CHAPTER 193

Nearshore Circulations on a Wavy Coast

Ming-Chung Lin* and Sheng-Yeh Hwang**

1. Introduction

Nearshore circulations, produced by wave-induced radiation stress gradicents, form different circulation patterns under different wave characteristics and topographical conditions. Although numerous studies of nearshore circulations, such as Bowen (1969), Miller and Barcilon (1978), Dalrymple & Lozano (1978) and so on, have been appeared in the literature, it seems that little attention is paid on the case of a non-straight shoreline. O'Rorske & Leblond (1972) have investigated the wave-induced longshore currents in a semicircular bay, while Uda (1982) has used the coordinate transformation to treat nearshore circulations on a circular-arc shaped coast. Lin and Lee (1982), introduced a small perturbation quantity of wave set-up and set-down, induced by the non-straight shoreline, into the mean total water depth to obtain a governing partial differential equation by which they investigated the nearshore circulations on a cuspate coast. In Lin and Liou (1986), a more general equation was deduced on the orthogonal curvlinear coordinate system to unify the diversities among the related theories, and moreover, to investigate the rearshore circulations on the arc-shaped coast.

2. Mathematical formulation

With the wave field there will be associated a mean transport velocity which we will lump together with the secondary mean currents that we wish to describe under the velocity vector U, assumed horizontal and depth purely independent. The mean momentum density of a water column is

$$M = \rho d U \tag{1}$$

where ρ is the water density and d the total water depth. For a steady wave field the mean mass conservation equation reduces to

$$\nabla \bullet M = 0 \tag{2}$$

and then the linearized mean momentum equation is expressed as (O'Rourke and Leblond, 1972)

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* Professor, Dept. of Naval Architecture, National Taiwan Univ., Taipei, Taiwan, Rep. of China.

** Graduate Institute of harbor and River Eng., National Taiwan College of marine Science and Technology, Keelung, Taiwan, Rep. of China.

$$\frac{1}{\rho d}\frac{\partial S_{ij}}{\partial X_i} = -g\frac{\partial \zeta}{\partial X_i} - \frac{fU_i}{\rho d} \tag{3}$$

where f is the friction coefficient, a function of total water depth and wave amplitude; S_{ij} is the radiation stress of the waves, g is the acceleration of gravity. In this equation, the lateral friction is neglected. Define

$$T_i = \frac{1}{\rho d}\frac{\partial S_{ij}}{\partial X_j}, \quad F_i = -\frac{fU_i}{\rho d} \tag{4}$$

then, in surf zone, equation (3) can be rewritten as

$$\nabla \times T = \nabla \times F \tag{5}$$

In linear model (O'Rourke and Leblond, 1972), in the outside surf zone:

$$\nabla \times F = 0 \tag{6}$$

and in the surf zone

$$\nabla \times T = \nabla \times F \tag{7}$$

The coordinate system used and the domain interested are duplicated as Fig. 1. in which ϕ is the angle of wave incidence, A and B, denote the region inside and outside the surf zone respectively.
the shoreline is assumed as

$$Y = \alpha \cos kx, \quad k = 2\pi/L \tag{8}$$

where L is spatial wave length of shoreline, and α shoreline wave amplitude. Applying conformal mapping (Lyne, 1971)

$$z = x+iy, \quad \zeta = \mu_1 + i\mu_2 = z - i\alpha e^{ikz} \tag{9}$$

it yields, up to $O(\alpha k)$

$$\begin{aligned} \mu_1 &= x+\alpha e^{-ky} \sin kx \\ \mu_2 &= y-\alpha e^{-ky} \cos kx \end{aligned} \tag{10}$$

The continuity equation (2) then becomes

$$\frac{\partial}{\partial \mu_1}(J^{-1/2}\, du) + \frac{\partial}{\partial \mu_2}(J^{-1/2}\, dv) = 0 \tag{11}$$

where J is Jacobian:

$$J = \left|\frac{d\zeta}{dz}\right|^2 = 1+2\alpha k e^{-ky} \cos kx + (\alpha k)^2 e^{-2ky} \tag{12}$$

By equation (11) the velocity components u and v are

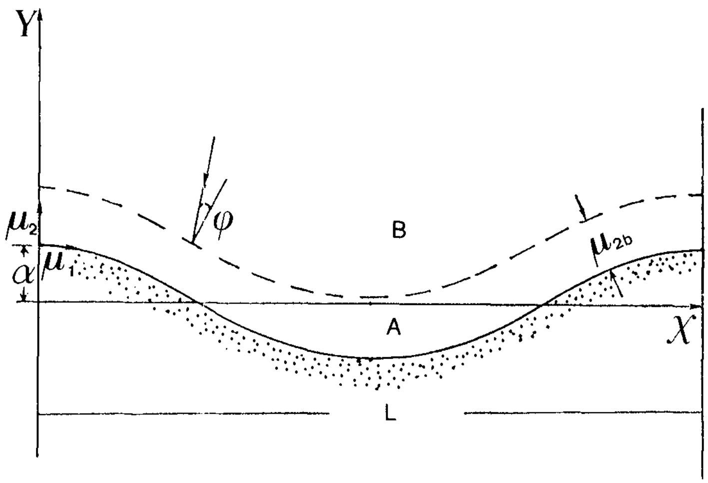

Fig. 1 Schematic diagram of coordinate system.

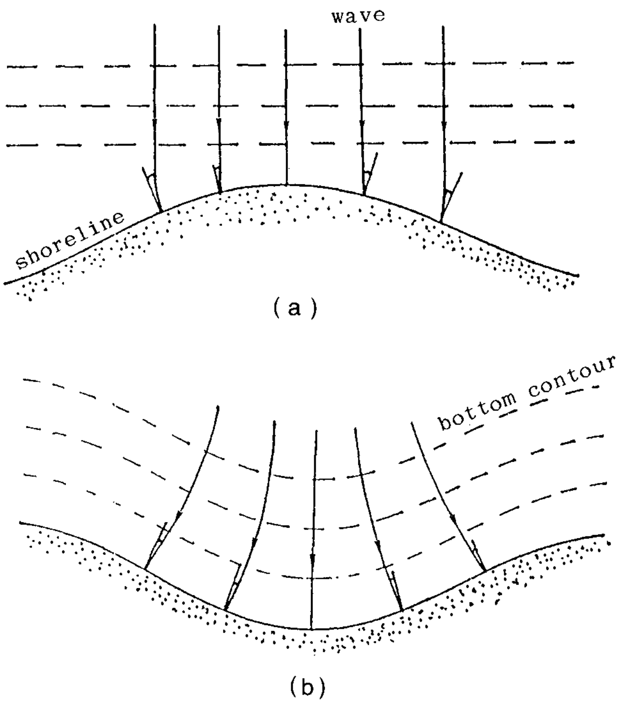

Fig. 2 Cases designed for calculations.

represented in terms of the transport stream function x as

$$u = \frac{J^{1/2}}{d} \frac{\partial X}{\partial \mu_2}, \quad v = \frac{J^{1/2}}{d} \frac{\partial X}{\partial \mu_1} \tag{13}$$

In surf zone, we express radiation stress components in (μ_1, μ_2) system to be ($T\mu_1$, $T\mu_2$) then

$$\nabla \times T = J[\frac{\partial}{\partial \mu_1} (J^{-1/2}.T\mu_2) - \frac{\partial}{\partial \mu_2} (J^{-1/2} \mu_1)] \tag{14}$$

where $T\mu_1$, $T\mu_2$ are defined as

$$T\mu_1 = \frac{J}{\rho d} [\frac{\partial}{\partial \mu_1}(J^{-1/2}.S\mu_1\mu_1) + \frac{\partial}{\partial \mu_2}(J^{-1/2} S\mu_1\mu_1)]$$

$$T\mu_2 = \frac{J}{\rho d} [\frac{\partial}{\partial \mu_1}(J^{-1/2}.S\mu_2\mu_1) + \frac{\partial}{\partial \mu_2}(J^{-1/2} S\mu_2\mu_2)] \tag{15}$$

Substituting equation (15) into equation (5), and expanding into Taylor's series, up to O(αk), we obtain

$$J [\frac{\partial}{\partial \mu_1}(\frac{f}{\rho d^2} \frac{\partial x}{\partial \mu_1}) + \frac{\partial}{\partial \mu_2}(\frac{f}{\rho d^2} \frac{\partial x}{\partial \mu_2})] =$$

$$J \{\frac{\partial}{\partial \mu_1} [\frac{J}{\rho d} \frac{\partial}{\partial \mu_1}(J^{-1/2}.S\mu_1\mu_2) + \frac{J}{\rho d} \frac{\partial}{\partial \mu_2}$$

$$(J^{-1/2}.S\mu_2\mu_2)] - \frac{\partial}{\partial \mu_2} [\frac{J}{\rho d} \frac{\partial}{\partial \mu_1} (J^{-1/2}.S\mu_1\mu_1)$$

$$+ \frac{J}{\rho d} \frac{\partial}{\partial \mu_2} (J^{-1/2}. S\mu_2\mu_2)]\} \tag{16}$$

where J_1 can be expanded by Taylor's series, up to O(α k)

$$J^{-1/2} = 1-\alpha k e^{k\mu_2} \cos k\mu_1 \tag{17}$$

In the region outside the surf zone, equation (6) can be similarly transformed to be

$$\nabla \times F = J [\frac{\partial}{\partial \mu_1} (\frac{f}{\rho d^2} \frac{\partial x}{\partial \mu_1}) + \frac{\partial}{\partial \mu_2} (\frac{f}{\rho d^2} \frac{\partial x}{\partial \mu_2})]$$

$$=0 \tag{18}$$

we further assume that in the seaward zone the mean currents do not reach to the bottom, but to about the water depth at the breaker line, as assumed by Liu and Lennon (1978). Thus the total water depth $d = \zeta + h_b$, where h_b is the water depth at the breaker line, will be taken as a constant. Assume $d = d_b$, we get

$$\frac{\partial^2 X}{\partial \mu_1^2} + \frac{\partial^2 X}{\partial \mu_2^2} = 0 \tag{19}$$

Let

$$\mu_1 = \frac{2\pi}{k}\xi, \quad \mu_2 = \frac{2\pi}{k}\eta \tag{20}$$

We can get the nondimensional equation:

$$\frac{\partial^2 X}{\partial \xi^2} + \frac{\partial^2 X}{\partial \eta^2} = 0 \tag{21}$$

3. General solution

In the seaward zone, the stream function satisfies $\nabla^2 X = 0$. We will label the stream function in each one of the two regions with a corresponding letter subscript. Now we consider some boundary conditions. The normal velocity at the shore line must vanish, that is

$$X_A(\xi, 0) = 0 \tag{22}$$

Further, the normal velocity must be matched at the boundaries between the two regions. By matching the stream function across those boundaries, this requirement is achieved:

$$\begin{aligned} X_A(\xi, 1) &= X_B(\xi, \delta) \\ X_{A,B}(0, \eta) &= 0 \\ X_{A,B}(1, \eta) &= 0 \\ X_B(\xi, \infty) &= X_\infty \end{aligned} \tag{23}$$

Set

$$X_A = X_1 + \delta X_2 \tag{24}$$

for δ is a small perturbation magnitude. In surf zone, we shall assume that the total mean depth is a linear function of the distance from shore

$$d = m(\xi).\eta \tag{25}$$

where $m(\xi) = m_o(1 + p\cos 2\pi\xi)$, m_o, p are small quantity. Set a simple, but reasonable form for ϕ_b, we take

$$\phi_b = -\phi_o \sin 2\pi\xi \tag{26}$$

Substituting equation (25) and (26) into equation (16) and make it nondimensional with

$$X_{A,B} = \frac{\pi g^{1/2} \gamma m_b^{5/2}}{C} \cdot X^*_{A,B} \tag{27}$$

we finally get the solutions for the region inside the surf zone:

$$X_A^* = \frac{\phi_o}{6} \left(\sin 2\pi\xi + \frac{5}{4} p \sin 4\pi\xi \right) \cdot \eta^3$$

$$+ \frac{2\pi}{7} \delta p \left(\sin 2\pi\xi + \frac{3}{4} p \sin 4\pi\xi \right) \cdot \eta^{\frac{7}{2}}$$

$$+ \frac{7\pi \phi_o^2}{24} \delta \cdot \sin 4\pi\xi \cdot \eta^4 + (\alpha k) \{ \frac{\phi_o}{4} (\sin 4\pi\xi$$

$$+ \frac{5}{4} p \sin 2\pi\xi + \frac{5}{4} p \sin 6\pi\xi) \cdot [\frac{\eta^3}{3} - \frac{k \mu_{2b}}{4} \cdot \eta^4$$

$$+ \frac{(k\mu_{2b})^2}{10} \cdot \eta^5] - \frac{5\pi}{4} \delta (\sin 2\pi\xi + \frac{5}{4} p \sin 2\pi\xi)$$

$$\cdot [\frac{2}{7} \eta^{\frac{7}{2}} - \frac{2k\mu_{2b}}{9} \cdot \eta^{\frac{9}{2}} + \frac{(k\mu_{2b})^2}{11} \cdot \eta^{\frac{11}{2}}]$$

$$+ \frac{\pi \delta p}{2} (\sin 4\pi\xi + \frac{3p}{4} \sin 2\pi\xi + \frac{3}{4} p \sin 6\pi\xi)$$

$$\cdot [\frac{2}{7} \eta^{\frac{7}{2}} - \frac{2k\mu_{2b}}{9} \cdot \eta^{\frac{9}{2}} + \frac{(k\mu_{2b})^2}{11} \cdot \eta^{\frac{11}{2}}] + \pi \phi_o^2 \delta$$

$$\cdot (\sin 2\pi\xi + \sin 6\pi\xi) \cdot [\frac{\eta^4}{6} - \frac{2k\mu_{2b}}{25} \cdot \eta^5 + \frac{(k\mu_{2b})^2}{42} \cdot \eta^6]$$

$$+ \frac{\pi \phi_o^2 \delta}{4} (\sin 2\pi\xi + \sin 6\pi\xi) \cdot [\frac{\eta^4}{4} - \frac{k\mu_{2b}}{5} \cdot \eta^5$$

$$+ \frac{(k\mu_{2b})^2}{12} \cdot \eta^6] + \frac{\pi \phi_o \delta}{4} \cdot (\sin 4\pi\xi + \frac{5}{4} p \sin 2\pi\xi$$

$$+ \frac{5}{4} p \sin 6\pi\xi) \cdot [\frac{\eta^4}{4} - \frac{k\mu_{2b}}{5} \cdot \eta^5 + \frac{(k\mu_{2b})^2}{12} \cdot \eta^6] \} \tag{28}$$

Equation (21) for the region outside the surf zone is the Laplace equation which can be solved easily. The final nondimensional form of the solution is

$$X_B{}^* = e^{-2\pi(\eta-\delta)} \cdot \{ \frac{\phi_o}{6} [\operatorname{Sin} 2\pi\xi + \frac{5}{4} p \operatorname{Sin} 4\pi\xi$$

$$\cdot e^{-2\pi(\eta-\delta)}] + \frac{2\pi}{7} \delta p [\operatorname{Sin} 2\pi\xi + \frac{3}{4} p \operatorname{Sin} 4\pi\xi$$

$$\cdot e^{-2\pi(\eta-\delta)}] + \frac{7\pi}{24} \delta \phi_o{}^2 \cdot \operatorname{Sin} 4\pi\xi \cdot e^{-2\pi(\eta-\delta)}$$

$$+ (\alpha k) \{ \frac{\phi_o}{4} [\operatorname{Sin} 4\pi\xi \cdot e^{-2\pi(\eta-\delta)}$$

$$+ \frac{5}{4} p \operatorname{Sin} 6\pi\xi \cdot e^{-4\pi(\eta-\delta)} + \frac{5}{4} p \operatorname{Sin} 2\pi\xi]$$

$$\cdot [\frac{1}{3} - \frac{k\mu_{2b}}{4} + \frac{(k\mu_{2b})^2}{10}] - \frac{5\pi\delta}{4} \cdot [\operatorname{Sin} 2\pi\xi$$

$$+ \frac{5}{4} p \operatorname{Sin} 4\pi\xi \cdot e^{-2\pi(\eta-\delta)}] \cdot [\frac{2}{7} - \frac{2}{9} k\mu_{2b} + \frac{(k\mu_{2b})^2}{11}]$$

$$+ \frac{\pi}{2} \delta p [\operatorname{Sin} 4\pi\xi \cdot e^{-2\pi(\eta-\delta)} + \frac{3}{4} p \operatorname{Sin} 2\pi\xi$$

$$+ \frac{3}{4} p \operatorname{Sin} 6\pi\xi \cdot e^{-4\pi(\eta-\delta)}] \cdot [\frac{2}{7} - \frac{2k\mu_{2b}}{9}$$

$$+ \frac{(k\mu_{2b})^2}{11}] + \pi\phi_o{}^2\delta [\operatorname{Sin} 2\pi\xi + \operatorname{Sin} 6\pi\xi \cdot e^{-4\pi(\eta-\delta)}]$$

$$\cdot [\frac{1}{6} - \frac{2}{25} k\mu_{2b} + \frac{(k\mu_{2b})^2}{42}] + \frac{\pi\phi_o{}^2\delta}{4} [\operatorname{Sin} 2\pi\xi$$

$$+ \mathrm{Sin}\ 6\pi\xi \cdot e^{-4\pi(\eta-\delta)}] \cdot [\frac{1}{4} - \frac{k\mu_{2b}}{5} + \frac{(k\mu_{2b})^2}{12}]$$

$$+ \frac{\pi\phi_o\delta}{4} \cdot [\mathrm{Sin}\ 4\pi\xi \cdot e^{-2\pi(\eta-\delta)} + \frac{5}{4} p\ \mathrm{Sin}\ 6\pi\xi$$

$$\cdot e^{-4\pi(\eta-\delta)} + \frac{5}{4} p\ \mathrm{Sin}\ 2\pi\xi] \cdot [\frac{1}{4} - \frac{k\mu_{2b}}{5}$$

$$+ \frac{(k\mu_{2b})^2}{12}] \} \} \tag{29}$$

The solutions X_A and X_B (A indicates inside surf zone; B outside) finally obtained include two parts, one concerned with (αk) and the other independent on (αk), where (α k) denotes the steepness of wavy shoreline.

4. Numerical results

If we neglect O(αk), then

$$X^*_A = \eta^3 \{ \frac{\phi_o}{6} [\sin 2\pi\xi + \frac{5}{4} p \sin 4\pi\xi] + \frac{2\pi}{7} \delta p \sin 2\pi\xi \cdot \eta^{1/2} + \frac{7\pi}{24} \delta\phi_o^2 \sin 4\pi\xi \cdot \eta \} \tag{30}$$

$$X^*_B = e^{-2\pi(\eta-\delta)} \{ \frac{\phi_o}{6} [\sin 2\pi\xi + \frac{5}{4} p \sin 4\pi\xi \cdot e^{-2\pi(\eta-\delta)}] + \frac{2\pi}{7} \delta p \sin 2\pi\xi + \frac{7\pi}{24} \delta\phi_o^2 \sin 4\pi\xi \cdot e^{-2\pi(\eta-\delta)} \} \tag{31}$$

Set

$$\psi^*_{A,B} = X^*_{A,B} \times 10^2 \tag{32}$$

we take the term up to O(α k), in order to find the difference from that up to $O(\alpha k)^o$ done by Uda (1982). Two cases of topographical conditions, as shown in Fig. 2, are designed. In case A, we take ϕ_o =0.35, p=1/3; in case B, ϕ_o =-0.35 p=1/3 are used. Then take the wave length of wavy shoreline L=1000m, 500m and 300m in turns, the amplitude α =50m and 80m, and μ_{2b}=100m. Some typical results are shown in Fig. 3-10. Table 1 shows the corresponding between those Figures and cases considered.

Table 1. Corresponding between Figures and Varions Cases.

	α =50m			
	up to $O(\alpha k)$		up to $O(\alpha k)^{o}$	
case / L=	A	B	A	B
1000m ($\alpha k/2$ =0.05)	Fig. 3	Fig. 6	Fig. 7	
500m ($\alpha k/2$ =0.1)	Fig. 4			
300m ($\alpha k/2$ =0.16)	Fig. 5		Fig. 8	
	α =80m			
	up to $O(\alpha k)$		up to $O(\alpha k)^{o}$	
case / L=	A	B	A	B
1000m ($\alpha k/2$ =0.08)	Fig. 9	Fig. 10		

From Fig. 3, Fig. 4 and Fig. 5, we can find that the shorter the length of one wavy shoreline is, the smaller the magnitude of circulations are going to be; in similarity, from Fig. 3 and Fig. 9, only reducing the amplitude α, the magnitude of the scope of circulations are smaller. Consequently, we can indicate that the steepness of a giant cuspate shoreline has influence to nearshore circulations. By comparing Fig. 3 with Fig. 7. It appears that term $O(\alpha k)$ has not much influence, while from Fig. 5 and Fig. 8, the term $O(\alpha k)$ has more influence. In addition it is supposed the positive values of $X^{*}_{A,B}$ denote counterclockwise of circulations, while minus values of $X^{*}_{A,B}$ mean that the circulations is in clockwise. Comparing to the definition of ϕ_{o}, we get the results that in case (A), the direction of circulations are counterclockwise, while in case (B), it is in the clockwise, since we just take a half of one wave length of shoreline into account, with symmetry we can get the whole associated nearshore currents.

5. Concluding remarks

Although the situations considered here may be too simple, two important conclusions are made concerning with the effects of the steepness of a wavy shoreline on the nearshore circulations:

(1) The smaller the steepness of a periodic shoreline, the larger the scope of the nearshore circulation.

(2) It is common that there appear two even three circulations, when the steepness becomes larger to some

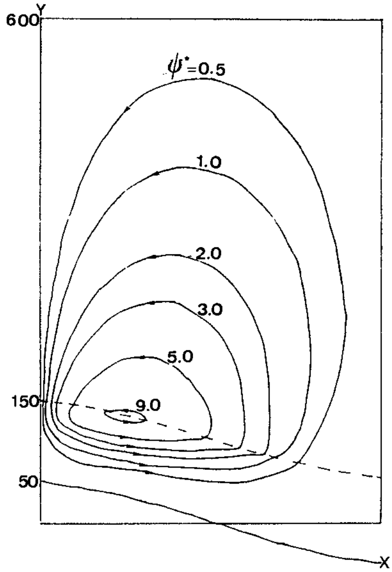

Fig. 3 Circulation pattern, Case(A), $\alpha k/2\pi$=0.05, L=1000M, α=50M.

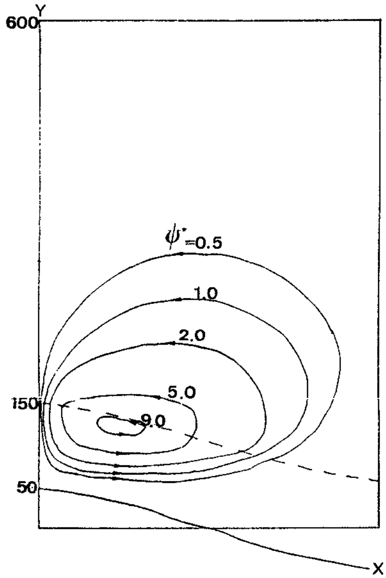

Fig. 4 Circulation pattern, Case (B), $\alpha k/2\pi$=0.1, L=500M, α=50M.

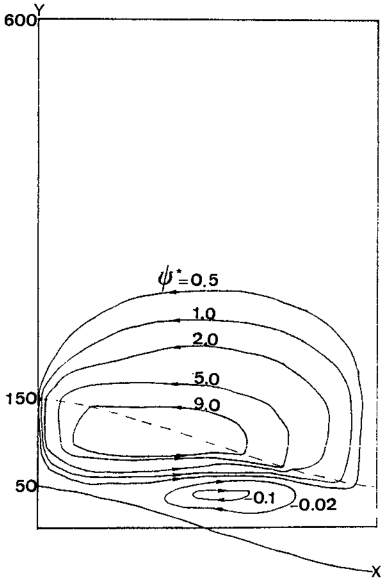

Fig. 5 Circulation pattern, Case(A), $\alpha k/2\pi$=0.16, L=300M, α=50M.

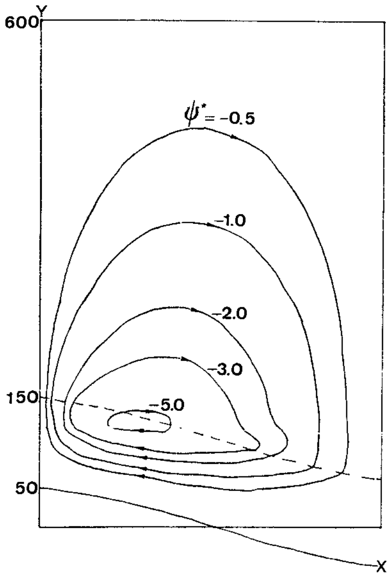

Fig. 6 Circulation pattern, Case(B), $\alpha k/2\pi$=0.05, L=1000M, α=50M.

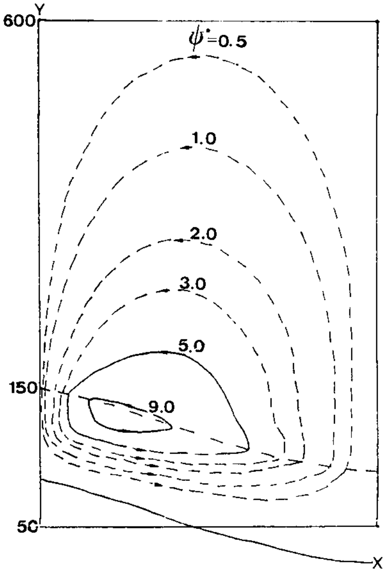

Fig. 7 Circulation pattern, Case(A), O(αk) term neglected, αk/2π=0.05, L=1000M, α=50M.

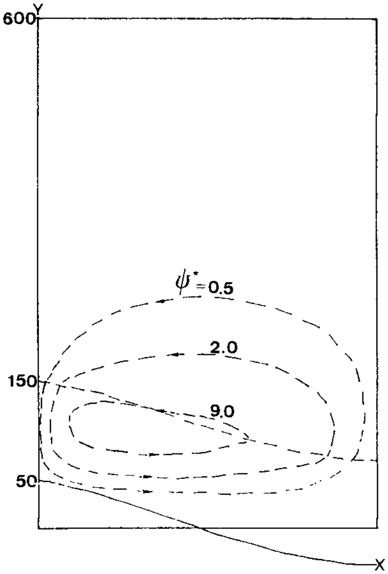

Fig. 8 Circulation pattern, Case(A), O(αk) term neglected, αk/2π=0.16, L=300M, α=50M.

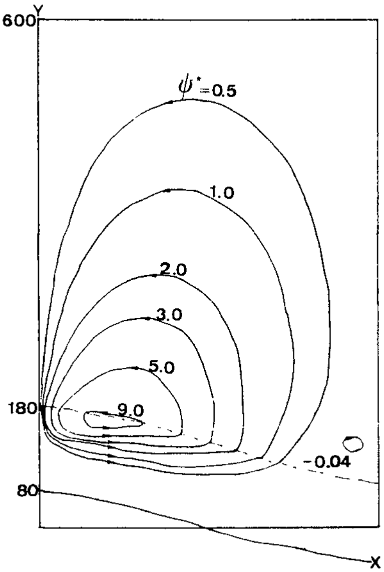

Fig. 9 Circulation pattern Case(A), αk/2π=0.08, L=1000M, α=80M.

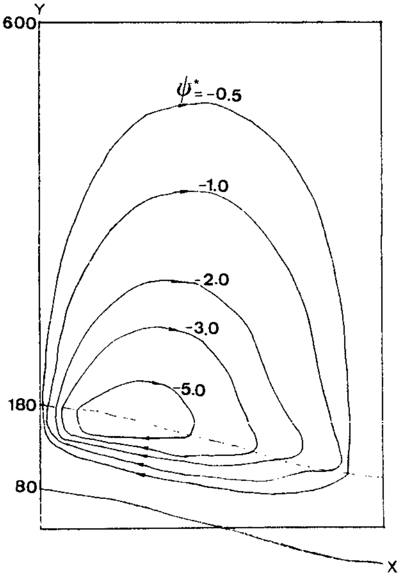

Fig. 10 Circulation pattern, Case(B), αk/2π=0.08, L=1000M, α=80M.

amount at extent.

6. Reference

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