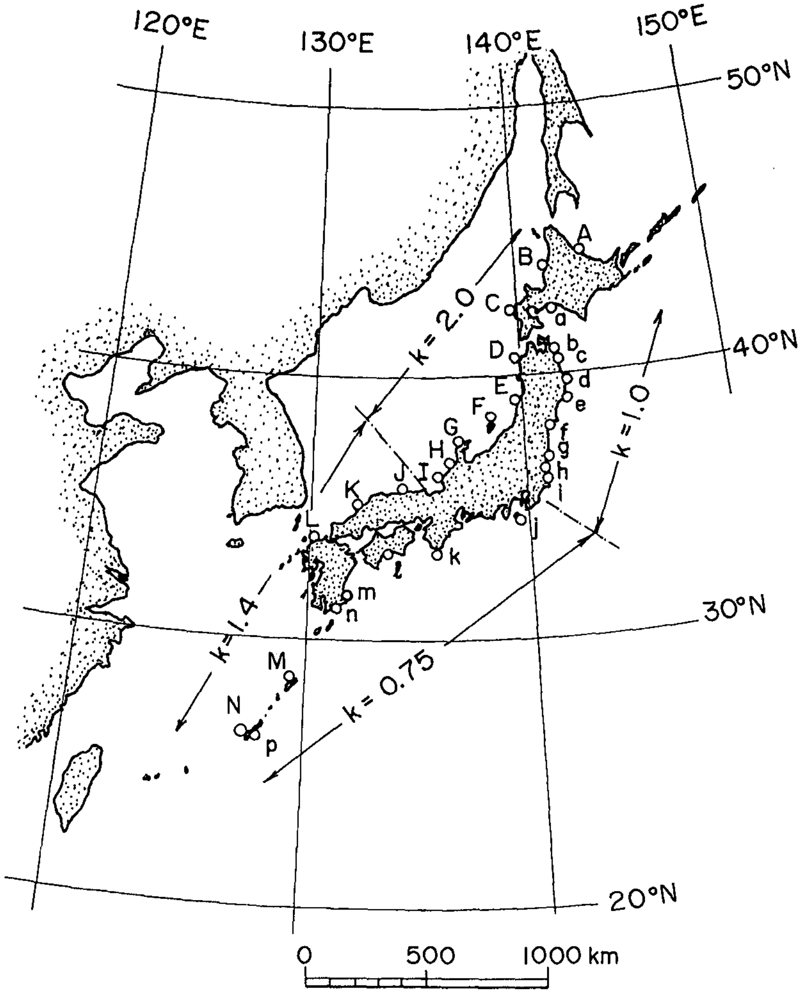

Fig. 8 Location map of wave stations used in the analysis.

The dimensionless deviation ξ of the largest data at each station is listed in the column (4) of Table 6, and the rejection test of the distribution functions of Eqs. (1) to (3) has been made with this ξ value and the residue of correlation coefficient Δr. The result of rejection test is marked with various symbols for each distribution. The upper DOL criterion is applied when the sample ξ value exceeds the $\xi_{95\%}$ value of the distribution being fitted, while the lower DOL criterion is for the case of $\xi < \xi_{5\%}$. The marks except for the open circle indicate that the distribution fitting should be rejected with the probability of misjudgment being less than 5%. In other words, one sample out of 20 samples would be rejected even if all the 20 samples are drawn from the same population.

Although the rejection test for the storm wave data of individual stations yields versatile answers, the results point out the presence of a characteristic distribution which is unrejectable in a group of stations located in the same region. For example, the stations 'A' to 'I' along the Sea of Okhotsk and the upper part of Japan Sea seems to support the Weibull distribution with $k = 2.0$ as the common distribution. For the stations 'J' to 'N', the Weibull with $k = 1.4$ seems more appropriate, probably because of occasional high waves generated by typhoons. In the region covering the stations from 'a' to 'i', the Weibull distribution with $k = 1.0$ has the least frequency of rejection. This area is exposed to swell from typhoons, but direct attack of typhoon waves is infrequent. The region for the station 'j' to 'p' where typhoon-generated waves dominate extreme wave climate seems to accept three distributions of the Weibull with $k = 0.75$ and the FT-II with $k = 5.0$ and 10.0 as the candidates for the population distributions.

The analysis of Table 6 will require further examination with additional data after 1985, especially from the viewpoint of the data homogenuity. Analysis of seasonal or monthly extremal wave data will be the easiest way to separate storm waves generated by different types of meteorological disturbances. Nevertheless, the result of Table 6 indicates the possibility of identifying the parent distribution for the regional population of storm wave data by filtering out various candidate functions with the new rejection criteria.

Concluding Remarks

Choosing a distribution function for a sample of extremal data sample is always a troublesome task, because no steadfast guidelines exist. It will be very difficult and almost impossible to establish such guidelines for affirmative recommendation. The present paper is intended to enable to issue negative recommendations for distribution fitting by establishing objective rejection criteria for extreme statistics. Two empirical criteria introduced herein well function in identifying possible regional population distributions of storm waves around Japan.

A question often asked is the minimum size of extremal sample at which an analyst can answer with confidence which one of candidate distributions is the true distribution of the population for that sample. An answer can be obtained by examining the sample size at which the lower DOL criterion $\xi_{5\%}$ of a longer-tailed distribution becomes equal to or greater than the upper DOL criterion $\xi_{95\%}$ of a shorter-tailed distribution. Beyond that sample size, the two distributions could be safely discerned. However, this sample size turns out to be quite large. For example, the FT-I and FT-II with k = 3.3 are only discernible at the sample size greater that 900 for an uncensored sample and greater than 1900 for a censored sample with ν = 0.25. The FT-I and the Weibull with k = 1.4 are very difficult to distinguish each other by means of the DOL criterion alone.

In the extremal analysis of storm wave data, emphasis should be placed on the effort of collecting as many data as possible over a period as long as feasible, so that the danger of choosing a distribution not belonging to the population will be minimized and the range of confidence interval of the return wave height will be lowered.

References

Andrew, M.E. and Hemsley, J.M. [1990]: Resampling approach to extreme wave-height analysis, *J. Waterway, Port, Coastal, and Ocean Engg.*, ASCE, Vol.116, No.4, pp.444-458.

Goda, Y. [1988]: On the methodology of selecting design wave height, *Proc. 21st Int. Conf. Coastal Engg.*, Malaga, pp. 899-913.

Goda, Y. [1990]: Rejection criteria for outliers and their applications in extreme statistics, *Proc. Japan Soc. Civil Engrs.*, No.417/II-13, pp.245-254 (*in Japanese*).

Goda, Y. and Onozawa, M.[1990]: Characteristics of the Fisher-Tippett type II distribution and their confidence intervals, *Proc. Japan Soc. Civil Engrs.*, No.417/II-13, pp.289-289 (*in Japanese*).

Jenkinson, A.F. [1955]: The frequency distribution of the annual maximum (or minimum) values of meteorological elements, *Quart. J. Roy. Meteor. Soc.*, Vol.81, pp.158-171.

Rossouw, J. [1988]: Design waves and their probability density functions, *Proc. 21st Int. Coastal Engg.*, Malaga, pp.822-834.