CHAPTER 189

Entrainment due to Mean Flow in Two-Layered Fluid

Masaru Ura*
and
Nobuhiro Matsunaga**

Abstract

The entrainment phenomena have been investigated across an interface between two-layered stratified flow induced by wind shear stress. The velocities of mean flow, turbulence and entrainment have been measured under three different conditions of water surface by using a wind-wave tank. When the entrainment velocity u_e is expressed on the basis of the turbulent quantities at the interface, the turbulent entrainment coefficient E ($\equiv u_e/u$) is given by $E = A\cdot(\varepsilon gl/u^2)^{-3/2}$ (A = 0.7). Here εg, u and l are the effective buoyancy, the turbulence intensity and the integral lengthscale of turbulence at the interface, respectively. This result coincides with the relationship of entrainment due to oscillating grid turbulence, in which the mean flow does not exist. When, for the practical purpose, the estimation of u_e is made by using the mean velocity U_m and the depth h of mixed layer, E_m ($\equiv u_e/U_m$) $= A_m\cdot(\varepsilon gh/U_m{}^2)^{-3/2}$ is derived from the transformation of $E = A\cdot(\varepsilon gl/u^2)^{-3/2}$. There holds $A_m = A\cdot T_f$ between A_m and T_f, T_f being a turbulence factor given by $(u/U_m)^4\cdot(l/h)^{-3/2}$. It has been found that this relationship is also valid in various types of two-layered stratified flows as well as the wind-induced two-layered flows.

Introduction

The density stratification is often observed in lakes or estuaries. When the wind blows over the surface of such a water body, the mixing phenomena occur frequently owing to turbulent flow induced by wind shear stress. Many laboratory studies on two-layered stratified flows have been made in order to understand this mixing phenomena. The mixing is characterized by entrainment process. Let us focus attention on the two-layered flows in which the shear stress τ is exerted on the water surface. The previous experimental works may be classified into four categories

* Professor, Department of Civil Engineering, Kyushu Institute of Technology, Tobata-ku Kitakyushu 804, Japan.
** Research Asocciate, Department of Civil Engineering, Kyushu University, Higashi-ku Fukuoka city 812, Japan.

as shown in Figure 1. They are as follows:

(i) the case when flow near the interface has the same direction as that of action of τ and wind waves are present (Tsuruya et al., 1984),

(ii) the case when shear-induced flow has the same flow pattern as the case (i) without the presence of wind waves (Kantha et al., 1975; Deardorff and Willis, 1982),

(iii) the case when flow near the interface has the opposite direction to the surface shear stress and the wind waves are present (Wu, 1973; Kit et al., 1980; Ura, 1984; Kranenburg, 1985), and

(iv) the case when flow pattern is the same as the case (iii) without the presence of wind waves (Kit et al., 1980).

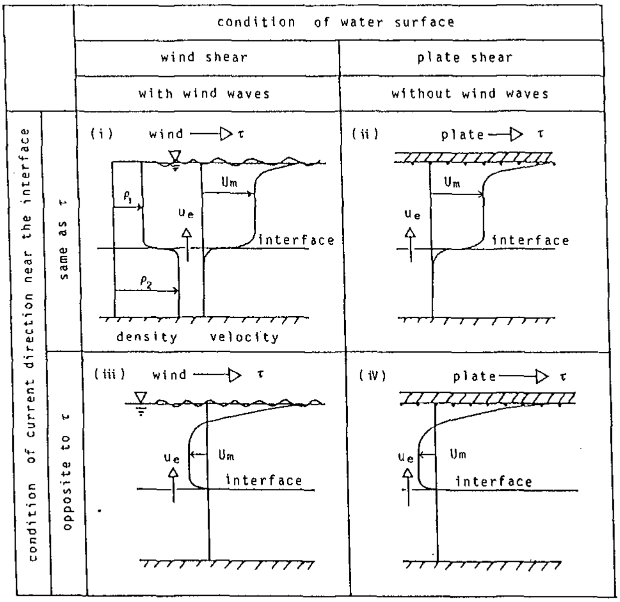


Figure 1. Classification of previous experimental works.

The researchers introduced in cases (i) to (iv) discussed the dependence of the entrainment coefficient E_* ($\equiv u_e/u_*$) on the Richardson number Ri_* ($\equiv \varepsilon gh/u_*^2$). Here, u_e, u_*, εg and h are the entrainment velocity, the friction velocity acting on the water surface, the effective buoyancy and the water depth of mixed layer, respectively. Their results are plotted together in Figure 2. We can support the relationship of $E_* = A_* Ri_*^{-3/2}$, but the proportional factor A_* varies from 700 to 0.13 corresponding to cases (i) to (iv). This wide variation is closely related to whether mean flow direction near the interface is same or opposite to the surface shear stress and whether wind waves are present or not. Using u_* as a characteristic velocity may not be proper in order to explain consistently these data. This reason is that four cases appear under a given magnitude of u_* at the least as far as referred here and that u_* includes no information about whether the reverse flow forms or not. Therefore, it seems appropriate to use the mean velocity U_m of uniform flow induced in the mixed layer rather than u_*.

On the other hand, it has been proposed by Ura et al. (1987b) that the entrainment process depends on the characteristic quantities of turbulence such as turbulence intensity u and integral lengthscale l of turbulence at the

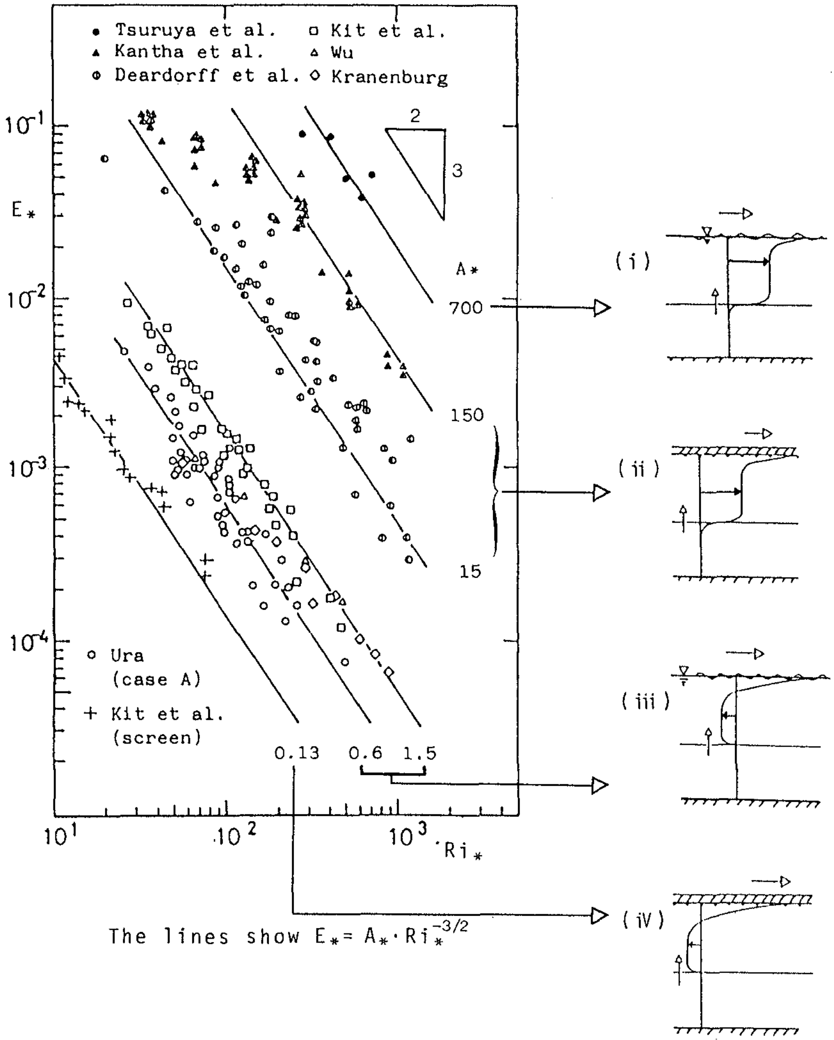


Figure 2. Ri_* - dependence of E_* .

interface. In fact, the turbulent entrainment coefficient $E\ (\equiv u_e/u)$ is given by

$$E = A\ (\varepsilon gl/u^2)^{-3/2} \qquad (1)$$

for various types of two-layered stratified flows. If equation (1) is rewritten by using h and U_m, the overall entrainment coefficient $E_m\ (\equiv u_e/U_m)$ becomes

$$\begin{aligned} E_m &= (u_e/u)\cdot(u/U_m) \\ &= A\ \left[(\varepsilon gl/u^2)\cdot(u/U_m)^2\cdot(h/l)\right]^{-3/2}\cdot(u/U_m) \\ &= A\ (u/U_m)^4\cdot(l/h)^{-3/2}\cdot(\varepsilon gh/U_m^2)^{-3/2} \\ &= A_m\cdot Ri_m^{-3/2} \quad , \qquad (2) \end{aligned}$$

where

$$A_m = A\cdot(u/U_m)^4\cdot(l/h)^{-3/2} \quad \text{and} \quad Ri_m = \varepsilon gh/U_m^2 \ .$$

It is seen from equation (2) that the proportional factor A_m is determined by the characteristic quantities of turbulence at the interface. Here, let us call $(u/U_m)^4\cdot(l/h)^{-3/2}$ the 'turbulence factor', i.e.,

$$T_f = (u/U_m)^4\cdot(l/h)^{-3/2} \quad . \qquad (3)$$

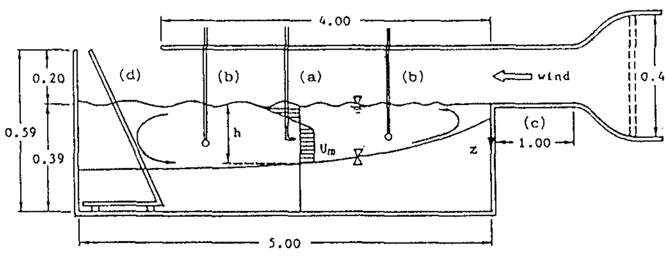
Equation (2) may also be applicable to general two-layered flows whose surface the shear stress does not act on.

The purpose of this paper is to explain consistently the entrainment coefficient of various types of two-layered flows on the basis of equation (2).

Experimental Methods

The wind-wave tank used in this experiment is schematically shown in Figure 3. It was 5.0m long, 0.2m wide and 0.59m high. The wind tunnel was 0.2m high with an approach section 1.0m long. A fetch of the wind on the water surface was 4.0m. The tank was filled with a two-layered fluid. The ordinate z is taken vertically downward from the mean water level.

A 4-pole conductivity probe has been used for the measurement of the fluid density. Density profiles were



(dimensions in m)

Figure 3. Experimental apparatus for wind-induced flow.
(a) Hot-film anemometer; (b) conductivity probes;
(c) approach section; (d) wave absorber.

measured at the positions 1.42, 2.42 and 3.40m from the upstream end of the tank. The position of an interface is defined as the location where the mean density of fluid is equal to $(\rho_1+\rho_2)/2$, where ρ_1 and ρ_2 are the fluid densities of upper and lower layers, respectively. Entrainment velocity u_e has been estimated from time variations of the depth h from the water surface to the interface level (i.e., $u_e = \partial h/\partial t$).

The wind shear stress, which induces a drift current and compensating return flow as sketched in Figure 3, acts on the free surface by blowing the wind into the tank. The vertical velocity distributions of the wind-induced flow were obtained at the middle of the tank in the longitudinal direction by using a single-component hot-film anemometer. The friction velocity u_* was evaluated from both the log-law profile of mean velocity under the water surface and the relation

$$\rho_1 u_*^2 = \gamma \rho_a u_{*a}^2 \quad ,$$

where ρ_a is the density of air, u_{*a} is the air friction velocity and γ is a coefficient, which take a value between 1.0 to 0.9 (Ura, 1984). The intensity u and lengthscale l were calculated from the horizontal velocity fluctuation. The lengthscale was based on either the auto-correlation technique or the empirical formula proposed by Townsend (1976), i.e.,

$$l = 0.8\ u^3/\varepsilon_d \ .$$

Here, ε_d is the rate of energy dissipation calculated from the -5/3-power law of wavenumber spectrum.

Three kinds of experiments were conducted by changing the condition of water surface as shown in Figure 4.

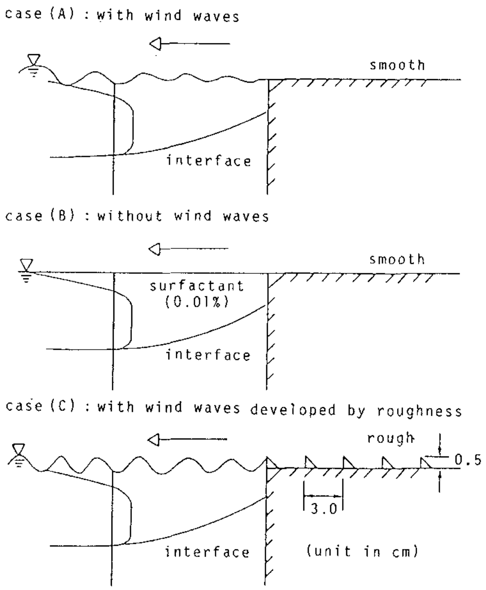


Figure 4. Experimental conditions of water surface.

Case (A): the two-layered flow with wind waves, which developed gradually along the fetch because of the smooth approach section (i.e.,case(iii)).

Case (B): the formation of wind waves was suppressed by adding surfactant(sodium dodecyle sulfate, concentration 1.1×10^{-2} %) in the water (similar to case(iv)).

Case (C): the wind waves formed at the beginning of the fetch because of the existence of roughness elements on the approach section (i.e., case (iii)).

Flow Properties

Figures 5 (a) to (c) show vertical distributions of mean velocity, turbulence intensity and integral lengthscale, respectively for cases (A), (B) and (C). In these figures z/h = 0 indicates water surface and z/h = 1.0 the

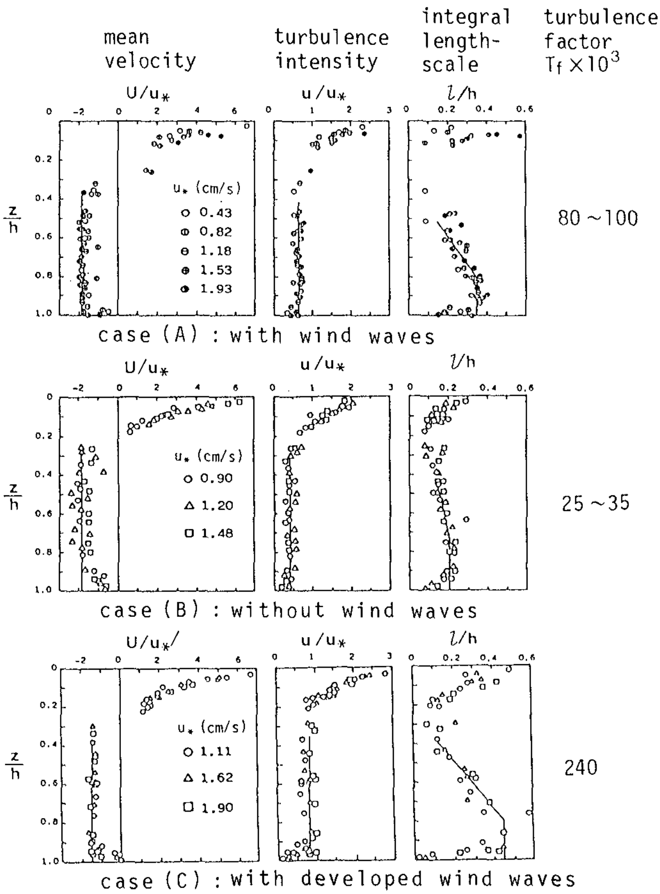


Figure 5. Vertical distributions of mean velocity, turbulence intensity and integral lengthscale.

interface. The values of these physical quantities at z/h = 0.9 are used as the representative values, because on the interface they are influenced significantly by the buoyancy effect. It is seen from the mean velocity profiles that a strong drift current occurs near the water surface by the wind, and that return flow with nearly uniform velocity forms in the range 1.0 > z/h > 0.25. Denoting this uniform velocity by U_m, U_m/u_* equals about 1.85 in cases (A) and (B) and 1.50 in case (C). The relative turbulence intensities u/u_* have almost constant values in the return flow region and the intensities are approximately given by

$u = 0.65u_* = 0.35U_m$ for case (A),
$u = 0.39u_* = 0.21U_m$ for case (B), and
$u = 0.82u_* = 0.52U_m$ for case (C).

The integral lengthscales near the interface are given by

$l \fallingdotseq 0.35h$ for case (A),
$l \fallingdotseq 0.20h$ for case (B), and
$l \fallingdotseq 0.45h$ for case (C).

From the results of these measurements, the values of the turbulence factor T_f are found to take 0.08 ~ 0.10, 0.025 ~ 0.035 and 0.24 in cases (A), (B) and (C), respectively. These results reveal that the condition of the water surface influences the value of T_f considerably.

Entrainment Coefficient

Figures 6 (a) and (b) are typical examples of the vertical density profiles, measured as the time proceeds, at the fetches F of 3.40 and 2.42m. The time-variations of h based on Figure 6 are shown in Figure 7. It is seen that the entrainment velocity $\partial h/\partial t$ is constant during the time of the measurement.

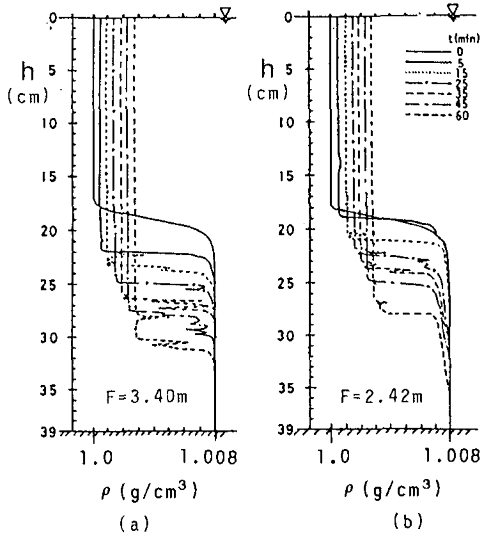


Figure 6. Time variations of vertical density profiles.

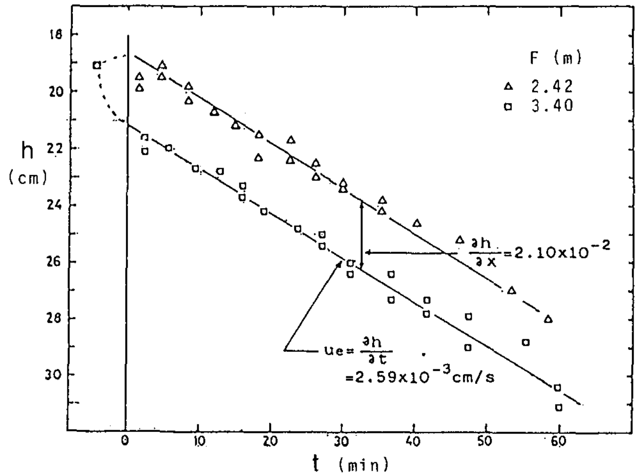


Figure 7. Time variations of the depth of the mixed layer.

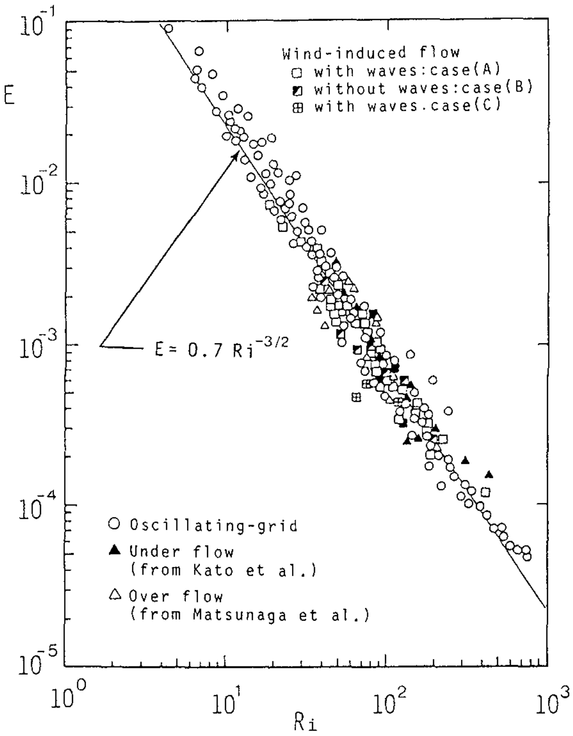


Figure 8. Ri - dependence of E.

In Figure 8, the turbulent entrainment coefficients $E = u_e/u$ are plotted against the local Richardson number $Ri = \epsilon gl/u^2$ for the three cases shown in Figure 4. The experimental results obtained from the oscillating grid turbulence (Ura et al., 1987a) and the over flow (Matsunaga et al., 1984) are also plotted in this figure. We confirm the unified expression of E based on Ri (Ura et al., 1987b), i.e.,

$$E = 0.7 \cdot Ri^{-3/2} \tag{4}$$

for various types of two-layered flows.

Figure 9 shows the relationships between bulk parameters E_m and Ri_m, which are obtained by many other researchers and authors. The values of T_f are also represented, being calculated on the basis of the data by the authors and Kato

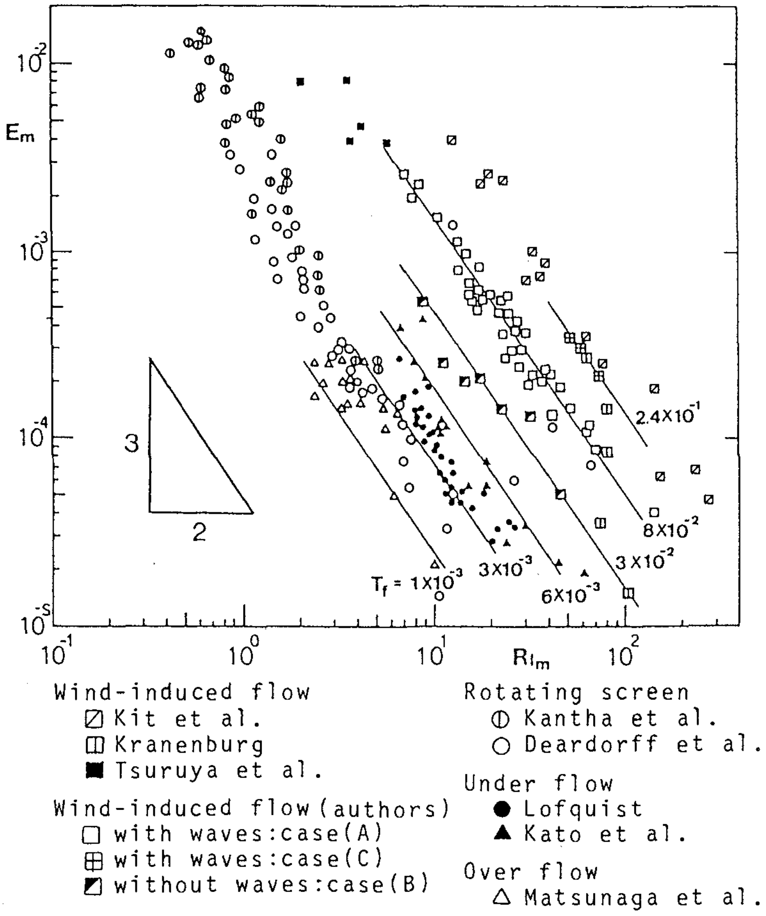


Figure 9. Relationships between E_m and Ri_m obtained by considering turbulence factor T_f.

et al. (1981). The values of T_f by the other researchers are unknown since they did not report the turbulence quantities. From this figure, however, we find that E_m is proportional to $Ri_m^{-3/2}$ and the proportional factor A_m consistently depends on T_f.

In order to confirm the validity of equation (2), it is necessary to show that A_m is proportional to T_f. Figure 10 shows the relationship between A_m and T_f. A_m is clearly proportional to T_f and the proportional constant takes 0.7. Therefore, we conclude that the entrainment relationship

$$E_m = A_m \cdot Ri_m^{-3/2} \quad ,$$

$$A_m = 0.7 \cdot T_f$$

holds for the various types of two-layered stratified flows.

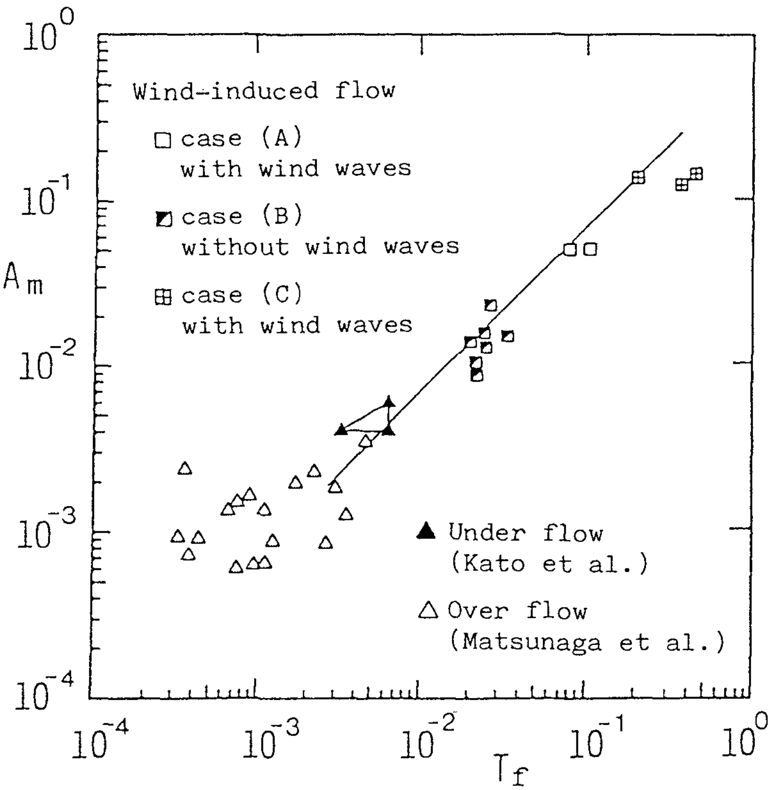


Figure 10. Relationships between A_m and T_f.

Acknowledgment

The authors thank Professors T. Tsubaki and T. Komatsu for stimulating discussions and Mr T. Awaya for technical help.

References

Deardorff, J. W. and Willis, G. E. (1982). "Dependence of mixed-layer entrainment on shear stress and velocity jump." J. Fluid Mech., 115, 123-149.
Kantha,L. H., Phillips, O. M. and Azad, R. S. (1975). "On the turbulent entrainment at a stable density interface." J. Fluid Mech., 79, 753-768.
Kato, H. and Ikeda, T. (1981). "An experiment of lower density current(1)." The 28th Japanese Conf. Coastal Eng., 520-524,(in Japanese).
Kit, E., Berent, E. and Vajda, M. (1980). "Vertical mixing induced by wind and rotating screen in a channel." J. Hydr. Res., 18(1), 35-58.
Kranenburg, C. (1985). "Mixed-layer deepening in lakes after wind setup." J. Hydr. Div., ASCE, 111(HY9), 1279-1297.
Lofquist, K. (1960). "Flow and stress near an interface between stratified liquids." Phys. Fluids, 3, 158-175.
Matsunaga, N., Ura, M., Namikawa, T. and Tsubaki, T.(1984). "Flow properties near an interface in upper-layer flow." Proc. 28th Japanese Conf. Hydr., 345-351(in Japanese).
Thompson, S. M. and Turner, J. S. (1975). "Mixing across an interface due to turbulence generated by an oscillating grid." J. Fluid Mech., 67, 349-368.
Townsend, A., A. (1976). The structure of turbulence shear flow (2nd ed.), Cambridge University Press. Cambridge, London.
Tsuruya, H., Nakano, S. and Ichinohe, H. (1984). "Vertical mixing and turbulent characteristics of wind-driven two-layered flows." Proc. 31st Japanese Conf. Coastal Eng. 655-659, (in Japanese).
Ura, M. (1984). "Interfacial waves and entrainment in a wind-induced stratified flow." Proc. 4th Cong. APD- IAHR, 403-417.
Ura, M., Komatsu, T. and Matsunaga, N. (1987a). "Entrainment due to oscillating grid turbulence in two-layered fluid." Turbulence Measurements and Flow Modeling, Chen, C. J., Chen, L. D. and Holly, F. M. Ed. Hemisphere Publishing Corporation, New York, N.Y., 109-118.
Ura, M., Tsubaki, T., Matsunaga, N. and Namikawa, T. (1987b)."A turbulence-based expression for the entrainment coefficient for two-layered stratified flows." J. Hydroscience and Hydr. Eng., 5(1),22-37.
Wu, J. (1973). "Wind-induced turbulent entrainment across a stable density interface." J. Fluid Mech., 61, 275-287.